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Numerical solution of Fuzzy Hybrid Differential Equation by Third order Runge Kutta Nystrom Method

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Abstract

In this paper we study numerical method for hybrid fuzzy differential equations by an application of Runge–Kutta Nystrom method of order three. Here we state a convergence result and give a numerical example to illustrate the theory. This method is discussed in detail and this is followed by a complete error analysis.

Keywords: Hybrid systems; Fuzzy differential equations; Runge–Kutta Nystrom method

1. Introduction

Hybrid systems are devoted to modeling, design, and validation of interactive systems of computer programs and continuous systems. That is, control systems that are capable of controlling complex systems which have discrete event dynamics as well as continuous time dynamics can be modeled by hybrid systems. The differential systems containing fuzzy valued functions and interaction with a discrete time controller are named hybrid fuzzy differential systems.

In this article we develop numerical methods for solving hybrid fuzzy differential equations by an application of the Runge–Kutta Nystrom method [6]. In Section 2 we list some basic definitions for fuzzy valued functions. Section 3 reviews hybrid fuzzy differential systems. Section 4 contains the Runge–Kutta Nystrom method for approaching hybrid fuzzy differential equations. Section 5 contains a numerical example to illustrate the theory.

2. **Preliminaries**

By $\mathcal{R}$ we denote the set of all real numbers. A fuzzy number is a mapping $u: \mathcal{R} \to [0, 1]$ with the following properties:

(a) u is upper semicontinous,

(b) u is fuzzy convex, i.e., u $(\lambda x + (1-\lambda)y) \geq \min\{u(x), u(y)\}$ for all x, y $\in \mathcal{R}$,$\lambda \in [0,1]$,

(c) u is normal, i.e.,$\exists x_0 \in \mathcal{R}$ for which $u(x_0) = 1$,

(d) Supp u = { $\mathcal{R}/u(x) > 0\}$ is the support of u, and its closure cl (supp u) is compact.

Let $\in$ be the set of all fuzzy number on r.The r-level set of a fuzzy number $u \in \in, 0 \leq r \leq 1$, denoted by$[u]_r$, is defined as

$$[u]_r = \begin{cases} \{x \in \mathcal{R} / u(x) \geq r\}, & 0 < r \leq 1 \\ cl(supp\ u), & r = 0 \end{cases}$$

It is clear that the r- level set of a fuzzy number is a closed and bounded interval $[\underline{u}(r), \overline{u}(r)]$, where $\underline{u}(r)$ denotes the left-hand end point of $[u]_r$ and $\overline{u}(r)$ denotes the right- hand side end point of $[u]_r$.since each $y \in \mathcal{R}$ can be regarded as a fuzzy number $\tilde{y}$ is defined by

$$\tilde{y}(t) = \begin{cases} 1, t = y \\ 0, t \neq y \end{cases}$$

Remark 2.1

Let X be the Cartesian product of universes$X = X_1 \times \dots \times X_n$, and $A_1 \dots A_n$ be n fuzzy numbers in $X_1 \times \dots \times X_n$ respectively. f is a mapping from X to a universe Y,

$y = f(x_1, x_2, \dots x_n)$ .Then the extension principle allows us to define a fuzzy set B in Y by $B = \{y, u(y) / y = f(x_1 \dots x_n), (x_1 \dots x_n) \in X\}$, where

$$u_B(y) = \begin{cases} \sup_{(x_1, \dots, x_n) \in f^{-1}(y)} \min\{u_{A1}(x_1), \dots \dots u_{An}(x_n)\} & \text{if } f^{-1}(y) \neq 0, \\ 0 & \text{if otherwise.} \end{cases}$$

Where f^{-1} is the inverse of f.For n=1, the extension principle, of course, reduces to

$$B = \{y, u_B(y) / y = f(x), x \in X\}$$

Where $$u_B(y) = \begin{cases} \sup x \in f^{-1}(y)\ u_A(x), & f^{-1}(y) \neq 0, \\ 0 & \text{otherwise.} \end{cases}$$

According to Zadeh's extension principle, operation of addition on € is defined by

$$(u \oplus v)(x) = \sup_{y \in \mathcal{R}} \min\{u(y), v(x - y)\}, \quad x \in \mathcal{R}$$

and scalar multiplication of a fuzzy number is given by

$$(k \odot u)(x) = \begin{cases} u(x/k), & k > 0, \\ \hat{0}, & k = 0, \end{cases}$$

Where $\hat{0} \in €$. The Hausdorff distance between fuzzy numbers given by $D: € \times € \to \mathcal{R}_+ \cup \{0\}$,

$$D(u, v) = \sup_{r \in [0,1]} \max\{|\underline{u}(r) - \underline{v}(r)|, |\overline{u}(r) - \overline{v}(r)|\},$$

It is easy to see that D is a metric in € and has the following properties

(i) $D(u \oplus w, v \oplus w) = D(u, v)$, $\forall u, v, w \in €$,
(ii) $D(k \odot u, k \odot v) = |k| D(u, v)$, $\forall\ k \in \mathcal{R}$, $u, v \in €$,
(iii) $D(u \oplus v, w \oplus e) \leq D(u, w) + D(v, e)$, $\forall\ u, v, w, e \in €$.
(iv) (D, €) is a complete metric space.

Next consider the initial value problem (IVP)

$$(2.1) \qquad \begin{cases} x'(t) = f(t, x(t)), \\ x(0) = x_0 \end{cases}$$

Where f is continuous mapping from $\mathcal{R}_+ \times \mathcal{R}$ into $\mathcal{R}$ and $x_0 \in €$ with r level sets

$$[x_0]_r = [\underline{x}_0(r), \overline{x}_0(r)], r \in (0,1].$$

The extension principle of Zadeh leads to the following definition of f (t, x) when x = x (t) is a fuzzy number

$$f(t,x)(s) = \sup\{x(t) \backslash s = f(t,r)\}, \quad s \in \mathcal{R}.$$

It follows that $[f(t,x)]_r = [\underline{f}(t,x,r), \overline{f}(t,x,r)] \quad r \in (0,1],$

Where $\underline{f}(t,x,r) = \min\{f(t,u) \backslash u \in [\underline{x}(r), \overline{x}(r)]\},$

$$\overline{f}(t,x,r) = \max\{f(t,u) \backslash u \in [\underline{x}(r), \overline{x}(r)]\}.$$

Theorem 2.1

Let f satisfy $|f(t,v) - f(t,\overline{v})| \leq g(t, |v - \overline{v}|), t \geq 0, \ v, \overline{v} \in \mathcal{R},$
Where g:$\mathcal{R}_+ \times \mathcal{R}_+$is a continuous mapping such that $r \rightarrow g(t,r)$ is non decreasing and the initial value problem

$$u'(t) = g(t, u(t)), \ u(0) = u_0, \qquad (2.2)$$

has a solution on $\mathcal{R}_+$ for $u_0 > 0$ and that $u(t) = 0$ is the only solution of (2.2) for $u_0 = 0$.Then the fuzzy initial value problem (2.1) has a unique solution.

3. The hybrid fuzzy differential system

Consider the hybrid fuzzy differential equation

$$(3.1) \begin{cases} x'(t) = f\left(t, x(t), \lambda_k\left(x(t_k)\right)\right), & t \in [t_k, t_{k+1}] \\ x(t_k) = x_k \end{cases}$$

Where ' denotes the seikkala derivative, $0 \leq t_0 < t_1 < \ldots < t_k < \ldots, \ t_k \rightarrow \infty,$

$f \in C[\mathcal{R}^+ \times € \times €, €], \ \lambda_k \in C[€, €].$

To be specific the system will be as follows

$$x'(t) = \begin{cases} x_0'(t) = f(t, x_0(t), \lambda_0(x_0)), & x_0(t_0) = x_0, \quad t_0 \leq t \leq t_1, \\ x_1'(t) = f(t, x_1(t), \lambda_1(x_1)), & x_1(t_1) = x_1, \quad t_1 \leq t \leq t_2, \\ \ldots\ldots\ldots & \\ x_k'(t) = f(t, x_k(t), \lambda_k(x_k)), & x_k(t_k) = x_k, \quad t_k \leq t \leq t_{k+1}, \\ \ldots & \end{cases}$$

With respect to the solution of (3.1), we determine the following function:

$$x(t) = x(t, t_0, x_0) = \begin{cases} x_0(t), & t_0 \leq t \leq t_1, \\ x_1(t), & t_1 \leq t \leq t_2, \\ . & \\ . & \\ x_k(t), & t_k \leq t \leq t_{k+1}, \\ . & \end{cases}$$

We note that the solutions of (3.1) are piecewise differentiable in each interval for $t \in [t_k, t_{k+1}]$ for a fixed $x_k \in €$ and k=0,1,2,….

Therefore we may replace (3.1) by an equivalent system

(3.2) $$\begin{cases} \underline{x}'(t) = \underline{f}(t, x, \lambda_k(x_k)) \equiv F_k(t, \underline{x}, \overline{x}), & \underline{x}(t_k) = \underline{x_k}, \\ \overline{x}'(t) = \overline{f}(t, x, \lambda_k(x_k)) \equiv G_k(t, \underline{x}, \overline{x}), & \overline{x}(t_k) = \overline{x_k} \end{cases}$$

which possesses a unique solution $(\underline{x}, \overline{x})$ which is a fuzzy function. That is for each t, the pair $\left(\underline{x}(t;r), \overline{x}(t;r)\right)$ is a fuzzy number, where, $\underline{x}(t;r), \overline{x}(t;r)$ are respectively the solutions of the parametric form given by

(3.3) $$\begin{cases} \underline{x}'(t;r) = F_k\left(t, \underline{x}(t;r), \overline{x}(t;r)\right), & \underline{x}(t_k;r) = \underline{x_k}(r), \\ \overline{x}'(t;r) = G_k\left(t, \underline{x}(t;r), \overline{x}(t;r)\right), & \overline{x}(t_k;r) = \overline{x_k}(r), \text{ for } r \in [0,1] \end{cases}$$

4. **The Runge–Kutta Nystrom method**

In this section ,for a hybrid fuzzy differential equation (3.1) we develop a Runge kutta Nystrom method of order three via an application of the Runge kutta Nystrom method for fuzzy differential equation in [6].We assume that the existence and the uniqueness of the solutions of (3.1) hold for each $[t_k, t_{k+1}]$.

For a fixed r, to integrate the system (3.3) in $[t_0, t_1], [t_1, t_2], \dots, [t_k, t_{k+1}], \dots$,we replace each interval by a set of $N_k + 1$ discrete equally spaced grid points at which the exact solution $x(t, r) = \left(\underline{x}(t;r), \overline{x}(t;r)\right)$ is approximated by some ($\underline{y_k}(t;r), \overline{y_k}(t;r)$).

For the chosen grid points on $[t_k, t_{k+1}]$

$$t_{k,n} = t_k + nh_k, h_k = \frac{t_{k+1} - t_k}{N_k}, 0 \le n \le N_k,$$

Let $\left(\underline{Y_k}(t;r), \overline{Y_k}(t;r)\right) \equiv \left(\underline{x}(t;r), \overline{x}(t;r)\right)$.

$\left(\underline{Y_k}(t;r), \overline{Y_k}(t;r)\right)$ and $(\underline{y_k}(t;r), \overline{y_k}(t;r))$ may be denoted respectively by $\left(\underline{Y_{k,n}}(r), \overline{Y_{k,n}}(r)\right)$ and $\left(\underline{y_{k,n}}(r), \overline{y_{k,n}}(r)\right)$.We allow the N_k's to vary over the $[t_k, t_{k+1}]$'s so that the h_k's may be comparable.To develop the Runge kutta method of order three for (3.1),we follow[6] and define

$$\underline{y_{k,n+1}}(r) - \underline{y_{k,n}}(r) = \sum_{i=1}^{3} w_i \underline{k_i}\left(t_{k,n}; y_{k,n}(r)\right)$$

$$\overline{y_{k,n+1}}(r) - \overline{y_{k,n}}(r) = \sum_{i=1}^{3} w_i \overline{k_i}\left(t_{k,n}; y_{k,n}(r)\right),$$

Where w_1, w_2, w_3 are constants and

$$\underline{k_1}\left(t_{k,n}; y_{k,n}(r)\right) = \min\left\{h_k f\left(t_{k,n}, u, \lambda_k(u_k)\right) \setminus u \in \left[\underline{y_{k,n}}(r), \overline{y_{k,n}}(r)\right], u_k \in \left[\underline{y_{k,0}}(r), \overline{y_{k,0}}(r)\right]\right\},$$

$$\overline{k_1}\left(t_{k,n}; y_{k,n}(r)\right) = \max\left\{h_k f\left(t_{k,n}, u, \lambda_k(u_k)\right) \setminus u \in \left[\underline{y_{k,n}}(r), \overline{y_{k,n}}(r)\right], u_k \in \left[\underline{y_{k,0}}(r), \overline{y_{k,0}}(r)\right]\right\},$$

$$\underline{k_2}\left(t_{k,n}; y_{k,n}(r)\right) = \min\left\{h_k f\left(t_{k,n} + \frac{2}{3}h_k, u, \lambda_k(u_k)\right) \setminus u \in [\underline{z_{k1}}(t_{k,n}, y_{k,n}(r)), \overline{z_{k1}}(t_{k,n}, y_{k,n}(r)), u_k \in \left[\underline{y_{k,0}}(r), \overline{y_{k,0}}(r)\right]\right\}$$

$$\overline{k_2}\left(t_{k,n}; y_{k,n}(r)\right) = \max\left\{h_k f\left(t_{k,n} + \frac{2}{3}h_k, u, \lambda_k(u_k)\right) \setminus u \in [\underline{z_{k1}}(t_{k,n}, y_{k,n}(r)), \overline{z_{k1}}(t_{k,n}, y_{k,n}(r)), u_k \in \left[\underline{y_{k,0}}(r), \overline{y_{k,0}}(r)\right]\right\}$$

$$\underline{k_3}\left(t_{k,n}; y_{k,n}(r)\right) = \min\left\{h_k f\left(t_{k,n} + \frac{2}{3}h_k, u, \lambda_k(u_k)\right) \setminus u \in [\underline{z_{k2}}(t_{k,n}, y_{k,n}(r)), \overline{z_{k2}}(t_{k,n}, y_{k,n}(r)), u_k \in \left[\underline{y_{k,0}}(r), \overline{y_{k,0}}(r)\right]\right\}$$

$$\overline{k_3}\left(t_{k,n}; y_{k,n}(r)\right) = \max\left\{h_k f\left(t_{k,n} + \frac{2}{3}h_k, u, \lambda_k(u_k)\right) \setminus u \in [\underline{z_{k2}}(t_{k,n}, y_{k,n}(r)), \overline{z_{k2}}(t_{k,n}, y_{k,n}(r)), u_k \in \left[\underline{y_{k,0}}(r), \overline{y_{k,0}}(r)\right]\right\}$$

Where in Runge kutta method of order three

$$\underline{z_{k1}}\left(t_{k,n}, y_{k,n}(r)\right) = \underline{y_{k,n}}(r) + \frac{2}{3}\underline{k_1}(t_{k,n}, y_{k,n}(r))$$

$$\overline{z_{k1}}\left(t_{k,n}, y_{k,n}(r)\right) = \overline{y_{k,n}}(r) + \frac{2}{3}\overline{k_1}(t_{k,n}, y_{k,n}(r))$$

$$\underline{z_{k2}}\left(t_{k,n}, y_{k,n}(r)\right) = \underline{y_{k,n}}(r) + \frac{2}{3}\underline{k_2}(t_{k,n}, y_{k,n}(r))$$

$$\overline{\overline{z_{k2}}}\left(t_{k,n}, y_{k,n}(r)\right) = \overline{y_{k,n}}(r) + \frac{2}{3}\overline{k_2}(t_{k,n}, y_{k,n}(r))$$

Next we define

$$S_k[t_{k,n}, \underline{y_{k,n}}(r), \overline{y_{k,n}}(r)] = 2\underline{k_1}(t_{k,n}, y_{k,n}(r)) + 3\underline{k_2}\left(t_{k,n}, y_{k,n}(r)\right) + 3\underline{k_3}\left(t_{k,n}; y_{k,n}(r)\right)$$

$$T_k[t_{k,n}, \underline{y_{k,n}}(r), \overline{y_{k,n}}(r)] = \overline{2k_1}(t_{k,n}, y_{k,n}(r)) + 3\overline{k_2}\left(t_{k,n}, y_{k,n}(r)\right) + 3\overline{k_3}\left(t_{k,n}; y_{k,n}(r)\right)$$

The exact solution at $t_{k,n+1}$ is given by

$$\begin{cases} \underline{Y_{k,n+1}}(r) \approx \underline{Y_{k,n}}(r) + \frac{1}{8} S_k[t_{k,n}, \underline{y_{k,n}}(r), \overline{y_{k,n}}(r)], \\ \overline{Y_{k,n+1}}(r) \approx \overline{Y_{k,n}}(r) + \frac{1}{8} T_k[t_{k,n}, \underline{y_{k,n}}(r), \overline{y_{k,n}}(r)] \end{cases}$$

The approximate solution is given by

$$(4.1)\begin{cases} \underline{y_{k,n+1}}(r) \approx \underline{y_{k,n}}(r) + \frac{1}{8} S_k[t_{k,n}, \underline{y_{k,n}}(r), \overline{y_{k,n}}(r)], \\ \overline{y_{k,n+1}}(r) \approx \overline{y_{k,n}}(r) + \frac{1}{8} T_k[t_{k,n}, \underline{y_{k,n}}(r), \overline{y_{k,n}}(r)] \end{cases}$$

Theorem 4.1

Consider the systems (3.2) and (4.1), for a fixed $k \in \mathbb{Z}^+$ and $r \in [0,1]$,

$$\lim_{h_0,\dots,h_k \to 0} \underline{y_{k,N_k}}(r) = \underline{x}(t_{k+1}; r)$$

$$\lim_{h_0,\dots,h_k \to 0} \overline{y_{k,N_k}}(r) = \overline{x}(t_{k+1}; r)$$

5. **Numerical example**

Before illustrating the numerical solution of a hybrid fuzzy IVP, first we recall the fuzzy IVP:

(5.1) $x'(t) = x(t)$, $x(0; r) = [0.75 + 0.25r, 1.125 - 0.125r], 0 \le r \le 1.$

The exact solution is given by

$x(t; r) = [(0.75 + 0.25r)\, e^t, (1.125 - 0.125r)e^t], 0 \le r \le 1.$ We see that

$x(1;r) = [(0.75+0.25r)e, (1.125-0.125r)e], 0 \le r \le 1.$ By the Runge kutta Nystrom method with N = 2 in [6],(5.1) gives

$y(1.0;r) = [(0.75+0.25r)(c_{0,1})^2, (1.125-0.125r)(c_{0,1})^2], 0 \le r \le 1.$where

$$c_{0,1} = 1 + 0.5 + \frac{(0.5)^2}{2} + \frac{(0.5)^3}{6}$$

Comparing the Euler in [10] and Runge kutta Nystrom method in [6] we see that Runge-kutta is much closer to the true solution.

Example 1

Next consider the following hybrid fuzzy IVP

$$(5.2) \begin{cases} x'(t) = x(t) + m(t)\lambda_k(x(t_k)), & t\epsilon[t_k, t_{k+1}], t_k = k, k = 0,1,2, \ldots\ldots\ldots. \\ x(0;r) = [0.75+0.25r, 1.125-0.125r], & 0 \le r \le 1, \end{cases}$$

where $m(t) = \begin{cases} 2(t(\text{mod } 1)), & \text{if } t(\text{mod } 1) \le 0.5 \\ 2(1-t(\text{mod } 1)), & \text{if } t(\text{mod } 1) > 0.5, \end{cases}$

$$\lambda_k(\mu) = \begin{cases} \hat{0}, & \text{if } k = 0 \\ \mu, & \text{if } k\epsilon\{1,2,\ldots\} \end{cases}$$

In(5.2) , $x(t)+m(t)\lambda_k(x(t_k))$ is a continuous function of t, x, and $\lambda_k(x(t_k))$. Therefore by Example 6.1 of Kaleva [5] and Theorem 4.2 of Buckley and Feuring [2] for each k=0,1,2,…, the fuzzy IVP

$$\begin{cases} x'(t) = x(t) + m(t)\lambda_k(x(t_k)), & t\epsilon[t_k, t_{k+1}], t_k = k, k = 0,1,2, \ldots\ldots\ldots. \\ x(t_k) = x_{tk}, \end{cases}$$

has a unique solution on $[t_k, t_{k+1}]$. To numerically solve the hybrid fuzzy IVP (5.2) we will apply the Runge–Kutta method for hybrid fuzzy differential equations from Section 4 with N=2 to obtain $y_{1,2}(r)$ approximating x(2.0;r). Let $f: [0,\infty) \times \mathcal{R} \times \mathcal{R} \to \mathcal{R}$ be given by

$f(t, x, \lambda_k(x(t_k))) = x(t) + m(t)\lambda_k(x(t_k)) \quad t_k = k, k = 1,2,3, \ldots\ldots$where $\lambda_k: \mathcal{R} \to \mathcal{R}$

is given by $\lambda_k(x) = \begin{cases} 0, & \text{if } k = 0 \\ x, & \text{if } k\epsilon\{1,2,\ldots\} \end{cases}$

Since the exact solution of (5.4) for t $\in$[1,1.5] is $x(t;r) = x(1;r)(3e^{t-1} - 2t), 0 \le r \le 1$,

$x(1.5;r) = x(1;r)(3\sqrt{e}-3), 0 \le r \le 1$.Then x(1.5;1) is approximately 5.29 and $y_{1,1}(1)$ is approximately 5.248.Since the exact solution of (5.4)for

$t \in [1.5,2]$ is $x(t;r) = x(1;r)(2t-2+e^{t-1.5}(3\sqrt{e}-4)), 0 \le r \le 1$,

$x(2.0;r) = x(1;r)(2+3e-4\sqrt{e}), 0 \le r \le 1$, Then x(2.0;1) is approximately 9.68 and $y_{1,2}(1)$ is approximately 9.65 .These observations are summarized in Table 5.1 For additional comparison, Fig 5.1 shows the graphs of x(2.0), $y_{1,2}$, and the corresponding Euler approximation.

References

[1] Abbasbandy.S and Allahviranloo.T (2002), “Numerical Solutions of fuzzy differential equations”, *Mathematical and computational applications* 7 pp 41-52

[2] Buckley J.J and Feuring T.,(2000), “Fuzzy differential equations”, *Fuzzy Sets and Systems* 110 , pp. 43–54.

[3] Friedman M.Ma and .Kandel.A, (1999) “Numerical Solutions of fuzzy differential equations,” *Fuzzy sets and systems* 105(1999),pp 133-138.

[4] Goetschel.R and Voxman.W.,(1986), “ Elementary fuzzy calculus”, *Fuzzy Sets and Systems* 18 , pp. 31–43.

[5] Kaleva.O, (1987),”Fuzzy differential equations”, *Fuzzy Sets and Systems* 24 , pp. 301–317.

[6] Kanagarajan.K and Sampath.M, (2010)“Runge kutta Nystrom method of order three for solving fuzzy differential equations”,*Computational methods in Applied Mathematics*, Vol 10(2),pp 195-203.

[7] Nirmala.V, Chenthur Pandian.S, (2011)“Numerical solution of fuzzy differential equation by fourth order runge kutta method with higher order derivative approximations” *European Journal of Scientific Research* Vol 62 No 2 pp 198-206.

[8] Nirmala.V, Saveetha.N, Chenthur Pandian.S, (2010) “Numerical solution of fuzzy differential equation by Runge kutta method with higher order derivative approximations” *Proceedings of the international conference on emerging trends in mathematics and computer applications*,MEPCO Schlenk Engineering college Sivakasi.Dec 16-18, PP 131-134(ISBN:978-81-8424-649-0)

[9] Palligkinis S.Ch, Papageorgiou.G, Famelis I.TH.,(2009) “Runge-Kutta methods for fuzzy differential equations”, *Applied Mathematics and Computation* 209 pp 97-105.

[10] Pederson.S and Sambandham.M.,(2007) “Numerical solution to hybrid fuzzy systems”, *Mathematical and Computer Modelling* 45 , pp. 1133–1144.

[11] Pederson.S and Sambandham.M.,(2006) “The Runge kutta method for hybrid fuzzy differential equations”, Nonlinear Analysis: Hybrid Systems 2 pp 626–634

[12] Sambandham.M, (2002) “Perturbed Lyaponav-like functions and hybrid fuzzy diferrential equations”,*International journal of hybrid systems* 2),pp.23-34.

[13] Saveetha.N, Nirmala.V, Chenthur Pandian.S, (2011) “The Runge kutta method of order three for hybrid fuzzy differential equations” *European Journal of Scientific Research* Vol 62 No 3 pp 335-343

[14] Seikkala.S, (1987)“On the fuzzy initial value problem”,*Fuzzy sets and systems* 24 ,pp.319-330.

[15] Wu C.-X. and M. Ma., (1991), “Embedding problem of fuzzy number space: Part I”, *Fuzzy Sets and Systems* 44 , pp. 33–38.

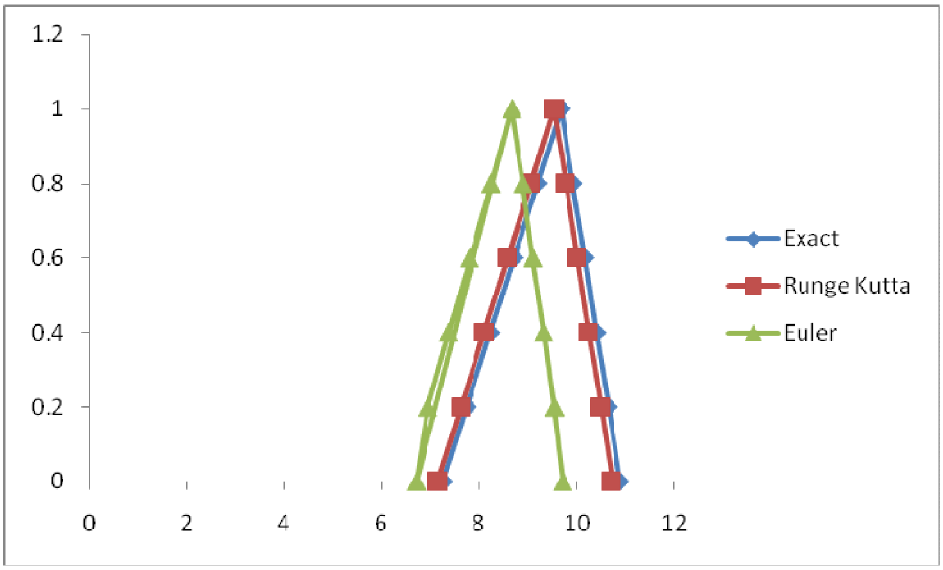

Figure 5.1. Comparison of Euler and Runge Kutta Nystrom method with the Exact Solution

Table 5.1:

Comparison of Exact and Approximate Solution

At t=1.5

r	Exact solution		Approximate solution	
	$\underline{y}$	$\overline{y}$	$\underline{y}$	$\overline{y}$
1	5.290221725	5.290221725	5.248236760	5.248236760
0.8	5.025710639	5.422477268	4.985824922	5.379442679
0.6	4.761199553	5.554732811	4.723413084	5.510648598
0.4	4.496688467	5.686988354	4.461001246	5.641854517
0.2	4.23217738	5.819243898	4.198589408	5.773060436
0	3.967666294	5.95149941	3.936177570	5.904266355

At t=2

r	Exact solution		Approximate solution	
	$\underline{y}$	$\overline{y}$	$\underline{y}$	$\overline{y}$
1	9.676975672	9.676975672	9.653510761	9.653510761
0.8	9.193126888	9.918900064	9.170835223	9.89484853
0.6	8.709278105	10.16082446	8.688159685	10.13618629
0.4	8.225429321	10.40274885	8.205484147	10.37752406
0.2	7.741580538	10.64467324	7.722808609	10.61886183
0	7.257731754	10.88659763	7.240133071	10.86019960

Error for different values of t

r	t=1	t=1.5	t=2
1	0.009514468	0.041984965	0.023464911
0.8	0.009038745	0.039885717	0.022291665
0.6	0.008563022	0.037786469	0.021118420
0.4	0.008087298	0.035687221	0.019945174
0.2	0.007611575	0.033587972	0.018771929
0	0.007135851	0.031488724	0.017598683