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γ - sαg*-Semi T_i Spaces In Topological Spaces

S. Maragathavalli

Department of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi,

Coimbatore District, Tamil Nadu, India

*smvalli@rediffmail.com

Abstract

In this paper we introduce the concept of γ-sαg*-open sets and discuss some of their basic properties.

Key words: γ-sαg*-semi T_i spaces (γ, β)-sαg*-semi continuous maps.

1. Introduction

The study of semi open set and semi continuity in topological space was initiated by Levine[14]. Bhattacharya and Lahiri[3] introduced the concept of semi generalized closed sets in the topological spaces analogous to generalized closed gets introduced by Levine[15]. Further they introduced the semi generalized continuous functions and investigated their properties. Kasahara[11] defined the concept of an operation on topological spaces and introduced the concept of α-closed graphs of a function. Jankovic[10] defined the concept of α-closed sets. Ogata [21] introduced the notion of τ_γ which is the collection of all γ-open sets in topological space (X, τ) and investigated the relation between γ-closure and τ_γ-closure.

We introduce the notion γ-sαg*-semi T_i (I = 0, ½, 1, 2) spaces. In section 4, we introduce (γ, β)-sαg*-semi continuous map which analogous to (γ, β)-continuous maps and investigate some important properties. Finally we introduce (γ, β)-sαg*-semi homeomorphism in (X, τ) and study some of their properties.

2. Premilinaries

Throughout this paper (X,) represent non-empty topological space on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X,), cl(A), int(A) denote the closure and interior of A respectively. The intersection of all -closed sets containing a subset A of (X,) is called the -closure of A and is denoted by cl(A).

2.1 Definition [11]

Let (X, τ) be a topological space. An operation γ on the topology τ is a mapping from τ on to power set P(X) of X such that $V \subseteq V^\gamma$ for each $V \in \tau$, where V^γ denote the value of γ at V. It is denoted by $\gamma: \tau \to P(X)$.

2.2 Definition [21]

A subset A of a topological space (X, τ) is called γ-open set if for each $x \in A$ there exists a open set U such that $x \in U$ and $U^\gamma \subseteq A$. τ_γ denotes set of all γ-open sets in (X, τ).

2.3 Definition [21]

The point $x \in X$ is in the γ-closure of a set $A \subseteq X$ if $U^\gamma \cap A \neq \phi$ for each open set U of x. The γ-closure of set A is denoted by $cl_\gamma(A)$.

2.4 Definition [21]

Let (X, τ) be a topological space and A be subset of X then τ_γ-l(A) = $\cap \{F : A \subseteq F, X - F \in \tau_\gamma\}$

2.5 Definition [21]

Let (X, τ) be topological space. An operation γ is said to be regular if, for every open neighborhood U and V of each $x \in X$, there exists an open neighborhood W of x such that $W^\gamma \subseteq U^\gamma \cap V^\gamma$.

2.6 Definition [21]
A topological space (X, τ) is said to be γ-regular, where γ is an operation of τ, if for each $x \in X$ and for each open neighborhood V of x, there exists an open neighborhood U of x such that U^γ contained in V.

2.7 Remark [21]
Let (X, τ) be a topological space, then for any subset A of X, $A \subseteq cl(A) \subseteq cl_\gamma(A) \subset \tau_\gamma\text{-}cl(A)$.

2.8 Definition [24]
A subset A of (X, τ) is said to be a γ-semi open set if and only if there exists a γ-open set U such that $U \subseteq A \subseteq cl_\gamma(U)$.

2.9 Definition [24]
Let A be any subset of X. Then τ_γ-int (A) is defined as $\tau_\gamma\text{-int}(A) = \cup\{U : U$ is a γ-open set and $U \subseteq A\}$

2.10 Definition[24]
A subset A of X is said to be γ-semi closed if and only if $X - A$ is γ-semi open.

2.11 Definition[24]
Let A be a subset of X. There $\tau_\gamma\text{-scl}(A) = \cap\{F : F$ is γ-semi closed and $A \subseteq F\}$.

2.12 Definition[20]
A subset A of (X, τ) is said to be a strongly αg^*-closed set if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is g^*-open in (X,τ).

2.13 Definition[20]
If a subset A of (X, τ) is a strongly αg^*-closed set then $X - A$ is a strongly αg^*-open set.

2.14 Definition[20]
A space (X, τ) is said to be a ${}_{s*}T_c$-space if every strongly αg^*-closed set of (X, τ) is closed in it.

2.15 Definition [20]
A space (X, τ) is called
(i) a γ-semi T_o space if for each distinct points $x, y \in X$, there exists a γ-semi open set U such that $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$.
(ii) a γ-semi T_1 space if for each distinct points $x, y \in X$, these exist γ-semi open sets U, V containing x and y respectively such that $y \notin U$ and $x \notin V$.
(iii) a γ - semi T_2 space if for each $x, y \in X$ there exists a γ-semi open sets U, V such that $x \in U$ and $y \in V$ and $U \cap V = \phi$.

2.16 Definition [24]
A subset A of (X, τ) is said is be γ-semi g-closed if $\tau_\gamma\text{-scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is a γ-semi open set in (X, τ).

2.17 Definition [24]
A space (X, τ) is said to be γ-semi $T_{1/2}$-space if every semi g-closed set in (X, τ) is γ-semi closed.

2.18 Definition[24]
A mapping $f: (X, \tau) \to (y, \sigma)$ is said to be (γ, β) -semi continuous if for each x of X and each β-semi open set V containing f(x) there exists a γ-semi open set U such that $x \in U$ and $f(U) \subseteq V$.

2.19 Definition [24]

A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be (γ, β)-semi closed if for any γ-semi closed set A of (X, τ), f(A) is a β-semi closed.

2.20 Definition [24]
A mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ is said to be (γ, β)-semi homeomorphism, if f is bijective, (γ, β)-semi-continuous and f^{-1} is (β, γ)-semi continuous.

2.21 Definition
A subset A of (X, τ) is said to be a γ-sαg*-semi open set if and only if there exists a γ-sαg*-open set U such that $U \subseteq A \subseteq cl_{\gamma}(U)$.

2.22 Theorem
If A is a γ-semi open set in (X, τ), then A is a γ-sαg*-semi open set.

2.23 Definition
A subset A of X is said to be γ-sαg*-semi closed if and only if X – A is γ-sαg*-semi open.

2.24 Definition
Let A be a subset of X. Then $\tau_{\gamma s*}$-scl(A) = $\cap$ {F : F is γ-sαg* semi closed and $A \subseteq F$} .

2.25 Theorem
For a point $x \in X$, $x \in \tau_{\gamma s*}$-scl(A) if and only if $V \cap A \neq \phi$ for any $V \in \tau_{\gamma s*}$-SO(X) such that $x \in V$.

2.26 Remark
From the Theorem 3.12 and the Definition 3.25 we have $A \subseteq \tau_{\gamma s*}$-scl(A) $\subseteq \tau_{\gamma s*}$-cl(A) for any subset A of (X, τ).

2.27 Remark
Let $\gamma: \tau \rightarrow P(X)$ be a operation. Then a subset A of (X, τ) is γ-sαg*-semi closed if and only if $\tau_{\gamma s*}$-scl(A)=A

3. γ-sαg*-Semi T_i Spaces

In this section, we investigate a general operation approaches on T_i spaces where i = 0, ½, 1,2. Let $\gamma : \tau \rightarrow P(X)$ be a operation on a topology τ.

3.1 Definition
A space (X, τ) is called γ-sαg*-semi T_0 space if for each distinct points $x, y \in X$ there exists a γ-sαg*-semi open set U such that $x \in U$ and $y \notin U$ or $y \in U$ and $x \notin U$.

3.2 Definition
A space (X, τ) is called γ-sαg* semi T_1 space if for each distinct points $x, y \in X$ there exists γ-sαg* semi open sets U, V containing x and y respectively such that $y \notin U$ and $x \notin V$.

3.3 Definition
A space (X, τ) is called a γ-sαg*-semi T_2 space if for each $x, y \in X$ there exist γ-sαg*-semi open sets U, V such that $x \in U$ and $y \in V$ and $U \cap V = \phi$.

3.4 Definition
A subset A of (X, τ) is said to be γ-sαg*-semi g-closed if τ_{γ}-scl(A) $\subseteq$ U whenever $A \subseteq U$ and U is a γ-sαg*-semi open set in (X, τ).

3.5 Remark
From Theorem 3.16 and Remark 3.28 we have every γ-sαg*-semi g-closed set is γ-semi g-closed.

3.6 Definition

A space (X, τ) is γ-sαg*-semi $T_{1/2}$ space if every γ-sαg*-semi g-closed set in (X, τ) is γ-semi closed.

3.7 Remark

Let A be a subset of X. Then $\tau_{\gamma s*}$-scl(A) ⊆ τ_γ-scl (A).

Proof

Let x ∉ τ_γ-scl(A)

⇒ x ∉ ∩ {F:F is γ - semi closed and A ⊆ F}

⇒ x ∉ F where F is γ - semi closed and A⊆ F

⇒ x ∉ F where F is γ - sαg* -semi closed and A⊆ F

⇒ x ∉ ∩ {F : F is γ - sαg*-semi closed and A⊆ F}

⇒ x ∉ $\tau_{\gamma s*}$-scl(A)

Therefore, τ_γ-scl(A) ⊆ $\tau_{\gamma s*}$-scl(A).

3.8 Theorem

A subset A of (X, τ) is γ-sαg*-semi g-closed if and only if $\tau_{\gamma s*}$-scl({x}) ∩ A ≠ ϕ holds for every x ∈ τ_γ -scl(A).

Proof

Let U be γ-sαg*-semi open set such that A ⊆ U. Let x ∈ τ_γ-scl(A). By assumption there exists a z ∈ $\tau_{\gamma s*}$-scl({x}) and z ∈ A ⊆ U. It follows from Theorem 3.27 that U ∩ {x} ≠ ϕ. Hence x ∈ U. This implies τ_γ-scl (A) ⊆ U. Therefore, A is γ-sαg*-semi g-closed set in (X, τ).

Conversely, suppose x ∈ τ_γ-scl(A) such that $\tau_{\gamma s*}$-scl({x}) ∩ A = ϕ. Since $\tau_{\gamma s*}$-scl ({x}) is γ-sαg*-semi closed set in (X, τ), from the Definition 3.24, $(\tau_{\gamma s*}\text{-scl}(\{x\})^c$ is a γ-sαg*-semi open set. Since A ⊆ $\tau_{\gamma s*}$-scl$(\{x\})^c$ and A is γ-sαg*-semi-g-closed set, we have τ_γ-scl(A) ⊆ $\tau_{\gamma s*}$-scl $(\{x\})^c$. Hence x ∉ τ_γ-scl(A) . This is a contradiction. Hence $\tau_{\gamma s*}$-scl({x}) ∩ A ≠ ϕ.

3.9 Theorem

If $\tau_{\gamma s*}$-scl({x}) ∩ A ≠ ϕ holds for every x ∈ $\tau_{\gamma s*}$-scl(A), then $\tau_{\gamma s*}$-scl(A) – A does not contain a non empty γ-sαg*-semi closed set.

Proof

Suppose there exists a non empty γ-sαg*-semi closed set F such that F ⊆ $\tau_{\gamma s*}$-scl(A) – A. Let x ∈ F, x ∈ $\tau_{\gamma s*}$-scl(A) holds. It follows from Remark 3.28 and 3.29, ϕ ≠ F ∩ A = $\tau_{\gamma s*}$-scl(F) ∩ A ⊇ $\tau_{\gamma s*}$-scl ({x}) ∩ A which is a contradiction. Thus, $\tau_{\gamma s*}$-scl(A) – A does not contains a non empty γ-sαg*-semi closed set.

3.10 Theorem

Let γ : τ → P(X) be an operation. Then for each x ∈ X, {x} is γ-sαg*-semi closed or $\{x\}^c$ is γ-sαg*-semi g-closed set in (X, τ).

Proof

Suppose that {x} is not γ- sαg*-semi closed then X–{x} is not γ-sαg*-semi open. Let U be any γ-sαg*-semi open set such that $\{x\}^c$ ⊆ U. Since U = X, we have τ_γ-scl $(\{x\})^c$ ⊆ U. Therefore, $\{x\}^c$ is a γ-sαg*-semi g-closed set.

3.11Theorem

A space (X, τ) is γ-sαg*-semi-$T_{½}$ space if and only if {x} is γ-sαg*-semi closed or γ-sαg*- semi open in (X, τ).

Proof

Suppose {x} is not γ-sαg*-semi closed Then, it follows from assumption and Theorem 3.10, {x} is γ-sαg*-semi open.

Conversely, Let F be γ-sαg*-semi g-closed set in (X, τ). Let x be any point in $\tau_{\gamma s*}$-scl(F), then {x} is γ-sαg*-semi open or γ-sαg*-semi closed.

Case (i) : Suppose {x} is γ-sαg*-semi open. Then by Theorem 3.27, we have

$\{x\} \cap F \neq \phi$. Hence $x \in F$.
Case (ii): suppose $\{x\}$ is γ-sαg*-semi closed. Assume $x \notin F$, Then $x \in \tau_{\gamma s*}$-scl(F) – F. This is not possible by Theorem 3.9. Thus we have $x \in F$. Therefore, $\tau_{\gamma s*}$-scl(F) = F and hence F is γ-sαg*-semi closed.

3.13 Remark
Let X = {a, b, c}, τ = {φ, X, {a}, {b}, {a, b}, {a, c}}, define $\gamma : \tau \to P(X)$ be an operation such that for every $A \in \tau$, $A^{\gamma} = A$ if $b \in A$, $A^{\gamma} = cl(A)$ if $b \notin A$. Then (X, τ) is γ- sαg*- semi T_0 but it is neither γ-sαg*-semi T_2 nor γ-sαg*-semi $T_{½}$ nor γ-sαg*-semi T_1.

4. (γ, β)-sαg*-SEMI CONTINUOUS MAPS

Through out this chapter let (X, τ) and (Y, σ) the two topological spaces and let $\gamma : \tau \to P(X)$ and $\beta : \sigma \to P(Y)$ be operations on τ and σ respectively.

4.1 Definition
A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be (γ, β)-sαg*-semi continuous if for each x of X and each β-sαg*-semi open set V containing f(x) there exists a γ-sαg*-semi open set U such that $x \in U$ and $f(U) \subseteq V$.

4.2 Remark
If (X, τ) and (Y, σ) are both γ-sαg*-regular spaces then the concept of (γ, β)-sαg*-semi continuity and semi continuity are coincide.

4.3 Theorem
Let f: $(X, \tau) \to (Y, \sigma)$ be (γ, β)- sαg*-semi continuous mapping. Then,
(i) f ($\tau_{\gamma s*}$-scl(A)) $\subseteq$ $\tau_{\beta s*}$-scl (f(A)) holds for every subset A of (X, τ).
(ii) Let γ be an operation, then for every β-sαg*-semi closed set B of (Y, σ), $f^{-1}(B)$ is γ-sαg*-semi closed in (X, τ)
Proof
(i) Let $y \in f(\tau_{\gamma s*}$-scl(A)) and V be any β-sαg*-semi open set containing y. Then there exists a point $x \in X$ and γ-sαg*-semi open set U such that f(x) = y and $x \in U$ and $f(U) \subseteq V$. Since $x \in \tau_{\gamma s*}$-scl(A), We have $U \cap A \neq \phi$ and hence $\phi \neq f(U \cap A) \subseteq f(U) \cap f(A) \subseteq V \cap f(A)$. This implies $f(x) \in \tau_{\beta s*}$-scl(f(A)). Therefore, we have f ($\tau_{\gamma s*}$-scl(A)) $\subseteq \tau_{\beta s*}$-scl(f(A)).
(ii) Let B be a β-sαg*-semi closed set in (Y, σ). Therefore, $\tau_{\beta s*}$-scl(B) = B. By using (i) we have f($\tau_{\gamma s*}$-scl (f^{-1}(B))) $\subseteq \tau_{\beta s*}$-scl (B) = B. Therefore we have $\tau_{\gamma s*}$-scl(f^{-1}(B)) $\subseteq$ (f^{-1}(B)). Hence f^{-1}(B) is γ-sαg*-semi closed.

4.4 Definition
A mapping $f : (X, \tau) \to (Y, \sigma)$ is said to be (γ, β)-sαg*-semi closed if for any γ-sαg*-semi closed set A of (X, τ), f(A) is a β-sαg*-semi closed .

4.5 Theorem
Suppose that f is (γ, β)-sαg*-semi continuous mapping and f is (γ, β)- sαg*-semi closed. Then for every γ-sαg*-semi g-closed set A of (X, τ) the image f(A) is β-sαg*-semi-g-closed.
Proof
Let V be any β-sαg*-semi open set in (Y, σ) such that $f(A) \subseteq V$. By using Theorem 4.3 (ii), f^{-1}(V) is γ-sαg*-semi open. Since, A is γ-sαg*-semi g-closed and $A \subseteq f^{-1}(V)$, we have $\tau_{\gamma s*}$-scl(A) $\subseteq f^{-1}$(V), and hence f($\tau_{\gamma s*}$-scl(A)) $\subseteq$ V. It follows from the assumption that f($\tau_{\gamma s*}$-scl(A)) is a β-sαg*-semi closed set. Therefore, $\tau_{\beta s*}$-scl(f(A))) $\subseteq$ $\tau_{\beta s*}$-scl(f($\tau_{\gamma s*}$-scl(A)) = f($\tau_{\gamma s*}$-scl(A)) $\subseteq$ V. This implies f(A) is β-sαg*-semi-g-closed.

4.6 Theorem
Let f: $(X, \tau) \to (Y, \sigma)$ be (γ, β)-sαg*-semi continuous and (γ, β)-sαg*- semi closed. If f is injective and

(Y, σ) is β-sαg*-semi $T_{½}$, then (X, τ) is γ-sαg*-semi $T_{½}$ space.

Proof

Let A be γ-sαg*-semi-g-closed set in (X, τ). Now, to show that A is γ-sαg*-semi closed. By Theorem 4.5, (i) and assumption it is obtained that f(A) is β-sαg*-semi-g-closed and hence f(A) is β-sαg*-semi-g-closed. By Theorem 5.4(ii), f^{-1}(f(A)) is γ-sαg*-semi closed in (X, τ). Therefore, A is γ-sαg*-semi closed in (X, τ). Hence (X, τ) is γ-sαg*-semi $T_{½}$ space.

4.7 Definition

A mapping f : (X, τ) → (Y, σ) said to be (γ, β)-sαg*-semi homeomorphism, if f is bijective, (γ, β)-sαg*-semi continuous and f^{-1} is (β, γ)-sαg*-semi continuous.

4.8 Theorem

Let f: (X, τ) → (Y, σ) be (γ, β)-sαg*-semi homeomorphism and (γ, β)-sαg*-semi closed. If (Y, σ) is β-sαg*-semi $T_{½}$ then (X, τ) is γ-sαg*-semi $T_{½}$ space.

Proof

Follows from Theorem 4.5.

4.9 Theorem

Let f : (X, τ) → (Y, σ) be (γ, β)-sαg*-semi continuous injection. If (Y, σ) is β-sαg*-semi T_1 (resp. β-sαg*- semi T_2) then (X, τ) is γ-sαg*-semi T_1 (resp. γ-sαg*-semi T_2).

Proof

Suppose (Y, σ) is β-sαg*-semi T_2. Let x and y be distinct points in X. Then, there exists two γ-sαg*-semi open sets V and W of Y such that f(x) ∈ V, f(y) ∈ W and V ∩ W = ϕ. Since f is (γ, β)-sαg*-semi continuous for V and W there exists two γ-sαg*-semi open set U and S such that x ∈ U, y ∈ S, and f(U) ⊆ V and f(S) ⊆ W. Therefore, U ∩ S = ϕ. Hence (X, τ) is γ-semi-sαg*-T_2 space. Similarly, we can prove the case β-sαg*-semi T_1.

5. Conclusion

The γ-sαg*-open sets, γ-sαg*-semi T_i spaces, (γ, β)-sαg*-semi continuous maps may be used to find decomposition of γ-sαg*-semi T_i spaces. We can also define separation axioms for the γ-sαg*-semi T_i spaces.

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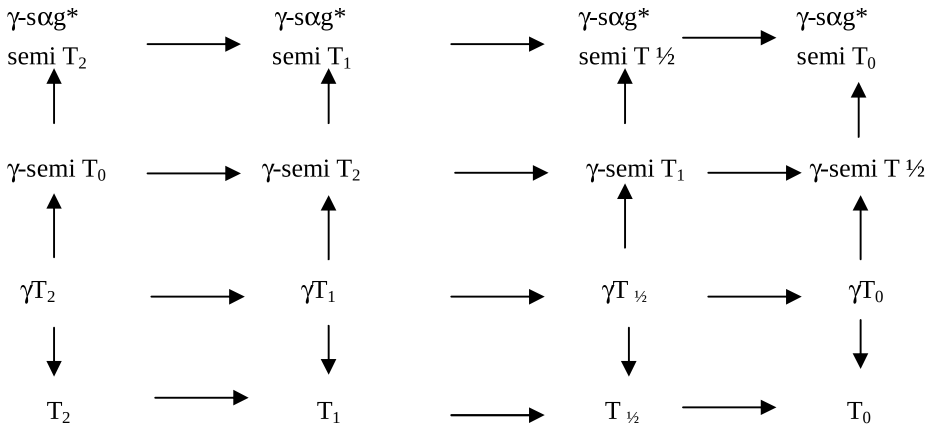
Note 1: From the Definitions, Theorem 3.11 and 3.12 and Remarks 3.13, 4.12 [24] we get



Where A $\rightarrow$ B represent A implies B but not conversely.

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