Comparison between five estimation methods for reliability function of weighted Rayleigh distribution by using simulation

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Abstract

The Rayleigh distribution and the distribution weighted are most important distributions used in the analysis of data modeling lifelong, in this paper we derive the weighted Rayleigh distribution (WRD) with estimation of its reliability using five methods are (mle , mom, jackknife , Bayes by using Jeffery information) . we compare between these methods by (MSE), using program (MATLAB 2011a), results will be displayed in tables especially for the purpose of to facilitate the comparison.

Keywords: Bayes Estimation, weighted distribution, Rayleigh distribution, Extension of Jeffery prior information, Maximum likelihood estimates .

1. Introduction:

Both distribution Rayleigh and weighted of the most important models and the most widely used in large-scale tests in life and reliability **[1].** These distributions are used other applications for modeling and analyzing the data in the various fields of science (medicine , Agriculture Engineering) **[2]**, the study of reliability is important to developed the efficient future plans to improve the quality and performance of the equipment.in recent times the research about the reliability of acceptance sampling plans has increased Johnson (1994) introduced the estimation of the parameters and the reliability of the distribution of Whipple and Rayleigh. Hammadi and Suhail najem (2011) discussed the Comparison between the capabilities of Bayes and non-Bayes parameter measurement and reliability of Rayleigh distribution with two parameters by using simulation **[3]** , Pandey and Nidhi Dwivedi comparison between Bayesian and maximum likelihood estimation of scale parameter weibull distribution with known shape under line loss function **[4]**.

In this research, we derive the weighted Rayleigh distribution and estimate its reliability using five estimation methods [Maximum Likelihood method (mle) , Method of Moments (mom), jackknife , Bayes by using Jeffery information] , there for we compare between these methods using the simulation by using MATLAB 2011a , we used in experiments the sizes of different samples and different values for the parameter β and repeat each experiment 1000 times, by depending on mean square error (MES) .

2. Theoretical side:

2.1. Weighted distribution:

Let $(\Omega, £, P)$ be a probability space, $T: \Omega \rightarrow H$ be a random variable (rv) where $H =$ (a , b) be an interval on real line with $0 < a, b$ with $a<b$ can be finite or infinite. When the distribution function(df) F(t) of T is absolutely continuous(discrete) with probability density function (pdf), f(t). And w(t) be a non-negative weight function satisfying $\mu_w = E(w(T)) < \infty$, then the rv T_w having pdf

$$f_w(x) = \frac{w(t)f(t)}{E[w(t)]}, a < t < b \qquad (1)$$

Where $E[w(x)] = \int_{-\infty}^{\infty} w(x)f(x)\, dx \quad , \quad -\infty < x < \infty$

2.2. Rayleigh distribution(RD):

Suppose T = $t_1, t_2, \ldots\ldots, t_n$; is an uncensored observation from a sample of n units or individuals under examination. Also assume that the uncensored observations (data) follow the Rayleigh model. Where the one -parameter Rayleigh failure time distribution of t has a probability density function (pdf) and a cumulative distribution function (cdf) given respectively by

$$f(t; \beta) = 2\beta t\, e^{-\beta t^2} \;, \; 0 < t < \infty, \; \beta > 0 \qquad (2)$$

2.3. Weighted Rayleigh distribution(WRD):

By Using equation (1) and (2) with the use of weight w(t)= e^{t^2} we get probability density function of Weighted Rayleigh distribution (WRD) is

$$f_w(t; \beta) = 2t(\beta - 1)\, e^{-t^2 (\beta-1)} \;, t > 0 , \beta > 1 \qquad (3)$$

That is WRD with $\beta_1 = (\beta - 1) > 0$, is similar to the RD with one parameter $\beta > 0$

And the cumulative distribution function of (WRD) is:

$$F_w\ (t;\beta) = 1 - \ e^{-t^2\ (\beta-1)} \qquad (4)$$

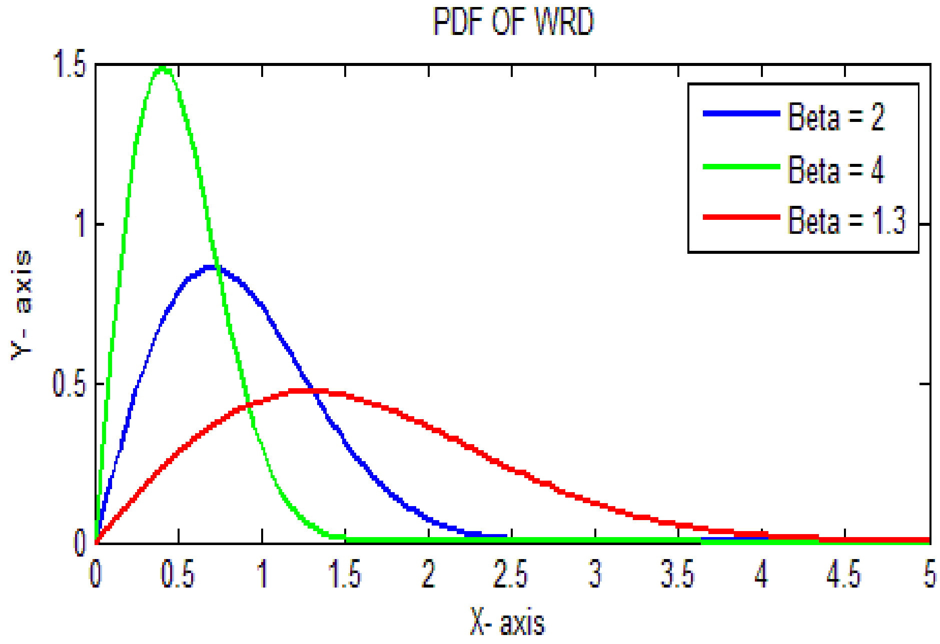

Figure (1):The plot of pdf of WRD

The reliability function $R_{f_w}(t;\beta)$ *is decreasing* ,and the hazard rate $h_{f_w}(t;\beta)$ also known as the instantaneous failure rate are given as

$$R_{f_w}(t;\beta) = 1 - F_w\ (t;\beta) = 1 - (\ 1 - e^{-t^2(\ \beta-1)}\ ) = e^{-t^2(\ \beta-1)} \qquad (5)$$

$$h_{f_w}(t;\beta) = \frac{fw\ (\ t;\beta)}{R_{f_w}(t;\beta)} = \frac{2t\ (\ \beta-1)\ e^{-t^2(\beta-1)}}{e^{-t^2(\ \beta-1)}} = \ 2t\ (\ \beta - 1) \qquad (6)$$

3. METHODES ESTIMATION:

3.1. Maximum Likelihood Estimation(MLE):

This is the way one of the most important methods of appreciation aims to make possible a function at the end of maximizes , All that is done to write down the likelihood function L(t; β), and then find the value $\hat{\beta}$ of β which maximizes L(t ; β).

The log-likelihood function based on the random sample $t_1, t_2, \dots, t_n$ of WRD is given by:

$$L = \prod_{i=1}^{n} f_w(t\,;\,\beta\,)$$

$$\log L(t_1, \dots\dots, t_n\,, \beta) = \prod_{i=1}^{n}(2t_i)\;(\beta-1)^n\, e^{\sum_{i=1}^{n} t_i^2 (\beta-1)} \tag{7}$$

$$\log L\,(\,t_1, \dots\dots, t_n, \beta) = n\log(\beta-1) + \sum_{i=1}^{n}\log(2t_i) - \sum_{i=1}^{n} t_i^2(\beta-1)$$

Then by take the partial derivative of log $L(x_1, \dots\dots, x_n\,, \beta)$ we get

$$\frac{\partial logL(t_1,\dots\dots,t_n\,,\beta)}{\partial\beta} = \frac{n}{\beta-1} - \sum_{i=1}^{n} {t_i}^2 \tag{8}$$

The solving of the equations $\frac{\partial logL(t_1,\dots\dots,t_n\,,\beta)}{\partial\beta} = 0$, yields the maximum likelihood

$$\hat{\beta}_{mle} = \frac{n}{\sum_{i=1}^{n} {t_i}^2} + 1 \tag{9}$$

Depending on property invariant by the Maximum Likelihood Estimation , then the estimator Maximum Likelihood to reliability function will be as follows

$$\hat{R}_{mle}(t) = e^{-t^2(\,\hat{\beta}_{mle}-1)} \tag{10}$$

3.2. Method of Moments

Let t_1, t_2,…, t_n be a random sample of size n from the WRD with p.d.f (4), then the moment estimator of $\hat{\beta}$ is obtained by setting the mean of the distribution equal to the sample mean, that is $E_{f_w}(T^k) = \mu$.

The MoM estimate $\hat{\beta}$ of β is obtained as

$$\frac{\Gamma(1+\frac{k}{2})}{(\beta-1)^{\,k/2}} = \frac{\sum_{i=1}^{n} t_i^k}{n} \tag{11}$$

For k=1 , since we have one parameter then estimate for parameter β is given by:

$$\hat{\beta}_{m.o.m} = \frac{\pi}{\sqrt{2\bar{t}}} + 1 \qquad (12)$$

Hence the estimated moments for reliability function will take the following formula [5]:

$$\hat{R}_{m.o.m}(t) = e^{-t^2(\ \hat{\beta}_{m.o.m} - 1)} \qquad (13)$$

3.3. Jackknife method [6],[7]:

This is the way one of the methods used to improve the values of estimators [6] , and depend on the value of the deleted one of the values of views , Let (t_i) and re-estimate Values estimators based on residual values that number (n -1) And are re-style on this all the values of the sample and sequentially. If the $\hat{\beta}_{mle}(j)$ represents the Maximum Likelihood Estimator resulting from the application of Maximum Likelihood Estimator of all data except the value (t_i), then estimator Jackknife to Maximum Likelihood Estimator for the parameter (β) , It is calculated by the following equation .

$$\hat{\beta}_{jakknife-mle} = n\hat{\beta}_{mle} - (n-1)\frac{\sum_{j=1}^{n} \hat{\beta}_{mle}(j)}{n} \qquad (14)$$

And the estimator reliability function will take the following formula

$$\hat{R}_{jakknife-mle}(t) = e^{-t^2(\hat{\beta}_{jakknife-mle} - 1)} \qquad (15)$$

3.4.Bayesian Estimation

This method depend on the assumption that the parameter that we want appreciation is a random variable requires access to Prior Information about him, Suppose T=(t_1,t_2,…., t_n) is a random sample of size n with distribution function $F_w\ (t;\beta)$ and the probability density function $f_w\ (t;\beta)$, in case the distribution WRD then density function is $f_w\ (\ t;\beta) = \ 2t(\beta-1)\ e^{-t^2\ (\beta-1)}$

To calculate Bayes estimator we can find by following steps

1- We write probability density function $f_w\ (\ t;\beta)$ as a conditional probability density function $f_w\ (\ t_i\ |\beta)$ that is
$f_w\ (\ t;\beta) = \ f_w\ (\ t_i\ |\beta)$

2- Then we find

a) $H(t,\beta)$ is the joint density function of T and is given by

$$H(t_i,\beta) = \prod_{i=1}^{n} f_w\,(t_i\,;\beta)\;\; g(\beta) = L(t_i\,;\beta)\,g(\beta)$$

b) $P\,(t,\beta)$ is the marginal density function is given by
$P(t_i\,,\beta) = \int_0^\infty H(t_i\,,\beta)\;d\beta$

c) density function of the posterior distribution of β is given by

$$\pi^*(\beta\,|t_1,t_2,\ldots.,t_n) = \frac{H(t_i,\beta)}{P(t_i,\beta)} = \frac{\prod_{i=1}^{n} f_w\,(t_i\,;\beta)\;\; g(\beta)}{\int_0^\infty \prod_{i=1}^{n} f_w\,(t_i\,;\beta)\;\; g(\beta)\;d\beta}$$

3- we find Bayes estimator for the parameter β by using the quadratic loss function as follows

$$E(\beta|t_i) = \int_0^\infty \beta\;\pi^*(\,\beta|t_1,t_2,\ldots.,t_n)\,d\beta$$

The following the most important the prior distributions used

a) <u>Jeffery Prior information:</u>

If we assume $g(\beta) = k\,\sqrt{I\,(\beta)}$

such that $I\,(\beta)\;represent\;fishre\;information$

$$\therefore\; g(\beta) = k\,\sqrt{-nE\left[\frac{\partial^2 \log f_w(t,\beta)}{\partial\beta^2}\right]} \;=\; \frac{k\,\sqrt{n}}{\beta-1}\;\;,\;\;k\;\;is\;constant$$

And the joint density function of T and is given by

$$H\big(t_i|\beta_1\big) = 2k\sqrt{n}\prod_{i=1}^{n} t_i\;\;{\beta_1}^{n-1}\;\;e^{-\sum_{i=1}^{n} t_i^2\,\beta_1} \qquad (16)$$

From (16) we find marginal density function of T and is given by

$$P(t_i|\beta_1) = \int_0^\infty H(t_i\,|\,\beta_1)\;\;d\beta_1 = k\sqrt{n}\int_0^\infty 2\prod_{i=1}^{n} t_i\;\;{\beta_1}^{n-1}\;\;e^{-\sum_{i=1}^{n} t_i^2\beta_1}\;\;d\beta_1$$

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Where $\beta_1 = \beta - 1$, and by using the transformation

$$u = \sum_{i=1}^{n} t_i^2 \, \beta_1 \implies \beta_1 = \frac{u}{\sum_{i=1}^{n} t_i^2} \implies d\beta_1 = \frac{du}{\sum_{i=1}^{n} t_i^2}$$

$$\therefore P(t_i | \, \beta_1) = \frac{2\, k\sqrt{n}\, \prod_{i=1}^{n} t_i \quad \Gamma\,(n)}{\left(\sum_{i=1}^{n} t_i^2\right)^n} \tag{17}$$

Hence, density function of the posterior distribution of β is given by

$$\pi^*(\beta_1 \,|t_1, t_2, \dots, t_n) = \frac{2\prod_{i=1}^{n} t_i \;\; {\beta_1}^{n-1} \;\; e^{-\sum_{i=1}^{n} t_i^2 \, \beta_1}}{\dfrac{2\, k\sqrt{n}\, \prod_{i=1}^{n} t_i \quad \Gamma\,(n)}{\left(\sum_{i=1}^{n} t_i^2\right)^n}}$$

$$\therefore \; \pi^*(\beta_1 \,|t_i) = \frac{{\beta_1}^{n-1} \;\; e^{-\sum_{i=1}^{n} t_i^2 \,(\beta-1)} \;\; \left(\sum_{i=1}^{n} t_i^2\right)^n}{\Gamma\,(n)} \tag{18}$$

By using the quadratic loss function

$$S(\beta_1 , \hat{\beta}) = \left[\frac{\beta_1 - \hat{\beta}}{\beta_1} \right]^2 \tag{19}$$

Bayes' estimator will be the estimator that minimizes the posterior risk given by

$$Risk\;(\beta_1) = E\left[S(\beta_1 , \hat{\beta})\right] = \int_0^{\infty} \left(\frac{\beta_1 - \hat{\beta}}{\beta_1} \right)^2 \;\; \pi^*(\beta_1 \,|t_i)\;\, d\beta_1$$

$$\frac{\partial\; Risk\;(\;\beta_1\;)}{\partial \hat{\beta}} = -2E\left(\frac{1}{\beta_1}\right) + 2\hat{\beta}\;\; E\left(\frac{1}{{\beta_1}^2}\right)$$

$$\therefore \;\; \hat{\beta}_{\,BS\,1} = \frac{E\left(\dfrac{1}{\beta_1}\right)}{E\left(\dfrac{1}{{\beta_1}^2}\right)}$$

For these expectations are equal

$$E\left(\frac{1}{\beta_1}\right) = \int_0^{\infty} \frac{1}{\beta_1}\; \pi^*(\beta_1 \,|t_i)\;\, d\beta_1$$

$$= \int_0^\infty \frac{{\beta_1}^{n-2}\ e^{-\sum_{i=1}^n t_i^2 \beta_1}\ \left(\sum_{i=1}^n t_i^2\right)^n}{\Gamma(n)}\, d\beta_1$$

Using the transformation

$$u = \sum_{i=1}^n t_i^2 \beta_1 \ \Rightarrow \ \beta_1 = \frac{u}{\sum_{i=1}^n t_i^2} \ \Rightarrow \ d\beta_1 = \frac{du}{\sum_{i=1}^n t_i^2}$$

$$\therefore E\left(\frac{1}{\beta_1}\right) = \frac{\sum_{i=1}^n t_i^2\ \ \Gamma(n-1)}{\Gamma(n)}$$

$$E\left(\frac{1}{{\beta_1}^2}\right) = \int_0^\infty \frac{{\beta_1}^{n-3}\ e^{-\sum_{i=1}^n t_i^2 \beta_1}\ \left(\sum_{i=1}^n t_i^2\right)^n}{\Gamma(n)}\, d\beta_1$$

By using the same transformation the former of $E\left(\frac{1}{\beta_1}\right)$ then

$$E\left(\frac{1}{{\beta_1}^2}\right) = \frac{\left(\sum_{i=1}^n t_i^2\right)^2\ \ \Gamma(n-2)}{\Gamma(n)}$$

$$\therefore \hat{\beta}_{BS\,1} = \frac{\dfrac{\sum_{i=1}^n t_i^2\ \ \Gamma(n-1)}{\Gamma(n)}}{\dfrac{\left(\sum_{i=1}^n t_i^2\right)^2\ \ \Gamma(n-2)}{\Gamma(n)}} = \frac{\Gamma(n-1)}{\sum_{i=1}^n t_i^2\ \ \Gamma(n-2)} \qquad (20)$$

And by using of (5) and (18), the Bayes estimator for the reliability function R(t) is given by

$$\hat{R}_{BS\,1}(t) = \int_0^\infty e^{-t^2\beta_1}\ \pi^*(\beta_1\,|t_i)\ d\beta_1$$

$$\hat{R}_{BS\,1}(t) = \int_0^\infty \frac{{\beta_1}^{n-1}\ \left(\sum_{i=1}^n t_i^2\right)^n\ e^{-\beta_1\left(\sum_{i=1}^n t_i^2 + t^2\right)}}{\Gamma(n)}\ d\beta_1$$

By using the transformation

$$u = \beta_1 \left(\sum_{i=1}^{n} t_i^2 + t^2 \right) \Longrightarrow \beta_1 = \frac{u}{\sum_{i=1}^{n} t_i^2 + t^2} \Longrightarrow d\beta_1 = \frac{du}{\sum_{i=1}^{n} t_i^2 + t^2}$$

$$\therefore \ \widehat{R}_{BS\,1}(t) = \left(\frac{\sum_{i=1}^{n} t_i^2}{\sum_{i=1}^{n} t_i^2 + t^2} \right)^n \qquad (21)$$

b)Extension Jeffery Prior Information

There is generalization of base Jeffrey [8],[9]

$$g(\beta) \propto \ [\, I\,(\beta)\,]^c \ , \ c \in R^+$$

$$g(\beta) = \ k \ \frac{[n]^c}{[(\beta - 1)^2]^c} \quad , \quad such\ that\ k\ constant$$

the joint density function of T and is given by

$$H(t_i|\beta_1) = 2k\, n^c \prod_{i=1}^{n} t_i \ \ {\beta_1}^{n-2c} \ \ e^{-\sum_{i=1}^{n} t_i^2 \,\beta_1} \qquad (22)$$

From (22) we find marginal density function of T and is given by

$$P(t_i|\beta_1) = \int_0^{\infty} H(t_i \mid \beta_1) \ d\beta_1 = \int_0^{\infty} 2k\, n^c \prod_{i=1}^{n} t_i \ \ {\beta_1}^{n-2c} \ \ e^{-\sum_{i=1}^{n} t_i^2 \,\beta_1} \ d\beta_1$$

Using the transformation

$$u = \sum_{i=1}^{n} t_i^2 \,\beta_1 \ \Longrightarrow \ \beta_1 = \frac{u}{\sum_{i=1}^{n} t_i^2} \ \Longrightarrow \ d\beta_1 = \frac{du}{\sum_{i=1}^{n} t_i^2}$$

$$\therefore P(t_i|\,\beta_1) = \frac{2\,k\,n^c \ \prod_{i=1}^{n} t_i \ \ \Gamma\,(n - 2c + 1)}{\left(\sum_{i=1}^{n} t_i^2\right)^{n-2c+1}} \qquad (23)$$

Hence, density function of the posterior distribution of β is given by

$$\pi^*(\beta_1\,|t_i) = \frac{{\beta_1}^{n\,-\,2c} \ e^{-\sum_{i=1}^{n} t_i^2 \,\beta_1} \ \left(\sum_{i=1}^{n} t_i^2\right)^{n-2c+1}}{\Gamma\,(n - 2c + 1)} \qquad (24)$$

By using the quadratic loss function

$$S(\beta_1, \hat{\beta}) = c\left[\frac{\beta_1 - \hat{\beta}}{\beta_1}\right]^2 \qquad (25)$$

Bayes' estimator will be the estimator that minimizes the posterior risk given by

$$Risk\ (\beta_1) = E[S(\beta_1, \hat{\beta})] = \int_0^\infty c\left(\frac{\beta_1 - \hat{\beta}}{\beta_1}\right)^2 \pi^*(\beta_1 | t_i)\ d\beta_1$$

$$\frac{\partial\, Risk\ (\beta_1)}{\partial \hat{\beta}} = -2c\, E\left(\frac{1}{\beta_1}\right) + 2c\,\hat{\beta}\ E\left(\frac{1}{\beta_1{}^2}\right)$$

$$\therefore\ \hat{\beta}_{BS\,2} = E\left[\frac{1}{\beta_1}\right] \Big/ E\left[\frac{1}{\beta_1{}^2}\right]$$

For these expectations are equal

$$E\left(\frac{1}{\beta_1}\right) = \int_0^\infty \frac{1}{\beta_1}\ \pi^*(\beta_1 | t_i)\ d\beta_1$$

$$\therefore E\left(\frac{1}{\beta_1}\right) = \frac{\sum_{i=1}^n t_i^2\ \ \Gamma(n-2c)}{\Gamma(n-2c+1)}$$

$$E\left(\frac{1}{\beta_1{}^2}\right) = \frac{\left(\sum_{i=1}^n t_i^2\right)^2\ \ \Gamma(n-2c-1)}{\Gamma(n-2c+1)}$$

Hence

$$\hat{\beta}_{BS\,2} = \frac{\Gamma(n-2c)}{\sum_{i=1}^n t_i^2\ \ \Gamma(n-2c+1)} \qquad (26)$$

And by using of (5) and (24), the Bayes estimator for the reliability function R(t) is given by

$$\hat{R}_{BS\,2}(t) = \int_0^\infty e^{-t^2\beta_1}\ \pi^*(\beta_1 | t_i)\ d\beta_1 = \left(\frac{\sum_{i=1}^n t_i^2}{\sum_{i=1}^n t_i^2 + t^2}\right)^{n-2c+1} \qquad (27)$$

4. Practical aspect: (Simulation and Conclusions)

A computer simulation experiment is done to compare five methods of estimation of parameter of the weighted Rayleigh distribution (WRD). Simulations is performed for samples sizes n=10, 20,50,100 , with values of the parameter β = 1.5 , 2, and value of Jeffery's extension c = 1.5, and samples were simulated from the uniform (0, 1) and then used to generate random samples from the weighted Rayleigh distribution by using the probability transform. For each sample, the parameter and reliability function was estimated by the proposed five methods of estimation and then the mean- squared errors and the estimated means of the parameter and reliability function , the transformation to the weighted Rayleigh distribution (WRD), variable is given by:

$$F_w\,(t_i;\beta) = 1 - e^{-{t_i}^2\,(\beta-1)}$$

$$U_i = 1 - e^{-{t_i}^2\,(\beta-1)} \qquad (28)$$

and After operations a simple algebraic produces

$$t_i = \sqrt{\frac{log(1-U_i)^{-1}}{\beta-1}} \qquad (29)$$

where $F_w\,(t_i;\beta)$ = The distribution function given in Eq. 4

Ui = Uniformly distributed random variable on (0,1)

And using equation (29) in the data generation for the different sizes of the samples and the values assumed for the parameter β .

The simulation program is written by using **(Mathlab-2011a)** and it has been performed to compare of **(a)** (ML) **(b)** (mom) **(c)** jackknife method **(d)** Bayesian with Jeffrey’s prior (BJP) and **(e)** Bayesian Extension Jeffrey’s prior (BEJP) , depending on Mean Square Errors (MSE)) to compare the efficiency of the five estimators, as follows:

$$MSE\big(\,\hat{R}(t)\,\big) = \frac{\sum_{i=1}^{L}\Big(\,\hat{R}(t_i) - R(t_i)\Big)^2}{L}$$

Where L=1000 is the number of replications , the results of the simulation study are reported in the following tables(1) to (4):

5. Conclusions:

We conclude from tables (1) to (4), the following

(1) the **Bayes Jeffery Prior information (BJP)** estimator best in the small size samples because it has less (MSE) especially when β=1.5

(2) the **(MLE**) estimator second best in the small size samples because it has the second lowest mean square error(MSE), especially when β=2.

(3) the **(jackknife**) estimator worst in the small size samples because it did not achieve the lower (MSE) for all the samples and sizes studied.

(4) the **(MLE)** estimator best in case the samples medium and large because it has less(MSE), especially when β=2.

(5) the **(BJP)** estimator second best in the medium and large size samples because it has less (MSE), especially when β=2, and give estimated **(M.O.M)** worst method for all cases and sizes which tested.

(6) We noted decreasing values of (MES) with increasing sample size of all cases this corresponds to the statistical theory .

6. The Recommendations

(1) can be adopted Bayes Jeffery Prior (BJP) method in case samples small in research that requires the estimation reliability function of weighted Rayleigh distribution ,especially in the case of systems which are characterized by short life and a time of short samples.

(2) can be adopted (MLE) method in case samples medium and large in research that requires the estimation reliability function of weighted Rayleigh distribution ,especially in the case of systems which are characterized by medium and large life and a time of samples medium and large.

(3) Recommends the researcher the development of research to include the case of the lost and data under probation in detail

Table (1):MSE of Estimated Reliability Function of WRD when

n=10 , 20 , β = 1.5 , c = 1.5

N	t	Mle	m.o.m	jackknife	*BJP*	*BEJP*	Best
10	0.5000	0.0017	0.0760	0.0016	0.0018	0.0014	***BEJP***
	0.7500	0.0027	0.1844	0.0025	0.0026	0.0024	***BEJP***
	1.0000	0.0052	0.2210	0.0049	0.0050	0.0048	***BEJP***
	1.2500	0.0067	0.1706	0.0067	0.0063	0.0071	***BJP***
	1.5000	0.0065	0.0980	0.0069	0.0062	0.0079	***BJP***
	1.7500	0.0051	0.0458	0.0057	0.0050	0.0073	***BJP***
	2.0000	0.0033	0.0182	0.0040	0.0035	0.0058	**Mle**
	2.2500	0.0019	0.0063	0.0024	0.0022	0.0040	**Mle**
20	0.5000	0.0006	0.0756	0.0006	0.0006	0.0006	***BEJP***
	0.7500	0.0022	0.1836	0.0020	0.0022	0.0021	***BEJP***
	1.0000	0.0043	0.2204	0.0041	0.0040	0.0043	***BJP***
	1.2500	0.0057	0.1703	0.0057	0.0054	0.0064	***BJP***
	1.5000	0.0057	0.0980	0.0060	0.0054	0.0072	***BJP***
	1.7500	0.00453	0.0458	0.0051	0.00452	0.0068	***BJP***
	2.0000	0.0030	0.0182	0.0036	0.0032	0.0055	**Mle**
	2.2500	0.0017	0.0063	0.0022	0.0020	0.0038	**Mle**

Table (2): MSE of Estimated Reliability Function of WRD when n=10,20

n	t	Mle	m.o.m	jackknife	*BJP*	*BEJP*	Best
10	0.5000	0.0051	0.0517	0.0046	0.0048	0.0043	***BEJP***
	0.7500	0.0117	0.0939	0.0120	0.0105	0.0124	***BJP***
	1.0000	0.0132	0.0748	0.0162	0.0121	0.0193	***BJP***
	1.2500	0.0097	0.0339	0.0146	0.0098	0.0206	**Mle**
	1.5000	0.0053	0.0100	0.0101	0.0065	0.0165	**Mle**
	1.7500	0.0021	0.0023	0.0059	0.0036	0.0106	**Mle**
	2.0000	0.0009	0.0003	0.0032	0.0017	0.0057	**m.o.m**
	2.2500	0.0003	0.00001	0.0018	0.0007	0.0027	**m.o.m**
20	0.5000	0.0021	0.0511	0.0019	0.0020	0.0018	***BEJP***
	0.7500	0.0052	0.0935	0.0051	0.0049	0.0053	***BJP***
	1.0000	0.0064	0.0749	0.0067	0.0061	0.0077	***BJP***
	1.2500	0.0050	0.0340	0.0056	0.0049	0.0073	***BJP***
	1.5000	0.0026	0.0101	0.0033	0.0029	0.0051	**Mle**
	1.7500	0.0010	0.0021	0.0015	0.0014	0.0027	**Mle**
	2.0000	0.0003	0.0003	0.0005	0.0005	0.0012	**Mle**
	2.2500	0.0001	0.0000	0.0001	0.0002	0.0004	**m.o.m**

β = 2 , c =1.5

Table (3): MSE of Estimated Reliability Function of WRD when n= 50 , 100, β = 1.5 , c =1.5

N	t	Mle	m.o.m	jackknife	BJP	BEJP	Best
50	0.50	0.000263	0.075439	0.000247	0.000262	0.000241	**BEJP**
	0.75	0.000953	0.183527	0.000906	0.000938	0.000890	**BEJP**
	1.00	0.001894	0.220581	0.001824	0.001849	0.001818	**BEJP**
	1.25	0.002557	0.170560	0.002506	0.002483	0.002556	**BJP**
	1.50	0.002589	0.098069	0.002592	0.002517	0.002737	**BJP**
	1.75	0.002077	0.045778	0.002131	0.002044	0.002362	**BJP**
	2.00	0.001366	0.018217	0.001442	0.001382	0.001705	**Mle**
	2.25	0.000756	0.006322	0.000822	0.000800	0.001054	**Mle**
	2.50	0.000358	0.001930	0.000402	0.000405	0.000568	**Mle**
100	0.50	0.000130	0.075150	0.000126	0.000131	0.000125	**BEJP**
	0.75	0.000478	0.183067	0.000465	0.000475	0.000463	**BEJP**
	1.00	0.000964	0.220295	0.000943	0.000953	0.000946	**jackknife**
	1.25	0.001322	0.170479	0.001305	0.001304	0.001324	**BJP**
	1.50	0.001360	0.098061	0.001357	0.001342	0.001400	**BJP**
	1.75	0.001106	0.045779	0.001118	0.001098	0.001182	**BJP**
	2.00	0.000735	0.018218	0.000753	0.000740	0.000824	**Mle**
	2.25	0.000408	0.006322	0.000425	0.000421	0.000486	**Mle**
	2.50	0.000192	0.001930	0.000204	0.000206	0.000246	**Mle**

Table (4): MSE of Estimated Reliability Function of WRD when n = 50 , 100 , β = 2 , c =1.5

n	t	Mle	m.o.m	jackknife	BJP	BEJP	Best
50	0.50	0.000814	0.050891	0.000776	0.000802	0.000763	**BEJP**
	0.75	0.002124	0.093385	0.002064	0.002073	0.002075	**jackknife**
	1.00	0.002690	0.074992	0.002684	0.002618	0.002809	**BJP**
	1.25	0.002069	0.034089	0.002135	0.002046	0.002387	**BJP**
	1.50	0.001078	0.010088	0.001157	0.001116	0.001423	**Mle**
	1.75	0.000406	0.002119	0.000455	0.000457	0.000635	**Mle**
	2.00	0.000115	0.000332	0.000135	0.000147	0.000220	**Mle**
	2.25	0.000025	0.000039	0.000031	0.000038	0.000061	**Mle**
	2.50	0.000020	0.000035	0.000029	0.000029	0.000055	**Mle**
100	0.50	0.000350	0.050501	0.000344	0.000348	0.000346	**jackknife**
	0.75	0.000941	0.09290	0.000933	0.000932	0.000949	**BJP**
	1.00	0.001229	0.074780	0.001234	0.00121	0.001283	**BJP**
	1.25	0.000972	0.034050	0.000992	0.000971	0.001069	**BJP**
	1.50	0.000514	0.010085	0.000536	0.000528	0.000609	**Mle**
	1.75	0.000193	0.002119	0.000206	0.000208	0.000252	**Mle**
	2.00	0.000053	0.000332	0.000058	0.000062	0.000078	**Mle**
	2.25	0.000011	0.000039	0.000012	0.000014	0.000019	**Mle**
	2.50	0.000001	0.000003	0.000002	0.000002	0.000004	**Mle**

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