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Mathematical model of the point kinetic equations of a fast reactor in LabVIEW

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Abstract

The reactor point kinetics equations have been subjected to intense research in an effort to find simple yet accurate numerical solution methods. Many solution techniques have been presented on the point kinetics equations with varying degrees of complexity including Power Series Solutions, CORE, PCA, Ganapol and Taylor series methods. In this paper, fast and simple algorithms were developed based on the first and second order Taylor series expansion and simulated in LabVIEW to solve the Reactor Point Kinetics. Simulations for a fast reactor are presented. At 1×10^{-8} s the neutron population was 1.000220 neutrons / cm^3, at 1×10^{-2} s it was 2.007681 neutrons / cm^3 and at 1×10^{-1} s it was 2.075317 neutrons / cm^3.

Keywords: reactor point kinetics equations, taylor series approximations, neutron population density, fast reactor

1. Background

1.1 Point kinetics equations

The point kinetics equations are a system of coupled ordinary differential equations, which model the short-term transient behavior of a nuclear reactor. These equations, which model the neutron population density and delayed neutron precursor concentrations, are usually solved assuming at least six delayed precursor groups. This results in a system of seven coupled differential equations. In general, analytical solutions are difficult due to the complexity of the problem, hence the need for numerical methods. Obtaining accurate results is often problematic because the equations are stiff and with many techniques very small time steps may be required. (McMahon et al, 2010, Adzri E. P., 2012)

It is well known that it is the presence of delayed neutrons in the reactor fission process that enable reactors to be controlled. The point kinetic model takes the delayed neutrons into account in the following well known set of coupled equation:

$$\frac{dN(t)}{dt} = \frac{\rho(t) - \beta}{\Lambda} N(t) + \sum_{i=1}^{6} \lambda_i C_i \qquad (1)$$

$$\frac{dC_i(t)}{dt} = \frac{\beta_i}{\Lambda} N(t) - \lambda_i C_i \qquad (2)$$

where equation (1) represents the neutron population density and equation (2) the concentration of the delayed neutron precursors. Additionally, N is neutron population density, t is time, β is delayed neutron fraction, Λ is mean generation time, λ_i is decay constant of delayed neutrons, C_i is delayed neutron precursor density, ρ is reactivity, h is step size. (Adzri E. P., 2012)

1.2 Solution Methods

Computational solutions or models of the point-kinetics equations provide insight into the dynamics of nuclear

reactor operation and are useful, for example, in understanding the power fluctuations experienced during start-up or shut-down when the control rods are adjusted. (Aboanber A. E., 2006) A large number of kinetics studies have been reported, which modeled the time-dependent behavior of a nuclear reactor using the point-kinetics equations, including Stiffness Confinement Method (SCM), Power Series Solution (PSS), Analytical Inversion (AI), Constant Reactivity (CORE), Piecewise Constant Approximation (PCA), etc.

The main idea behind the SCM method is the observation that the stiffness in the point reactor equations is only present in the coupling of the prompt neutron equation with the delayed neutron precursor equations. (Van den Eynde G., 2012) So the equation was solved with an appropriate discretization method, reiterating an analytical solution until convergence is reached, but using large time steps. (Chao Y., et al, 1985)

The PSS method solved the point reactor equations using a series development for the different constituents and derived consequent recurrence relations for the different coefficients (Basken J. et al, 1996) The main drawback of the method is its difficulty in handling the development of the reactivity function.

In the AI method, a matrix representation of the system of ordinary differential equations and the formal solution expressed by the exponential of the system matrix was devised. This method works for a constant system matrix, i.e. the reactivity and other parameters are considered to be constant during one time step. But it's a rather very complex method, requiring root-finding or eigenvalue finding, assuming constant reactivity during time step and unfortunately there's no time step control. (Van den Eynde G., 2012)

The PCA method assumed that the reactivity is constant during one time step. Using the matrix approach, the solution at the next time step was written as the function of the solution at the current time step and the exponential of the system matrix. (Kinard M., et al, 2004) This method was simple to implement in MATLAB or OCTAVE. This method requires root-finding or eigenvalue finding, assuming reactivity constant during time step. It indicated no time step control and which time step is good. (Kinard M., et al, 2004)

The CORE numerical algorithm used the Laplace transform and the Heaviside expansion theorem to solve the point kinetic equation. It is iterated until the solution converges. (Quintero-Leyva, 2008)

McMahon et al, 2010 used Taylor series expansion to develop a numerical solution to the reactor point kinetics equations. It was shown that taking a first order expansion of the neutron density and precursor concentrations at each time step produced results that were comparable to those obtained using other popular yet more complicated methods. The algorithm was developed using a simple Taylor series expansion. The procedure is tested using a variety of initial conditions and input data, including sub, critical and super critical step reactivities, ramp reactivity, sinusoidal, pulse, and zigzag reactivity. These results are compared to those obtained using other methods and are found to be consistent. (McMahon et al, 2010, Adzri E. P., 2012)

2. Taylor series approximation of the point kinetic equations

Using the method of McMahon et al, 2010, the Taylor Series expansion of the neutron density in equation (1) was written as

$$N(t+h) = N(t) + h\frac{dN}{dt} + \frac{1}{2!}h^2\frac{d^2N}{dt^2} + \cdots + \frac{1}{n!}h^n\frac{d^{n-1}N}{dt^{n-1}} \quad (3)$$

The delayed neutron precursor concentration (equation 2) became:

$$C_i(t+h) = C_i(t) + h\frac{dC_i}{dt} + \frac{1}{2!}h^2\frac{d^2C_i}{dt^2} + \cdots + \frac{1}{n!}h^n\frac{d^{n-1}C_i}{dt^{n-1}} \quad (4)$$

where h is the step size.

Considering a first order of the terms only, using equation (3) an expression was derived to find the neutron population density at a later time N (t + h) from the neutron density at the earlier time N (t)

$$N(t+h) = N(t) + h\left[\frac{\rho(t) - \beta}{\Lambda}N(t) + h\sum_{i=1}^{6}\lambda_i C_i\right] \quad (5)$$

Each delayed neutron precursor was calculated using equations (2) and (4), taking terms to a first order,

$$C_i(t+h) = C_i(t) + h\left[\frac{\beta_i}{\Lambda}N(t) - \lambda_i C_i(t)\right] \quad (6)$$

A second order of the terms was now considering, using equation (3), an expression was written to find the neutron population density at a later time N (t + h) from the neutron population density at the earlier time N (t)

$$N(t+h) = N(t) + h\frac{\rho(t) - \beta}{\Lambda}N(t) + h\sum_{i=1}^{6}\lambda_i C_i + \frac{1}{2!}h^2\frac{d}{dt}\left[\frac{\rho(t)-\beta}{\Lambda}N(t) + \sum_{i=1}^{6}\lambda_i C_i\right] \quad (7)$$

The delayed neutron precursor was calculated using equation (2) and (4), but taking terms to a second order;

$$C_i(t+h) = C_i(t) + h\frac{\beta_i}{\Lambda}N(t) - h\lambda_i C_i(t) + \frac{1}{2!}h^2\frac{d}{dt}\left[\frac{\beta_i}{\Lambda}N(t) - \lambda_i C_i(t)\right] \quad (8)$$

A generic term was then deduced to compute the point kinetics equation, using equation (3);

$$N(t+h) = N(t) + h\frac{\rho(t)-\beta}{\Lambda}N(t) + h\sum_{i=1}^{6}\lambda_i C_i + \frac{1}{2!}h^2\frac{d}{dt}\left[\frac{\rho(t)-\beta}{\Lambda}N(t) + \sum_{i=1}^{6}\lambda_i C_i\right] + \cdots + \frac{1}{n!}h^n\frac{d^{n-1}}{dt^{n-1}}\left[\frac{\rho(t)-\beta}{\Lambda}N(t) + \sum_{i=1}^{6}\lambda_i C_i\right] \quad (9)$$

Equation (4) was also deduced as;

$$C_i(t+h) = C_i(t) + h\frac{\beta_i}{\Lambda}N(t) - h\lambda_i C_i(t) + \frac{1}{2!}h^2\frac{d}{dt}\left[\frac{\beta_i}{\Lambda}N(t) - \lambda_i C_i(t)\right] + \cdots + \frac{1}{n!}h^n\frac{d^{n-1}}{dt^{n-1}}\left[\frac{\beta_i}{\Lambda}N(t) - \lambda_i C_i(t)\right] \quad (10)$$

(Adzri E. P., 2012)

3.1 Algorithm for solution of second order approximation

An algorithm was developed to compute the second order solution of the Point kinetics equation.

1. Establish all constants and initialize parameters. (Specify all the necessary constants here: Decay Constants, Delay Neutron Fraction, etc).
2. Initialize *Current Time t=0,* neutron population *N(0)=1,* and intermediate sum *Int-Sum*=0, where *Int_Sum* is used as an accumulator to hold the sum of *s* set of data for later use.
3. Calculate the initial values for the concentration of delayed neutron precursors [Cn(0)]. These calculations are as follows:

$$C_n(0) = \frac{\beta_n N(0)}{\lambda_n \Lambda}$$

where n =1,2,3,4,5,6. Thus,

$$C_1(0) = \frac{\beta_1 N(0)}{\lambda_1 \Lambda}, C_2(0) = \frac{\beta_2 N(0)}{\lambda_2 \Lambda}$$,etc

4. Calculate $Int\_Sum = \sum_{i=1}^{6} \lambda_i C_i$

5. Calculate total delayed neutron fraction. That is, *Tot_DNeut_Frac* = $\sum_{i=1}^{6} \beta_i$

6. Plot initial values of neutron population N (0) and initial time t_0 (=0)

$$N(1) = N(0) + h\left[\frac{\rho - \beta}{\Lambda} N(0) + Int\_Sum\right]$$

7. Calculate the next iteration of neutron population:

a.

b. Increment Current Time by the Simulation Time Step (t = t + h)

c. Plot N(1) and t

8. DO WHILE *Current Time* < *Total Simulation Time*

a. Calculate the next set of values for the delayed neutron precursors.

b.

$$C_1(t+h) = C_1(t) + h\left[\frac{\beta_1 N(t)}{} - \lambda_1 C_1(t)\right] + \frac{1}{}h^2 \frac{d}{}\left[\frac{\beta_1 N(t)}{} - \lambda_1 C_1(t)\right]$$

c. $$C_2(t+h) = C_2(t) + h\left[\frac{\beta_2 N(t)}{\Lambda} - \lambda_2 C_2(t)\right] + \frac{1}{2!}h^2 \frac{d}{dt}\left[\frac{\beta_2 N(t)}{\Lambda} - \lambda_2 C_2(t)\right]$$

d. $$C_3(t+h) = C_3(t) + h\left[\frac{\beta_3 N(t)}{\Lambda} - \lambda_3 C_3(t)\right] + \frac{1}{2!}h^2 \frac{d}{dt}\left[\frac{\beta_3 N(t)}{\Lambda} - \lambda_3 C_3(t)\right]$$

e. $$C_4(t+h) = C_4(t) + h\left[\frac{\beta_4 N(t)}{\Lambda} - \lambda_4 C_4(t)\right] + \frac{1}{2!}h^2 \frac{d}{dt}\left[\frac{\beta_4 N(t)}{\Lambda} - \lambda_4 C_4(t)\right]$$

f. $$C_5(t+h) = C_5(t) + h\left[\frac{\beta_5 N(t)}{\Lambda} - \lambda_5 C_5(t)\right] + \frac{1}{2!}h^2 \frac{d}{dt}\left[\frac{\beta_5 N(t)}{\Lambda} - \lambda_5 C_5(t)\right]$$

g. $$C_6(t+h) = C_6(t) + h\left[\frac{\beta_6 N(t)}{\Lambda} - \lambda_6 C_6(t)\right] + \frac{1}{2!}h^2 \frac{d}{dt}\left[\frac{\beta_6 N(t)}{\Lambda} - \lambda_6 C_6(t)\right]$$

9. Reset *Int_Sum* to zero. Then calculate I*nt_Sum* $\sum_{i=1}^{6} \lambda_i C_i(t)$
10. Calculate

$$N(t+h) = N(t) + h\left[\frac{\rho-\beta}{\Lambda}N(t) + Int\_Sum\right] + \frac{1}{2!}h^2\frac{d}{dt}\left[\frac{\rho-\beta}{\Lambda}N(t) + Int\_Sum\right]$$

11. Increment *t* by *h* (i.e., $t = t + h$)
12. Plot N(t) and t

END WHILE

END

(Adzri E.P., 2012)

3.2 Code in LabVIEW

LabVIEW is a graphical programming language that uses icons and other graphical objects in a drag and drop environment instead of the text based environment used in traditional programming languages. LabVIEW employs dataflow programming, where data flows through nodes, a node in this case being a block of code represented in LabVIEW as an icon. LabVIEW uses two windows: the front panel; the controls (used for inputting data), and indicators (outputs the results) and the block Diagram Panel; the graphical source code that defines the functions and interconnections of the icons on the front panel. The user inputs relating to the kinetic parameters of the reactor and the indicators (graphs, neutron population density and delayed precursor concentration) are coded on the front panel. (Adzri E. P., 2012)

In fig 1, the front panel of the LabVIEW program is presented.

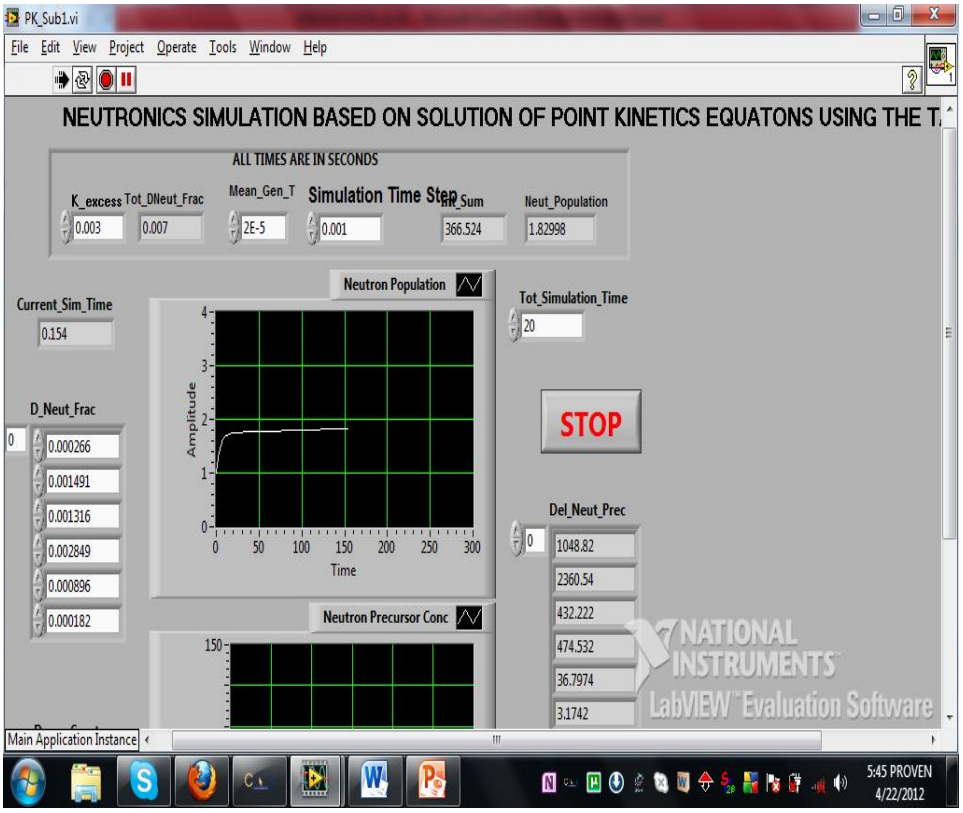

Fig. 1. Front panel of the LabVIEW program. (Adzri E. P., 2012)

The algorithm was implemented in stacked sequences in the block diagram. Fig 2 presents the initialization pane of the algorithm

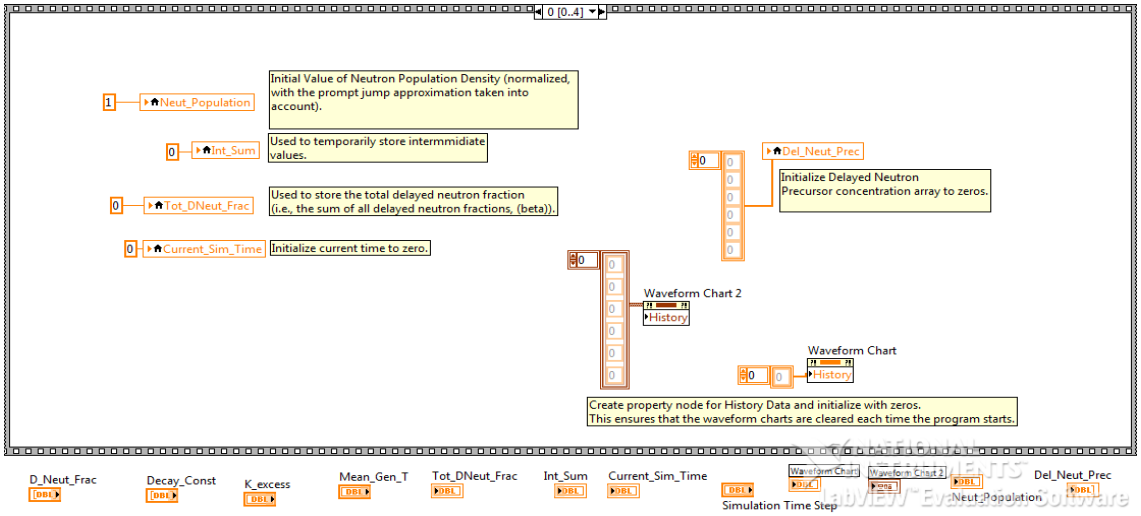

Fig. 2. Block diagram of the LabVIEW program [Frame 0(0..4)] (Adzri E. P., 2012)

Fig 3. presents the part of the algorithm that calculates the initial values of the concentration of delayed neutron precursors.

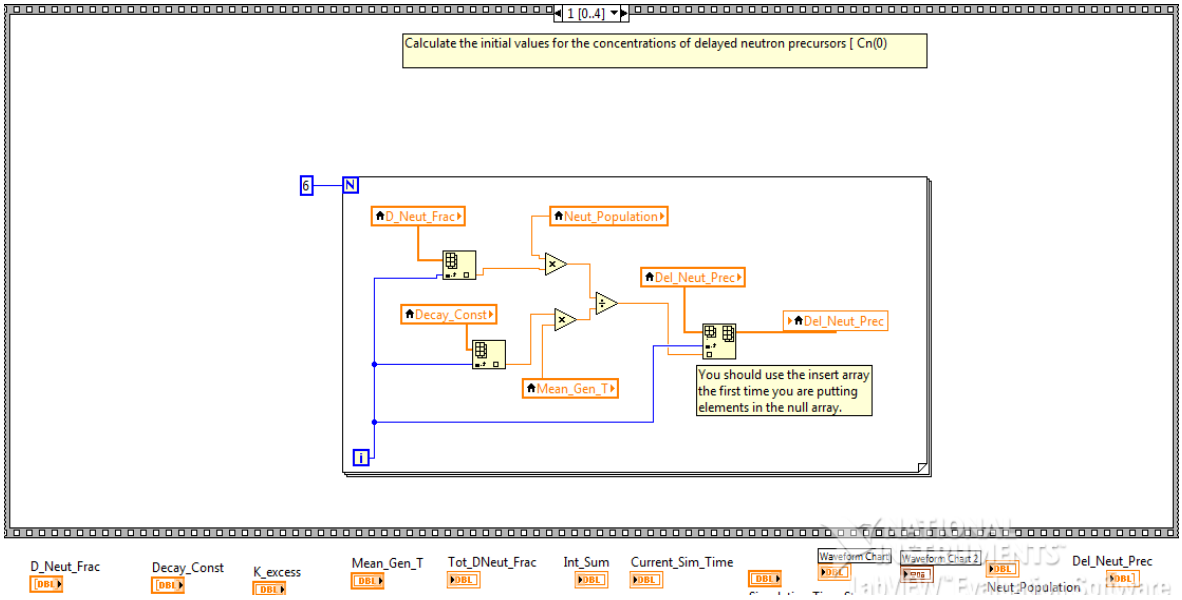

Fig. 3. Block diagram of the LabVIEW program [Frame 1(0..4)] (Adzri E. P., 2012)

4. Results and Discussions

Sub-Critical step reactivity for a fast reactor was considered with the following kinetic parameters as shown in Table 1. For fast reactors, typical times are of the order 10^{-6} seconds and step sizes are also small, h = 0.001 and neutron generation times are typically small, $\Lambda = 0.00002$

Table 1. Kinetic Parameters for a Sub-critical thermal reactor.

□ =0.003		
□ =0.007		
Λ = 0.00002		
ℏ=0.001		
i	***β_i***	***λ_i [s^{-1}]***
1	0.000266	0.0127
2	0.001491	0.0317
3	0.001316	0.0155
4	0.002849	0.3110
5	0.000896	1.4000
6	0.000182	3.8700

Normalized neutron population *□(0)=1* at time 0

Table 2 below presents the neutron population density using the kinetic parameters in table 1.
In table 2, the results of the neutron population density for sub- critical step reactivity of a fast reactor using the kinetic parameters in table 1 are presented. The results are denoted Proven[1] and Proven[2] for first and second order approximations respectively. These results are compared with Ganapol. (Ganapol B. D., 2009)
Table 2. Results for neutron population density for a fast reactor

Time (s)	Proven[1]	Proven[2]	Ganapol
1E-08	1.000220	1.000220	1.000220
1E-06	1.022000	1.021762	1.021760
1E -04	1.891910	1.889247	1.889220
1E -02	2.007680	2.007681	2.007681
0.1	2.075320	2.075317	2.075317
1.0	2.655860	2.655855	2.655853
10.0	12.74654	12.74657	12.74654

It is observed from Table 2 that, at 1×10^{-8} s the neutron population density was 1.000220, at 1×10^{-2} s it was 2.007681 and at 1×10^{-1} s it was 2.075317. These are the same results reported by Ganapol. While there are some differences between the Proven[1] and Proven[2] results, the difference is only slight in many cases.
The results obtained in this paper showed that LabVIEW can be used to develop efficient and accurate code to model fast reactors, which in general involve much smaller neutron generation times and smaller time steps.

5. Conclusion

A numerical solution for the reactor point kinetics equations was developed using the Taylor series approximations. A fast algorithm was also coded for its implementation in LabVIEW to run simulations for a fast reactor.

A computation of the point kinetic equations for any reactor can thus be computed easily considering the kinetic parameters for that reactor.

The mathematical model developed to simulate the neutron population density of nuclear reactors given the reactor parameters is designed for use on personal computers. The goal was therefore to develop a system that not only mimicked real reactor system behavior, but for developing reactor models in LabVIEW.

6. Future work

Further work can be carried out in predicting the effect of temperature feedback on the point kinetic equation of the nuclear reactor.

7. Acknowledgements

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