Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.2, No.3, 2012 www.iiste.org

Fully Invariant and Characteristic Interval-Valued Intuitionistic Fuzzy Dual Ideals of BF-algebras

B. Satyanarayana, D. Ramesh and R. Durga Prasad
Department of Applied Mathematics
Acharya Nagarjuna University Campus
Nuzvid-521201, Krishna (District)
Andhara Pradesh, INDIA.
E. Mail: *drbsn63@yahoo.co.in
ram.fuzzy@gmail.com
durgaprasad.fuzzy@gmail.com

Abstract

The notion of interval-valued intuitionistic fuzzy sets was first introduced by Atanassov and Gargov as a generalization of both interval-valued fuzzy sets and intuitionistic fuzzy sets. Satyanarayana et. al., applied the concept of interval-valued intuitionistic fuzzy ideals and interval-valued intuitionistic fuzzy dual ideals to BF-algebras. In this paper, we introduce the notion of fully invariant and characteristic interval-valued intuitionistic fuzzy dual ideals of BF-algebras and investigate some of its properties.

1. Introduction and preliminaries

For the first time Zadeh (1965) introduced the concept of fuzzy sets and also Zadeh (1975) introduced the concept of an interval-valued fuzzy sets, which is an extension of the concept of fuzzy set. Atanassov and Gargov, 1989) introduced the notion of interval-valued intuitionistic fuzzy sets, which is a generalization of both intuitionistic fuzzy sets and interval-valued fuzzy sets. Meng and Jun (1993) introduced fuzzy dual ideals in BCK-algebras. On other hand, Satyanarayana et al., (2010) and (2011) applied the concept of interval-valued intuitionistic fuzzy ideals and interval-valued intuitionistic fuzzy dual ideals to BF-algebras. In this paper we introduce the notion of fully invariant and characteristic interval-valued intuitionistic fuzzy dual ideals of BF-algebras and investigate some of its properties.

By a BF-algebra we mean an algebra satisfying the axioms:

(1). $x * x = 0$,

(2). $x * 0 = x$,

(3). $0 * (x * y) = y * x$, for all $x, y \in X$

Throughout this paper, X is a BF-algebra. If there is an element 1 of X satisfying $x \leq 1$, for all $x \in X$, then the element 1 is called unit of X. A BF-algebra with unit is called bounded. In a bounded

BF-algebra, we denote $1*x$ by Nx for brief. A bounded BF-algebra X called involutory if NNx=x, for all $x \in X$.

Definition 1.1 (Satyanarayana et al., 2011) A nonempty subset D in a BF-algebra X is said to be a dual ideal of X if it satisfies:

$(D_1)\ 1 \in D$,

$(D_2)\ N(Nx * Ny) \in D$ and $y \in D$ imply $x \in D$, for any $x, y \in X$.

Definition 1.2 Let X be a set. A fuzzy set in X is a function $\mu : X \to [0,1]$.

Definition 1.3 (Meng and Jun (1993)) A fuzzy subset of X is said to be a fuzzy dual ideal of X if it satisfies

(FDI 1) $\mu(1) \geq \mu(x)$

(FDI 2) $\mu(x) \geq \min\{\mu(N(Nx * Ny)), \mu(y)\}$ for all x, y in X.

We now review some fuzzy logic concepts. For fuzzy sets μ and λ of X and $s, t \in [0,1]$, the set

$U(\mu; t) = \{x \in X : \mu(x) \geq t\}$ is called upper t-level cut of μ and the set

$L(\mu; t) = \{x \in X : \lambda(x) \leq s\}$ is called lower s-level cut of λ. The fuzzy set μ in X is called a fuzzy dual sub algebra of X, if $\mu(N(Nx * Ny)) \geq \min\{\mu(x), \mu(y)\}$, for all $x, y \in X$.

Intuitionistic fuzzy sets: (Fatemi 2011 and Wang et. al., 2011) An intuitionistic fuzzy set (shortly IFS) in a non-empty set X is an object having the form $A = \{(x, \mu_A(x), \lambda_A(x)) : x \in X\}$, where the functions $\mu_A : X \to [0,1]$ and $\lambda_A : X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non membership (namely $\lambda_A(x)$) of each element $x \in X$ to the set A respectively such that for any $x \in X$. $0 \leq \mu_A(x) + \lambda_A(x) \leq 1$. For the sake of simplicity we use the symbol form $A = (\mu_A, \lambda_A)$ or $A = (X, \mu_A, \lambda_A)$.

By an interval number D on $[0,1]$ we mean an interval $[a^-, a^+]$ where $0 \leq a^- \leq a^+ \leq 1$. The set of all closed subintervals of $[0,1]$ is denoted by $D[0,1]$. For interval numbers $D_1 = [a_1^-, b_1^+]$,

$D_2 = [a_2^-, b_2^+]$. We define

- $D_1 \cap D_2 = \min(D_1, D_2) = \min([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\min\{a_1^-, a_2^-\}, \min\{b_1^+, b_2^+\}]$
- $D_1 \cup D_2 = \max(D_1, D_2) = \max([a_1^-, b_1^+], [a_2^-, b_2^+]) = [\max\{a_1^-, a_2^-\}, \max\{b_1^+, b_2^+\}]$

$D_1 + D_2 = [a_1^- + a_2^- - a_1^-.a_2^-, b_1^+ + b_2^+ - b_1^+.b_2^+]$

and put

- $D_1 \le D_2 \Leftrightarrow a_1^- \le a_2^-$ and $b_1^+ \le b_2^+$
- $D_1 = D_2 \Leftrightarrow a_1^- = a_2^-$ and $b_1^+ = b_2^+$,
- $D_1 < D_2 \Leftrightarrow D_1 \le D_2$ and $D_1 \ne D_2$
- $mD = m[a_1^-, b_1^+] = [ma_1^-, mb_1^+]$, where $0 \le m \le 1$.

Obviously $(D[0,1], \le, \vee, \wedge)$ form a complete lattice with $[0, 0]$ as its least element and $[1,1]$ as its greatest element. We now use D[0, 1] to denote the set of all closed subintervals of the interval [0, 1].

Let L be a given nonempty set. An interval-valued fuzzy set (briefly, i-v fuzzy set) B on L is defined by $B = \{(x, [\mu_B^-(x), \mu_B^+(x)]) : x \in L\}$, where $\mu_B^-(x)$ and $\mu_B^+(x)$ are fuzzy sets of L such that $\mu_B^-(x) \le \mu_B^+(x)$ for all $x \in L$. Let $\tilde{\mu}_B(x) = [\mu_B^-(x), \mu_B^+(x)]$, then

$B = \{(x, \tilde{\mu}_B(x)) : x \in L\}$ where $\tilde{\mu}_B : L \to D[0,1]$. A mapping $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ $: L \to D[0,1] \times D[0,1]$ is called an interval-valued intuitionistic fuzzy set (i-v IF set, in short) in L if $0 \le \mu_A^+(x) + \lambda_A^+(x) \le 1$ and $0 \le \mu_A^-(x) + \lambda_A^-(x) \le 1$ for all $x \in L$ (that is, $A^+ = (X, \mu_A^+, \lambda_A^+)$ and $A^- = (X, \mu_A^-, \lambda_A^-)$ are intuitionistic fuzzy sets), where the mappings $\tilde{\mu}_A(x) = [\mu_A^-(x), \mu_A^+(x)] : L \to D[0,1]$ and $\tilde{\lambda}_A(x) = [\lambda_A^-(x), \lambda_A^+(x)] : L \to D[0,1]$ denote the degree of membership (namely $\tilde{\mu}_A(x)$) and degree of non-membership (namely $\tilde{\lambda}_A(x)$) of each element $x \in L$ to A respectively.

2 Main Result

In this section we introduce fully invariant and characteristic interval–valued intuitionistic fuzzy dual ideals and prove some of its properties.

Definition 2.1 A dual ideal F of BF-algebra X is said to be a fully invariant dual ideal if $f(F) \subseteq F$ for all $f \in End(X)$ where $End(X)$ is set of all endomorphisms of BF-algebras X.

Definition 2.2 An interval-valued IFS $A = (X, \tilde{\mu}_A, \tilde{\lambda}_A)$ is called interval-valued intuitionistic fuzzy dual ideal (shortly i-v IF dual ideal) of BF-algebra X if satisfies the following inequality

(i-v IF1) $\tilde{\mu}_A(1) \geq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(1) \leq \tilde{\lambda}_A(x)$

(i-v IF2) $\tilde{\mu}_A(x) \geq \min\{\tilde{\mu}_A(N(Nx * Ny)), \tilde{\mu}_A(y)\}$

(i-v IF2) $\tilde{\lambda}_A(x) \leq \max\{\tilde{\lambda}_A(N(Nx * Ny)), \tilde{\lambda}_A(y)\}$, for all $x, y, z \in X$.

Example 2.3 Consider a BF-algebra $X = \{0, 1, 2, 3\}$ with following table.

*	0	1	2	3
0	0	1	2	3
1	1	0	3	2
2	2	3	0	1
3	3	2	1	0

Let A be an interval valued fuzzy set in X defined by $\tilde{\mu}_A(0) = \tilde{\mu}_A(1) = [0.6, 0.7]$, $\tilde{\mu}_A(2) = \tilde{\mu}_A(3) = [0.2, 0.3]$, $\tilde{\lambda}_A(0) = \tilde{\lambda}_A(1) = [0.1, 0.2]$ and $\tilde{\lambda}_A(2) = \tilde{\lambda}_A(3) = [0.5, 0.7]$.

It is easy to verify that A is an interval valued intuitionistic fuzzy dual ideal of X.

Definition 2.4 An interval-valued intuitionistic fuzzy dual ideal $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ of X is called a fully invariant if $\tilde{\mu}_A^f(x) = \tilde{\mu}_A(f(x)) \leq \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A^f(x) = \tilde{\lambda}_A(f(x)) \leq \tilde{\lambda}_A(x)$ for all $x \in X$ and

$f \in End(X)$.

Theorem 2.5 If $\{A_i \setminus i \in I\}$ is a family of i-v intuitionistic fuzzy fully invariant dual ideals of X, then $\bigcap_{i\in I} A_i = (\wedge_{i\in I}\tilde{\mu}_{A_i}, \vee_{i\in I}\tilde{\lambda}_{A_i})$ is an interval-valued intuitionistic fully invariant dual ideal of X, where

$$\underset{i\in I}{\wedge}\tilde{\mu}_{A_i}(x) = \inf\{\tilde{\mu}_{A_i}(x) \setminus i \in I, x \in X\} \text{ and } \underset{i\in I}{\vee}\tilde{\lambda}_{A_i}(x) = \sup\{\tilde{\lambda}_{A_i}(x) \setminus i \in I, x \in X\}.$$

Theorem 2.6 Let S be nonempty subsets of BF-algebra X and $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ an i-v intuitionistic fuzzy dual ideals defined by

$$\tilde{\mu}_A(x) = \begin{cases} [s_2, t_2] & \text{if } x \in S \\ [s_1, t_1] & \text{otherwise,} \end{cases} \text{ and } \tilde{\lambda}_A(x) = \begin{cases} [\alpha_2, \beta_2] & \text{if } x \in S \\ [\alpha_1, \beta_1] & \text{otherwise,} \end{cases}$$

Where $[0,0] \le [s_1, t_1] < [s_2, t_2] \le [1,1], [0,0] \le [\alpha_2, \beta_2] < [\alpha_1, \beta_1] \le [1,1]$,

$[0,0] \le [s_i, t_i] + [\alpha_i, \beta_i] \le [1,1]$ for $i = 1, 2$. If S is an interval-valued intuitionistic fully invariant dual ideal of X, then $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an interval-valued intuitionistic fully invariant dual ideal of X.

Proof: We can easily see that $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an i-v intuitionistic fuzzy dual ideal of X. Let $x \in X$ and $f \in End(X)$. If $x \in S$, then $f(x) \in f(S) \subseteq S$. Thus we have

$\tilde{\mu}_A^f(x) = \tilde{\mu}_A(f(x)) \le \tilde{\mu}_A(x) = [s_2, t_2]$ and $\tilde{\lambda}_A^f(x) = \tilde{\lambda}_A(f(x)) \le \tilde{\lambda}_A(x) = [\alpha_2, \beta_2]$. For if otherwise, we have

$\tilde{\mu}_A^f(x) = \tilde{\mu}_A(f(x)) \le \tilde{\mu}_A(x) = [s_1, t_1]$ and $\tilde{\lambda}_A^f(x) = \tilde{\lambda}_A(f(x)) \le \tilde{\lambda}_A(x) = [\alpha_1, \beta_1]$.

Thus, we have verified that $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is an interval-valued intuitionistic fully invariant dual ideal of X.

Definition 2.7 An i-v intuitionistic fuzzy dual ideal $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ of X has the same type as an i-v intuitionistic fuzzy dual ideal $B = (\tilde{\mu}_B, \tilde{\lambda}_B)$ of X if there exist $f \in End(X)$ such that

$A = B \circ f$ i. e $\tilde{\mu}_A(x) \geq \tilde{\mu}_B(f(x)), \tilde{\lambda}_A(x) \geq \tilde{\lambda}_B(f(x))$ for all $x \in X$.

Theorem 2.8 Interval-valued intuitionistic fuzzy dual ideals of X have same type if and only if they are isomorphic.

Proof: We only need to prove the necessity part because the sufficiency part is obvious. If an i-v intuitionistic fuzzy dual ideal $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ of X has the same type as $B = (\tilde{\mu}_B, \tilde{\lambda}_B)$, then there exist $\phi \in \mathrm{End}(X)$ such that $\tilde{\mu}_A(x) \geq \tilde{\mu}_B(\phi(x)), \tilde{\lambda}_A(x) \geq \tilde{\lambda}_B(\phi(x))$ for all $x \in X$. Let $f : A(X) \rightarrow B(X)$ be a mapping defined by $f(A(x) = B(\phi(x))$ for all $x \in X$, i.e,

$f(\tilde{\mu}_A(x)) = \tilde{\mu}_B(\phi(x)),\ f(\tilde{\lambda}_A(x)) = \tilde{\lambda}_B(\phi(x))$ for $x \in X$. Then, it is clear that f is a surjective homomorphism. Also, f is injective because $f(\tilde{\mu}_A(x)) = f(\tilde{\mu}_A(y))$ for all $x, y \in X$ implies $\tilde{\mu}_B(\phi(x)) = \tilde{\mu}_B(\phi(y))$. Hence $\tilde{\mu}_A(x) = \tilde{\mu}_B(y)$. Likewise, from $f(\tilde{\lambda}_A(x)) = f(\tilde{\lambda}_A(y))$ we conclude $\tilde{\lambda}_A(x) = \tilde{\lambda}_B(y)$ for all $x \in X$. Hence $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is isomorphic to $B = (\tilde{\mu}_B, \tilde{\lambda}_B)$. This completes the proof.

Definition 2.9 An ideal C of X is said to be characteristic if $f(C) = C$ for all $f \in \mathrm{Aut}(X)$ where $\mathrm{Aut}(x)$ is the set of all automorphisms of X. An i-v intuitionistic fuzzy dual ideal $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ of X is called characteristic if $\tilde{\mu}_A(f(x) = \tilde{\mu}_A(x)$ and $\tilde{\lambda}_A(f(x)) = \tilde{\lambda}_A(x)$ for all $x \in X$ and $f \in \mathrm{Aut}(X)$.

Lemma 2.10 Let $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be an i-v intuitionistic fuzzy dual ideal of X and let $x \in X$. Then $\tilde{\mu}_A(x) = \tilde{t}$, $\tilde{\lambda}_A(x) = \tilde{s}$ if and only if $x \in U(\tilde{\mu}_A; \tilde{t})$, $x \notin U(\tilde{\mu}_A; \tilde{s})$ and $x \in L(\tilde{\lambda}_A; \tilde{s})$, $x \notin L(\tilde{\lambda}_A; \tilde{t})$ for all $\tilde{s} > \tilde{t}$.

Proof: Straight forward

Theorem 2.11 An i-v intuitionistic fuzzy dual ideal is characteristic if and only if each its level set is a characteristic dual ideal.

Proof: Let an i-v intuitionistic fuzzy dual ideal $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ be characteristic,

$\tilde{t} \in Im(\tilde{\mu}_A), f \in Aut(X), x \in U(\tilde{\mu}_A; \tilde{t})$. Then $\tilde{\mu}_A(f(x)) = \tilde{\mu}_A(x) \geq \tilde{t}$, which means that

$f(x) \in U(\tilde{\mu}_A; \tilde{t})$. Thus $f(U(\tilde{\mu}_A; \tilde{t})) \subseteq U(\tilde{\mu}_A; \tilde{t})$. Since for each $x \in U(\tilde{\mu}_A; \tilde{t})$ there exist

$y \in X$ such that $f(y) = x$ we have $\tilde{\mu}_A(y) = \tilde{\mu}_A(f(y)) = \tilde{\mu}_A(x) \geq \tilde{t}$, hence we conclude

$y \in U(\tilde{\mu}_A; \tilde{t})$.

Consequently, $x = f(y) \in f(U(\tilde{\mu}_A; \tilde{t})$. Hence $f(U(\tilde{\mu}_A; \tilde{t}) = U(\tilde{\mu}_A; \tilde{t})$.

Similarly, $f(L(\tilde{\lambda}_A; \tilde{s})) = L(\tilde{\lambda}_A; \tilde{s})$. This proves that $U(\tilde{\mu}_A; \tilde{t})$ and $L(\tilde{\lambda}_A; \tilde{s})$ are characteristic.

Conversely, if all levels of $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ are characteristic dual ideals of X, then for $x \in X$,

$f \in Aut(X)$ and $\tilde{\mu}_A(x) = \tilde{t} < \tilde{s} = \tilde{\lambda}_A(x)$, by lemma 2.10, we have $x \in U(\tilde{\mu}_A; \tilde{t}), x \notin U(\tilde{\mu}_A; \tilde{s})$

and $x \in L(\tilde{\lambda}_A; \tilde{s}), x \notin L(\tilde{\lambda}_A; \tilde{t})$. Thus $f(x) \in f(U(\tilde{\mu}_A; \tilde{t})) = U(\tilde{\mu}_A; \tilde{t})$ and

$f(x) \in f(L(\tilde{\lambda}_A; \tilde{s}) = L(\tilde{\lambda}_A; \tilde{s})$, i.e. $\tilde{\mu}_A(f(x)) \geq \tilde{t}$ and $\tilde{\lambda}_A(f(x)) \leq \tilde{s}$. For $\tilde{\mu}_A(f(x)) = \tilde{t}_1 > \tilde{t}$,

$\tilde{\lambda}_A(f(x)) = \tilde{s}_1 < \tilde{s}$. We have $f(x) \in U(\tilde{\mu}_A; \tilde{t}_1) = f(U(\tilde{\mu}_A; \tilde{t}_1)$, $f(x) \in L(\tilde{\lambda}_A; \tilde{s}_1) = f(L(\tilde{\lambda}_A; \tilde{s}_1))$.

Hence $x \in U(\tilde{\mu}_A; \tilde{t}_1)$, $x \in L(\tilde{\lambda}_A; \tilde{s}_1)$ this is a contradiction. Thus $\tilde{\mu}_A(f(x)) = \tilde{\mu}_A(x)$ and

$\tilde{\lambda}_A(f(x)) = \tilde{\lambda}_A(x)$ .So, $A = (\tilde{\mu}_A, \tilde{\lambda}_A)$ is characteristic.

References

Atanassov, K.T. and G. Gargov, 1989. Interval valued intuitionistic fuzzy sets, Fuzzy sets and systems, 31: 343-349.

Fatemi A., 2011. Entropy of stochastic intuitionistic fuzzy sets, Journal of Applied Sci. 11: 748-751.

Meng, J. and Y.B. Jun, 1993. Fuzzy dual ideals in BCK-algebras, Comm. Korean Math. Soc., 8: 225-231.

Satyanarayana, B., D. Ramesh, M.V. Vijayakumar and R. Durga Prasad, 2010. On fuzzy ideal in BF-algebras. Int. J. Math. Sci. Engg. Apple., 4: 263-274.

Satyanarayana, B., D. Ramesh and R. Durga Prasad, 2011. On interval-valued intuitionistic fuzzy dual ideals of bf-algebras, annals of the constantin brancusi university of targu jiu. Eng. Series, 3: 116-126.

Wang, H., G. Qian and X. Feng, 2011. An intuitionistic fuzzy AHP based on synthesis of eigenvectors and its application. Inf. Technol. J., 10: 1850-1866.

Zadeh, L.A., 1965. Fuzzy sets, Informatiuon Control, 8: 338-353.

Zadeh, L.A., 1975. The concept of a linguistic variable and its application to aproximate reasoning, Information Sciences, 8: 199-249.