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Fuzzy Join Prime Semi L-Ideals

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Abstract

The concept of fuzzy join prime semi L-ideals in fuzzy join subsemilattices is introduced. The properties of fuzzy join prime semi L-ideals are discussed.

Keywords: Fuzzy join prime semi L-ideals, f-invariant fuzzy join prime semi L-ideals**,** fuzzy join prime semi L-ideal homomorphism.

Introduction

Zhang Yue[2] defined Prime L-Fuzzy Ideals in Fuzzy Sets. Rajesh Kumar[4] derived the concepts of Fuzzy Semiprime Ideals in Ring. Swami & Swamy[3] generalized the concepts of Fuzzy Prime Ideals of Rings.

Definition: 1

A fuzzy join semi L-ideal S(μ) of a fuzzy join semilattice A is said to be a fuzzy join prime semi L-ideal of A if

(i) S(μ) is not a constant function and

(ii) For any two fuzzy join semi L-ideals S(σ) and S(θ) in A if S(σ) $\vee$ S(θ) $\subseteq$ S(μ), then either

S(σ) $\subseteq$ S(μ) or S(θ) $\subseteq$ S(μ).

Example: 1

Let A = { 0, a, b, 1 } be a fuzzy join semilattice.
Consider S(μ) is a fuzzy join prime semi L-ideal of A.
Then S[μ(0)] = 0.6, S[μ(a)] = 0.5, S[μ(b)] = 0.4, S[μ(1)] = 0.7
Let S(σ) and S(θ) be any fuzzy join prime semi L-ideals of A.
Then S[σ(0)] = 0.4, S[σ(a)] = 0.2, S[σ(b)] = 0.3, S[σ(1)] = 0.8 and
S[θ(0)] = 0.5, S[θ(a)] = 0.3, S[θ(b)] = 0.4, S[θ(1)] = 0.7
Here S(σ) $\vee$ S(θ) $\subseteq$ S(μ), S(σ) $\not\subset$ S(μ) or S(θ) $\not\subset$ S(μ).

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Hence S(μ) is a fuzzy join prime semi L-ideal of A.

Note: 1

$S(\sigma) \subseteq S(\mu)$ means $S[\ \sigma(x)\ ] \leq S[\ \mu(x)\ ]$ for all $x \in A$.

Definition: 2

A fuzzy join prime semi L-ideal S(μ) of a fuzzy join semilattice A is called fuzzy level join semi L-prime if the fuzzy level join semi L-ideal $S(\mu_t)$, where $t = S[\ \mu(0)\ ]$ is a prime semi L-ideal of A.

Proposition: 1

Let S(μ) be any fuzzy join prime semi L-ideal of a fuzzy join semilattice A such that each fuzzy level join prime semi L-ideal $S(\mu_t)$, $t \in \text{Im } S(\mu)$ is prime. If $S[\ \mu(x)\ ] < S[\ \mu(y)\ ]$ for some $x, y \in A$ then $S[\ \mu(x \vee y)\ ] = S[\ \mu(y)\ ]$.

Proof:

Let $S[\ \mu(x)\ ] = t$; $S[\ \mu(y)\ ] = t'$; and $S[\ \mu(x \vee y)\ ] = s$

Given $S[\ \mu(x)\ ] < S[\ \mu(y)\ ]$

(i.e) $t < t'$

Now,

$s = S[\ \mu(x \vee y)\ ] \geq \max\{ S[\ \mu(x)\ ], S[\ \mu(y)\ ] \}$

$= \max\{ t, t' \}$

$= t'$

Therefore $t < t' \leq s$

Suppose that $t' < s$.

If $x \vee y \in S(\mu_s)$ then either $x \in S(\mu_s)$ or $y \in S(\mu_s)$, Since $S(\mu_s)$ is a fuzzy level join prime semi L-ideal of A.

Now, $x \in S(\mu_s) \Rightarrow S[\ \mu(x)\ ] = s$ or $y \in S(\mu_s) \Rightarrow S[\ \mu(y)\ ] = s$

Hence, $t = S[\ \mu(x)\ ] \geq s$ or $t' = S[\ \mu(y)\ ] \geq s$, which is not possible.

Therefore, $t' = s$

(i.e) $S[\ \mu(x \vee y)\ ] = S[\ \mu(y)\ ]$

Corollary: 1

If S(μ) is any fuzzy join prime semi L-ideal of a fuzzy join semilattice A then $S[\ \mu(x \vee y)\ ] = \max\{ S[\ \mu(x)\ ], S[\ \mu(y)\ ] \}$, for all $x, y \in A$.

Proof: Let $x, y \in A \Rightarrow x \vee y \in A$

S(μ) is a fuzzy join prime semi L-ideal.

$\Rightarrow S[\ \mu(x \vee y)\ ] \geq \max\{ S[\ \mu(x)\ ], S[\ \mu(y)\ ] \}$.

$\Rightarrow$ If $S[\ \mu(x)\ ] < S[\ \mu(y)\ ]$, then $S[\ \mu(x \vee y)\ ] = S[\ \mu(y)\ ]$

Similarly, if $S[\ \mu(x)\ ] > S[\ \mu(y)\ ]$, then $S[\ \mu(x \vee y)\ ] = S[\ \mu(x)\ ]$, by theorem 5.1.5

Therefore, $S[\ \mu(x \vee y)\ ] = \max\{ S[\ \mu(x)\ ], S[\ \mu(y)\ ] \}$.

Theorem: 1

Let S(μ) be a fuzzy join prime semi L-ideal of a fuzzy join semilattice A then Card Im S(μ) = 2.

Proof: Since S(μ) is non constant, card Im $S(\mu) \geq 2$.

Suppose that card Im $S(\mu) \geq 3$.

Let $S[\mu(0)] = s$ and $k = \text{Sup}\{ S[\mu(x)] / x \in A \}$.
Then there exists $t, m \in \text{Im } S(\mu)$ such that $t < m < s$ and $t \leq k$.
Let $S(\sigma)$ and $S(\theta)$ be two fuzzy join prime semi L-ideals of A such that
$S[\sigma(x)] = ½ (t + m)$, for all $x \in A$ and

$$S[\theta(x)] = \begin{cases} k, & \text{if } x \notin S(\mu_m) = \{ x \in A / S[\mu(x)] \geq m\} \\ s, & \text{if } x \in S(\mu_m) \end{cases}$$

Clearly, $S(\sigma)$ is a fuzzy join prime semi L-ideal of A.
To show that $S(\theta)$ is a fuzzy join prime semi L-ideal of A.
Let $x, y \in A$.

Case (i): If $x, y \in S(\mu_m)$, then $S[\theta(x)] = s$, $S[\theta(y)] = s$, $x \vee y \in S(\mu_m)$

Also, $S[\theta(x \vee y)] = s = \max\{ S[\theta(x)], S[\theta(y)] \}$
$\Rightarrow S[\theta(x \vee y)] \geq \max\{ S[\theta(x)], S[\theta(y)] \}$
Therefore $S(\theta)$ is s fuzzy join prime semi L-ideal of A.

Case (ii): If $x \in S(\mu_m)$ and $y \notin S(\mu_m)$, then $S[\theta(x)] = s$, $S[\theta(y)] = k$, $x \vee y \notin S(\mu_m)$

Also,
$S[\theta(x \vee y)] = k = \max\{ S[\theta(x)], S[\theta(y)] \}$
$= \max\{ s, k \}$
$\Rightarrow S[\theta(x \vee y)] \geq \max\{ S[\theta(x)], S[\theta(y)] \}$
Therefore, $S(\theta)$ is a fuzzy join prime semi L-ideal of A.

Case(iii): If $x \notin S(\mu_m)$ and $y \notin S(\mu_m)$, then $S[\theta(x)] = S[\theta(y)] = k$, $x \vee y \notin S(\mu_m)$

Also, $S[\theta(x \vee y)] = k = \max\{ S[\theta(x)], S[\theta(y)] \}$
$= \max\{ k, k \}$
$\Rightarrow S[\theta(x \vee y)] \geq \max\{ S[\theta(x)], S[\theta(y)] \}$
Therefore, $S(\theta)$ is a fuzzy join prime semi L-ideal of A.

Claim: $S(\sigma) \vee S(\theta) \subseteq S(\mu)$

Let $x \in A$.

Consider the following cases:

(i) Let $x = 0$.

$$\text{Then } [S(\sigma) \vee S(\theta)](x) = \max_{x = y \vee z}\{ \max\{ S[\sigma(y)], S[\theta(z)] \} \}$$
$$\leq ½ (t + m)$$
$$< s$$
$$= S[\mu(0)]$$

(ii) Let $x \neq 0$, $x \in S(\mu_m)$.

Then $S[\mu(x)] \geq m$ and

$$[S(\sigma) \vee S(\theta)](x) = \max\{ \max\{ S[\sigma(y)], S[\theta(z)] \} \}$$

$x = y \vee z$

$\leq ½ (t + m)$

$< m$

$= S[\mu(x)]$, Since $\max \{ S[\sigma(y)], S[\theta(z)] \} \leq S[\sigma(y)]$.

(iii) Let $x \neq 0$, $x \notin S(\mu_m)$.

Then for any $y, z \in A$, such that $x = y \vee z$, $y \notin S(\mu_m)$ and $z \notin S(\mu_m)$.

Thus, $S[\theta(y)] = k$ and $S[\theta(z)] = k$

Hence, $[S(\sigma) \vee S(\theta)] (x) = \max_{x = y \vee z} \{ \max \{ S[\sigma(y)], S[\theta(z)] \} \}$

$= \max \{ \max (k, k) \}$

$= k$

$\leq S[\mu(x)]$.

Thus in any case, $[S(\sigma) \vee S(\theta)] (x) \leq S[\mu(x)]$

Hence $S(\sigma) \vee S(\theta) \leq S(\mu)$

Now, there exists $y \in A$ such that $S[\mu(y)] = t$

Then $S[\sigma(y)] = ½ (t + m) > S[\mu(y)]$

$\Rightarrow S[\sigma(y)] > S[\mu(y)]$

Hence, $S(\sigma) \subseteq S(\mu)$

Also there exists $x \in A$ such that $S[\mu(x)] = t$.

Then, $x \in S(\mu_m)$ and

thus $S[\theta(x)] = s > m = S[\mu(x)]$

$\Rightarrow S[\theta(x)] > S[\mu(x)]$

Hence $S(\theta) \subseteq S(\mu)$

This shows that $S(\mu)$ is not a fuzzy join prime semi L-ideal of A, which is a contradiction to the hypothesis.

Hence, card Im $S(\mu) = 2$.

Theorem: 2

Let A be a fuzzy join semilattice and let $S(\mu)$ be a fuzzy join semi L-ideal of A such that Card Im

$S(\mu) = 2$, $S[\mu(0)] = 1$ and the set $S(\mu_0) = \{ x \in A / S[\mu(x)] = S[\mu(0)] \}$ is a fuzzy level join prime semi L-ideal of A. Then $S(\mu)$ is a fuzzy join prime semi L-ideal of A.

Proof: Let Im $S(\mu) = \{ t, 1\}$, $t < 1$.

Then $S[\mu(0)] = 1$.

Let $x, y \in A$.

Case (i):

If $x, y \in S(\mu_0)$ then $x \vee y \in S(\mu_0)$ and $S[\mu(x \vee y)] = 1 = \max \{ S[\mu(x)], S[\mu(y)] \}$.

Case (ii):

If $x \in S(\mu_0)$ and $y \notin S(\mu_0)$, then $x \vee y \notin S(\mu_0)$ and $S[\mu(x \vee y)] = t = \max \{ S[\mu(x)], S[\mu(y)] \}$,

Since $S[\mu(x)] = 1 > t = S[\mu(y)]$.

Case (iii):

If $x, y \in S(\mu_0)$ and $y \notin S(\mu_0)$, then $S[\mu(x)] = S[\mu(y)] = t$

Thus, $S[\mu(x \vee y)] \geq t = \max \{ S[\mu(x)], S[\mu(y)] \}$.

Hence, $S[\mu(x \vee y)] \geq \max \{ S[\mu(x)], S[\mu(y)] \}$, for all $x, y \in A$.

Now, if $x \in S(\mu_0)$, then $x \vee y, y \vee x \in S(\mu_0)$

Therefore, $S[\mu(x \vee y)] = S[\mu(y \vee x)] = 1 = S[\mu(x)]$.

If $x \notin S(\mu_0)$, then $S[\mu(x \vee y)] \geq t = S[\mu(x)]$ and $S[\mu(y \vee x)] \geq t = S[\mu(x)]$.

Hence, $S(\mu)$ is a fuzzy join prime semi L-ideal of A.

Let $S(\sigma)$ and $S(\theta)$ be fuzzy join prime semi L-ideals of A such that $S(\sigma) \vee S(\theta) \subseteq S(\mu)$.

Suppose that $S(\sigma) \subseteq S(\mu)$ and $S(\theta) \subseteq S(\mu)$.

Then there exists $x, y \in A$ such that $S[\sigma(x)] > S[\mu(x)]$ and $S[\theta(x)] > S[\mu(y)]$.

Since for all $a \in S(\mu_0)$, $S[\mu(a)] = 1 = S[\mu(0)]$, $x \notin S(\mu_0)$ and $y \notin S(\mu_0)$.

Now, since $S(\mu_0)$ is a fuzzy join prime semi L-ideal of A, there exists $z \in A$ such that $x \vee z \vee y \notin S(\mu_0)$.

Let $a = x \vee z \vee y$.

Then, $S[\mu(a)] = S[\mu(x)] = S[\mu(y)] = t$.

$$\text{Now, } [S(\sigma) \vee S(\theta)](x) = \max_{x = u \vee v} \{ \max \{ S[\sigma(u)] \vee S[\theta(v)] \} \}$$

$$\geq \max \{ S[\sigma(x)], S[\theta(z \vee y)] \}$$

> t = S[μ(a)], Since S[σ(x)] > S[μ(y)] = t and S[θ(z∨y)] ≥ S[θ(y)] > S[μ(y)] = t

That is , S(σ) ∨ S(θ) ⊆ S(μ).

This contradicts the assumption that S(σ) ∨ S(θ) ⊆ S(μ)

Thus, S(μ) is a fuzzy join prime semi L-ideal of A.

Theorem: 3

If f is a fuzzy join prime semi L-ideal homomorphism from a fuzzy join semi L-ideal of A onto a fuzzy join semi L-ideal of A′ and S(μ′) is any fuzzy join prime semi L-ideal of A′, then f^{-1}[S(μ′)] is a fuzzy join prime semi L-ideal of A.

Proof:

Let S(μ) and S(σ) be any two fuzzy join prime semi L-ideals of A such that S(μ) ∨ S(σ) ⊆ f^{-1}[S(μ′)].

⇒ f [S(μ) ∨ S(σ)] ⊆ ff^{-1}[S(μ)] = S(μ′)

⇒ f [S(μ)] ∨ f [S(σ)] ⊆ S(μ′), Since f is a fuzzy join prime semi L-ideal homomorphism.

⇒ Either f [S(μ)] ⊆ S(μ′) or f [S(σ)] ⊆ S(μ′), Since S(μ′) is fuzzy join prime semi L-ideal of A′.

⇒ Either f^{-1}f [S(μ)] ⊆ f^{-1}[S(μ′)] or f^{-1}f [S(σ)] ⊆ f^{-1}[S(μ′)].

⇒ Either S(μ) ⊆ f^{-1}[S(μ′)] or S(σ) ⊆ f^{-1}[S(μ′)]

Hence f^{-1}[S(μ′)] is a fuzzy join prime semi L-ideal of A.

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