

On Almost π - Generalized Semi Continuous Mappings in Intuitionistic Fuzzy Topological Spaces

S.Maragathavalli¹ K. Ramesh^{2*}

1. Department of Mathematics, Sree Saraswathi Thyagaraja College, Pollachi, Tamilnadu, India
2. Department of Mathematics, SVS College of Engineering, Coimbatore, Tamilnadu, India

* E-mail of the corresponding author: rameshfuzzy@gmail.com

Abstract

In this paper we have introduced intuitionistic fuzzy almost π - generalized semi continuous mappings and intuitionistic fuzzy almost contra π - generalized semi continuous mappings and some of their basic properties are studied.

Key words: Intuitionistic fuzzy topology, intuitionistic fuzzy π -generalized semi closed set, intuitionistic fuzzy almost π - generalized semi continuous mappings and intuitionistic fuzzy almost contra π -generalized semi continuous mappings, intuitionistic fuzzy $\pi T_{1/2}$ ($IF\pi T_{1/2}$) space and intuitionistic fuzzy $\pi gT_{1/2}$ ($IF\pi gT_{1/2}$) space

1. Introduction

The concept of intuitionistic fuzzy sets was introduced by Atanassov[1] and later Coker[4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we introduce intuitionistic fuzzy almost π - generalized semi continuous mappings, intuitionistic fuzzy almost contra π - generalized semi continuous mappings and studied some of their basic properties. We provide some characterizations of intuitionistic fuzzy almost π - generalized semi continuous mappings and intuitionistic fuzzy almost contra π - generalized semi continuous mappings.

2. Preliminaries

Definition 2.1:[1] An intuitionistic fuzzy set (IFS in short) A in X is an object having the form

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$$

where the functions $\mu_A(x): X \rightarrow [0, 1]$ and $\nu_A(x): X \rightarrow [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote by $IFS(X)$, the set of all intuitionistic fuzzy sets in X.

Definition 2.2:[1] Let A and B be IFSs of the form

$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

- (a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
- (b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
- (c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
- (d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
- (e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of

$A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$. The intuitionistic fuzzy sets $0_- = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_- = \{ \langle x, 1, 0 \rangle / x \in X \}$ are respectively the empty set and the whole set of X .

Definition 2.3:[3] An intuitionistic fuzzy topology (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

- (i) $0_-, 1_- \in \tau$
- (ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$
- (iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an intuitionistic fuzzy topological space (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X . The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X .

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X . Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by

$$\text{int}(A) = \cup \{ G / G \text{ is an IFOS in } X \text{ and } G \subseteq A \},$$

$$\text{cl}(A) = \cap \{ K / K \text{ is an IFCS in } X \text{ and } A \subseteq K \}.$$

Definition 2.5:[9] A subset of A of a space (X, τ) is called:

- (i) regular open if $A = \text{int}(\text{cl}(A))$.
- (ii) $\overline{\tau}$ open if A is the union of regular open sets.

Definition 2.6:[8] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

- (i) intuitionistic fuzzy semi open set (IFSOS in short) if $A \subseteq \text{cl}(\text{int}(A))$,
- (ii) intuitionistic fuzzy α -open set (IF α OS in short) if $A \subseteq \text{int}(\text{cl}(\text{int}(A)))$,
- (iii) intuitionistic fuzzy regular open set (IFROS in short) if $A = \text{int}(\text{cl}(A))$.

The family of all IFOS (respectively IFSOS, IF α OS, IFROS) of an IFTS (X, τ) is denoted by IFO(X) (respectively IFSO(X), IF α O(X), IFRO(X)).

Definition 2.7:[8] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an

- (i) intuitionistic fuzzy semi closed set (IFSCS in short) if $\text{int}(\text{cl}(A)) \subseteq A$,
- (ii) intuitionistic fuzzy α -closed set (IF α CS in short) if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$,
- (iii) intuitionistic fuzzy regular closed set (IFRCS in short) if $A = \text{cl}(\text{int}(A))$.

The family of all IFCS (respectively IFSCS, IF α CS, IFRCS) of an IFTS (X, τ) is denoted by IFC(X) (respectively IFSC(X), IF α C(X), IFRC(X)).

Definition 2.8:[11] Let A be an IFS in an IFTS (X, τ) . Then $\text{scl}(A) = \cap \{ K / K \text{ is an IFSCS in } X \text{ and } A \subseteq K \}$.

Definition 2.9:[10] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an intuitionistic fuzzy γ closed set (IF γ CS in short) if $\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)) \subseteq A$.

Definition 2.10:[6] An IFS A of an IFTS (X, τ) is an

- (i) intuitionistic fuzzy pre closed set (IFPCS in short) if $\text{cl}(\text{int}(A)) \subseteq A$,
- (ii) intuitionistic fuzzy pre open set (IFPOS in short) if $A \subseteq \text{int}(\text{cl}(A))$.

Definition 2.11:[9] An IFS A in an IFTS (X, τ) is an

- (i) intuitionistic fuzzy generalized closed set (IFGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X ,
- (ii) intuitionistic fuzzy regular generalized closed set (IFRGCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFROS in X ,
- (iii) intuitionistic fuzzy generalized pre closed set (IFGPCS in short) if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X .

Definition 2.12:[12] An IFS A is said to be an intuitionistic fuzzy alpha generalized open set (IF α GOS in short) in X if the complement A^c is an IF α GCS in X .

The family of all IF α GCSs (IF α GOSs) of an IFTS (X, τ) is denoted by IF α GC(X) (IF α GO(X)).

Definition 2.13:[9] An IFS A in an IFTS (X, τ) is said to be an intuitionistic fuzzy τ -generalized semi closed set (IF τ GSCS in short) if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and U is an IF τ OS in (X, τ) . An IFS A is said to be an intuitionistic fuzzy τ -generalized semi open set (IF τ GSOS in short) in X if the complement A^c is an IF τ GSCS in X . The family of all IF τ GSCSs of an IFTS (X, τ) is denoted by IF τ GSC(X).

Result 2.14:[9] Every IFCS, IFGCS, IFRCS, IF α CS, IF α GCS IFGSCS is an IF τ GSCS but the converses may not be true in general.

Definition 2.15:[6] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be intuitionistic fuzzy continuous (IF continuous in short) if $f^{-1}(B) \in \text{IFO}(X)$ for every $B \in \sigma$.

Definition 2.16:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be

- (i) intuitionistic fuzzy semi continuous (IFS continuous in short) if $f^{-1}(B) \in \text{IFSO}(X)$ for every $B \in \sigma$,
- (ii) intuitionistic fuzzy α -continuous (IF α continuous in short) if $f^{-1}(B) \in \text{IF}\alpha\text{O}(X)$ for every $B \in \sigma$.

Definition 2.17:[10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy γ -continuous (IF γ continuous in short) if $f^{-1}(B) \in \text{IF}\gamma\text{CS}(X)$ for every IFCS B in Y .

Definition 2.18:[11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy generalized continuous (IFG continuous in short) if $f^{-1}(B) \in \text{IFGCS}(X)$ for every IFCS B in Y .

Definition 2.19:[11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy α -generalized semi continuous (IF α GS continuous in short) if $f^{-1}(B) \in \text{IF}\alpha\text{GSCS}(X)$ for every IFCS B in Y .

Definition 2.20:[11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy generalized semi continuous (IFGS continuous in short) if $f^{-1}(B) \in \text{IFGSCS}(X)$ for

Definition 2.21:[14] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be intuitionistic fuzzy almost continuous (IFA continuous in short) if $f^{-1}(B) \in \text{IFC}(X)$ for every IFRCB B in Y .

Definition 2.22:[11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be intuitionistic fuzzy almost α -generalized continuous (IFA α G continuous in short) if $f^{-1}(B) \in \text{IF}\alpha\text{GC}(X)$ for every IFRCB B in Y .

Definition 2.23:[2] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be intuitionistic fuzzy contra continuous (IF contra continuous in short) if $f^{-1}(B) \in \text{IFC}(X)$ for every IFOS B in Y .

Definition 2.24:[2] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be intuitionistic fuzzy contra α -continuous (IFC α continuous in short) if $f^{-1}(B) \in \text{IF}\alpha\text{C}(X)$ for every IFOS B in Y .

Definition 2.25:[11] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy contra pre-continuous (IFCP continuous in short) if $f^{-1}(B) \in \text{IFPCS}(X)$ for every IFOS B in Y .

Definition 2.26:[2] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ) . Then f is said to be an intuitionistic fuzzy contra generalized continuous (IFCG continuous in short) if $f^{-1}(B) \in \text{IFGCS}(X)$ for every IFOS B in Y .

Definition 2.27:[12] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi T_{1/2}$ (IF $\pi T_{1/2}$ in short) space if every IF π GSCS in X is an IFCS in X .

Definition 2.28: [12] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi g T_{1/2}$ (IF $\pi g T_{1/2}$ in short) space if every IF π GSCS in X is an IFGCS in X .

Result 2.29:[9] (i) Every IF π OS is an IFOS in (X, τ) ,
 (ii) Every IF π CS is an IFCS in (X, τ) .

3. Intuitionistic fuzzy almost π - generalized semi continuous mappings

In this section we introduce intuitionistic fuzzy almost π - generalized semi continuous mapping and studied some of its properties.

Definition 3.1: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy almost π - generalized semi continuous* (IFA π GS continuous in short) if $f^{-1}(B)$ is an IF π GSCS in (X, τ) for every IFRCB B in (Y, σ) .

Example 3.2: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.2, 0.2), (0.6, 0.7) \rangle$, $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFA π GS continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IFA π GS continuous mapping but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IF continuous mapping. Let A be an IFRCS in Y. This implies A is an IFCS in Y. Since f is an IF continuous mapping, $f^{-1}(A)$ is an IFCS in X. Every IFCS is an IF π GSCS, $f^{-1}(A)$ is an IF π GSCS in X. Hence f is an IFA π GS continuous mapping.

Example 3.4: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.3, 0.2), (0.6, 0.7) \rangle$, $G_2 = \langle y, (0.4, 0.2), (0.5, 0.4) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFA π GS continuous mapping. But f is not an IF continuous mapping since $G_2^c = \langle y, (0.5, 0.4), (0.4, 0.2) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.5, 0.4), (0.4, 0.2) \rangle$ is not an IFCS in X.

Theorem 3.5: Every IFS continuous mapping is an IFA π GS continuous mapping but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFS continuous mapping. Let A be an IFRCS in Y. This implies A is an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFSCS in X. Every IFSCS is an IF π GSCS, $f^{-1}(A)$ is an IF π GSCS in X. Hence f is an IFA π GS continuous mapping.

Example 3.6: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.2, 0.3), (0.5, 0.7) \rangle$, $G_2 = \langle y, (0.6, 0.8), (0.1, 0.2) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFA π GS continuous mapping. But f is not an IFS continuous mapping since $G_2^c = \langle y, (0.1, 0.2), (0.6, 0.8) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.1, 0.2), (0.6, 0.8) \rangle$ is not an IFSCS in X.

Theorem 3.7: Every IF α continuous mapping is an IFA π GS continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF α continuous mapping. Let A be an IFRCS in Y. This implies A is an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IF α CS in X. Every IF α CS is an IF π GSCS, $f^{-1}(A)$ is an IF π GSCS in X. Hence f is an IFA π GS continuous mapping.

Example 3.8: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$, $G_2 = \langle y, (0.8, 0.8), (0.2, 0.2) \rangle$ and $G_3 = \langle x, (0.4, 0.4), (0.5, 0.6) \rangle$. Then $\tau = \{ 0_-, G_1, G_2, 1_- \}$ and $\sigma = \{ 0_-, G_3, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is IFA π GS continuous mapping but not an IF α continuous mapping. The IFS $G_3^c = \langle y, (0.5, 0.6), (0.4, 0.4) \rangle$ is IFRCS in Y but $f^{-1}(G_3^c) = \langle x, (0.5, 0.6), (0.4, 0.4) \rangle$ is not an IF α CS in X.

Theorem 3.9: Every IF α G continuous mapping is an IFA π GS continuous but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IF α G continuous mapping. Let A be an IFRCS in Y. This implies A is an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IF α GCS in X. Since every IF α GCS is an IFGSCS and every IFGSCS is an IF π GSCS, $f^{-1}(A)$ is an IF π GSCS in X. Hence f is an IFA π GS continuous mapping.

Example 3.10: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.7, 0.6), (0.3, 0.4) \rangle$, $G_2 = \langle y, (0.3, 0.4), (0.4, 0.2) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFA π GS continuous mapping but not an IF α G continuous mapping since $G_2^c = \langle y, (0.4, 0.2), (0.3, 0.4) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.4, 0.2), (0.3, 0.4) \rangle$ is not an IF α GCS in X.

Theorem 3.11: Every IFG continuous mapping is an IFA \mathbb{T} GS continuous but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFG continuous mapping. Let A be an IFRCS in Y. This implies A is an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFGCS in X. Since every IFGCS is an IF \mathbb{T} GS, f is an IFA \mathbb{T} GS continuous mapping.

Example 3.12: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.7, 0.6), (0.3, 0.4) \rangle$, $G_2 = \langle y, (0.3, 0.4), (0.4, 0.2) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFA \mathbb{T} GS continuous mapping. But f is not an IFG continuous mapping since $G_2^c = \langle y, (0.4, 0.2), (0.3, 0.4) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.4, 0.2), (0.3, 0.4) \rangle$ is not an IFGCS in X.

Theorem 3.13: Every IFGS continuous mapping is an IFA \mathbb{T} GS continuous but not conversely.

Proof: Assume that $f : (X, \tau) \rightarrow (Y, \sigma)$ is an IFGS continuous mapping. Let A be an IFRCS in Y. This implies A is an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFGSCS in X. Since every IFGSCS is an IF \mathbb{T} GS, f is an IFA \mathbb{T} GS continuous mapping.

Example 3.14: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.5, 0.6), (0.2, 0.2) \rangle$, $G_2 = \langle y, (0.6, 0.6), (0.3, 0.2) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFA \mathbb{T} GS continuous mapping but f is not an IFGSC continuous mapping since $G_2^c = \langle y, (0.3, 0.2), (0.6, 0.6) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.3, 0.2), (0.6, 0.6) \rangle$ is not an IFGSCS in X.

Theorem 3.15: Every IFA continuous mapping is an IFA \mathbb{T} GS continuous mapping but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFA continuous mapping. Let A be an IFRCS in Y. Since f is IFA continuous mapping, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IF \mathbb{T} GS, $f^{-1}(A)$ is an IF \mathbb{T} GS in X. Hence f is an IFA \mathbb{T} GS continuous mapping.

Example 3.16: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.3, 0.2), (0.6, 0.7) \rangle$, $G_2 = \langle y, (0.4, 0.2), (0.5, 0.4) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFA \mathbb{T} GS continuous mapping but f is not an IFA continuous mapping since $G_2^c = \langle y, (0.5, 0.4), (0.4, 0.2) \rangle$ is an IFRCS in Y but $f^{-1}(G_2^c) = \langle x, (0.5, 0.4), (0.4, 0.2) \rangle$ is not an IFCS in X.

Theorem 3.17: Every IFA α G continuous mapping is an IFA \mathbb{T} GS continuous but not conversely.

Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFA α G continuous mapping. Let A be an IFRCS in Y. Since f is IFA continuous mapping, Then by hypothesis $f^{-1}(A)$ is an IF α GCS in X. Since every IF α GCS is an IFGSCS and every IFGSCS is an IF \mathbb{T} GS, $f^{-1}(A)$ is an IF \mathbb{T} GS in X. Hence f is an IFA \mathbb{T} GS continuous mapping.

Example 3.18: Let $X = \{ a, b \}$, $Y = \{ u, v \}$ and $G_1 = \langle x, (0.8, 0.9), (0, 0.1) \rangle$, $G_2 = \langle y, (0, 0.3), (0.7, 0.7) \rangle$. Then $\tau = \{ 0_-, G_1, 1_- \}$ and $\sigma = \{ 0_-, G_2, 1_- \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFA \mathbb{T} GS continuous mapping. But f is not an IFA α G continuous mapping since $G_2^c = \langle y, (0.7, 0.7), (0, 0.3) \rangle$ is an IFRCS in Y but $f^{-1}(G_2^c) = \langle x, (0.7, 0.7), (0, 0.3) \rangle$ is not an IF α GCS in X.

Theorem 3.19: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping from an IFTS X into an IFTS Y . Then the following conditions are equivalent if X is an $IF\tau T_{1/2}$ space.

- (i) f is an IFA τ GS continuous mapping.
- (ii) If B is an IFROS in Y then $f^{-1}(B)$ is an IF τ GSOS in X .
- (iii) $f^{-1}(B) \subseteq \text{int}(\text{cl}(f^{-1}(B)))$ for every IFROS B in Y .

Proof: (i) \Rightarrow (ii): obviously.

(ii) \Rightarrow (iii): Let B be any IFROS in Y . Then by hypothesis $f^{-1}(B)$ is an IF τ GSOS in X . Since X is an $IF\tau T_{1/2}$ space, $f^{-1}(B)$ is an IFOS in X (Result 2.29). Therefore $f^{-1}(B) = \text{int}(f^{-1}(B)) \subseteq \text{int}(\text{cl}(f^{-1}(B)))$.

(iii) \Rightarrow (i): Let B be an IFRCS in Y . Then its complement B^c is an IFROS in Y . By hypothesis $f^{-1}(B^c) \subseteq \text{int}(\text{cl}(f^{-1}(B^c)))$. Hence $f^{-1}(B^c)$ is an IF τ OS in X . Since every IF τ OS is an IF τ GSOS, $f^{-1}(B^c)$ is an IF τ GSOS in X . Therefore $f^{-1}(B)$ is an IF τ GSCS in X . Hence f is an IFA τ GS continuous mapping.

Theorem 3.20: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a mapping. Then the following conditions are equivalent if X is an $IF\tau T_{1/2}$ space.

- (i) f is an IFA τ GS continuous mapping.
- (ii) $\text{int}(\text{cl}(f^{-1}(A))) \subseteq f^{-1}(A)$ for every IFRCS A in Y .

Proof: (i) \Rightarrow (ii): Let A be an IFRCS in Y . By hypothesis, $f^{-1}(A)$ is an IF τ GSCS in X . Since X is an $IF\tau T_{1/2}$, $f^{-1}(A)$ is an IFCS in X (Result 2.29). Therefore $\text{cl}(f^{-1}(A)) = f^{-1}(A)$. Now $\text{int}(\text{cl}(f^{-1}(A))) \subseteq \text{cl}(f^{-1}(A)) \subseteq f^{-1}(A)$.

(ii) \Rightarrow (i): Let A be an IFRCS in Y . By hypothesis $\text{int}(\text{cl}(f^{-1}(A))) \subseteq f^{-1}(A)$. This implies $f^{-1}(A)$ is an IF τ CS in X and hence $f^{-1}(A)$ is an IF τ GSCS in X . Therefore f is an IFA τ GS continuous mapping.

Theorem 3.21: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF τ GS continuous mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is IFA continuous, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFA τ GS continuous mapping.

Proof: Let A be an IFRCS in Z . Then $g^{-1}(A)$ is an IFCS in Y . Since f is an IF τ GS continuous mapping, $f^{-1}(g^{-1}(A))$ is an IF τ GSCS in X . Hence $g \circ f$ is an IFA τ GS continuous mapping.

4. Intuitionistic fuzzy almost contra τ - generalized semi continuous mappings

In this section we introduce intuitionistic fuzzy almost contra τ - generalized semi continuous mappings and studied some of its properties.

Definition 4.1: A mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ is called an *intuitionistic fuzzy almost contra τ - generalized semi continuous mappings* (IFAC τ GS continuous in short) if $f^{-1}(B)$ is an IF τ GSCS in (X, τ) for every IFROS B of (Y, σ) .

Example 4.2: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.2, 0.2), (0.6, 0.7) \rangle$, $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7) \rangle$. Then $\tau = \{0, G_1, 1\}$ and $\sigma = \{0, G_2, 1\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFAC τ GS continuous mapping.

Theorem 4.3: Every IF contra continuous mapping is an IFAC τ GS continuous mapping but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF contra continuous mapping. Let A be an IFROS in Y . This implies A

is an IFOS in Y. Since f is IF contra continuous mapping, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IF τ GSCS, $f^{-1}(A)$ is an IF τ GSCS in X. Hence f is an IFAC τ GS continuous mapping.

Example 4.4: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.3, 0.2), (0.6, 0.7) \rangle$, $G_2 = \langle y, (0.4, 0.2), (0.5, 0.4) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFAC τ GS continuous mapping but not an IF contra continuous mapping since $G_2 = \langle y, (0.4, 0.2), (0.5, 0.4) \rangle$ is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.5, 0.4) \rangle$ is not an IFCS in X.

Theorem 4.5: Every IFCGS continuous mapping is an IFAC τ GS continuous but not conversely.

Proof: Assume that $f: (X, \tau) \rightarrow (Y, \sigma)$ is an IFCGS continuous mapping. Let A be an IFROS in Y. This implies A is an IFOS in Y. Since f is IFCGS continuous mapping, $f^{-1}(A)$ is an IFGSCS in X. Since every IFGSCS is an IF τ GSCS, $f^{-1}(A)$ is an IF τ GSCS in X. Hence f is an IFAC τ GS continuous mapping.

Example 4.6: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.5, 0.6), (0.2, 0.2) \rangle$, $G_2 = \langle y, (0.3, 0.2), (0.6, 0.6) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is an IFAC τ GS continuous mapping but f is not an IFGSCS continuous mapping since $G_2 = \langle y, (0.3, 0.2), (0.6, 0.6) \rangle$ is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.3, 0.2), (0.6, 0.6) \rangle$ is not an IFGSCS in X.

Theorem 4.7: Every IFC α continuous mapping is an IFAC τ GS continuous mapping but not conversely.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFC α continuous mapping. Let A be an IFROS in Y. This implies A is IFOS in Y. Then by hypothesis $f^{-1}(A)$ is an IF α CS in X. Since every IF α CS is an IF τ GSCS, $f^{-1}(A)$ is an IF τ GSCS in X. Hence f is an IFAC τ GS continuous mapping.

Example 4.8: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$, $G_2 = \langle x, (0.8, 0.8), (0.2, 0.2) \rangle$ and $G_3 = \langle y, (0.4, 0.4), (0.5, 0.6) \rangle$. Then $\tau = \{0_-, G_1, G_2, 1_-\}$ and $\sigma = \{0_-, G_3, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is IFAC τ GS continuous mapping but not an IFC α continuous mapping since $G_3 = \langle y, (0.4, 0.4), (0.5, 0.6) \rangle$ is an IFROS in Y but $f^{-1}(G_3) = \langle x, (0.4, 0.4), (0.5, 0.6) \rangle$ not IF α CS in X.

Remark 4.9: IFCP continuous mapping and IFAC τ GS continuous mapping are independent to each other.

Example 4.10: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$, $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7) \rangle$. Then $\tau = \{0_-, G_1, 1_-\}$ and $\sigma = \{0_-, G_2, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is IFAC τ GS continuous mapping but not an IFCP continuous mapping since $G_2 = \langle y, (0.4, 0.2), (0.6, 0.7) \rangle$ is an IFOS in Y but $f^{-1}(G_2) = \langle x, (0.4, 0.2), (0.6, 0.7) \rangle$ is not an IFPCS in X.

Example 4.11: Let $X = \{a, b\}$, $Y = \{u, v\}$ and $G_1 = \langle x, (0.2, 0.4), (0.5, 0.4) \rangle$, $G_2 = \langle x, (0.1, 0.3), (0.3, 0.4) \rangle$, $G_3 = \langle x, (0.1, 0.3), (0.5, 0.4) \rangle$, $G_4 = \langle x, (0.2, 0.4), (0.3, 0.4) \rangle$, $G_5 = \langle x, (0.4, 0.4), (0.3, 0.4) \rangle$, $G_6 = \langle y, (0, 0.3), (0.5, 0.4) \rangle$. Then $\tau = \{0_-, G_1, G_2, G_3, G_4, G_5, 1_-\}$ and $\sigma = \{0_-, G_6, 1_-\}$ are IFTs on X and Y respectively. Define a mapping $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = u$ and $f(b) = v$. Then f is IFP continuous mapping but not an IFAC τ GS continuous mapping since $G_6 = \langle y, (0, 0.3), (0.5, 0.4) \rangle$ is an IFROS in Y but $f^{-1}(G_6) = \langle x, (0.0, 0.3), (0.5, 0.4) \rangle$ is not an IF τ GSCS in X.

Theorem 4.12: If a bijection mapping $f: X \rightarrow Y$ is IFAC π GS continuous then the inverse image of each IFRCs in Y is an IF π GSOS in X .

Proof: Let A be an IFRCs in Y . This implies A^c is IFROS in Y . Since f is IFAC π GS continuous, $f^{-1}(A^c)$ is IF π GSCS in X . Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IF π GSOS in X .

Theorem 4.13: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IFAC π GS continuous mapping, then f is an IFACGS continuous mapping if X is an IF π g $T_{1/2}$ space.

Proof: Let A be an IFROS in Y . Then $f^{-1}(A)$ is an IF π GSCS in X , by hypothesis. Since X is an IF π g $T_{1/2}$ space, $f^{-1}(A)$ is an IFGSCS in X . Hence f is an IFACGS continuous mapping.

Theorem 4.14: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an IF π GS continuous mapping and $g: (Y, \sigma) \rightarrow (Z, \delta)$ is IFA continuous, then $g \circ f: (X, \tau) \rightarrow (Z, \delta)$ is an IFAC π GS continuous.

Proof: Let A be an IFROS in Z . Then $g^{-1}(A)$ is an IFCS in Y , by hypothesis. Since f is an IF π GS continuous mapping, $f^{-1}(g^{-1}(A))$ is an IF π GSCS in X . Hence $g \circ f$ is an IFAC π GS continuous mapping.

4. CONCLUSION

In this paper we have introduced intuitionistic fuzzy almost π -generalized semi continuous mappings and studied some of its basic properties. Also we have studied the relationship between intuitionistic fuzzy generalized semi continuous mappings and some of the intuitionistic fuzzy continuous mappings already exist.

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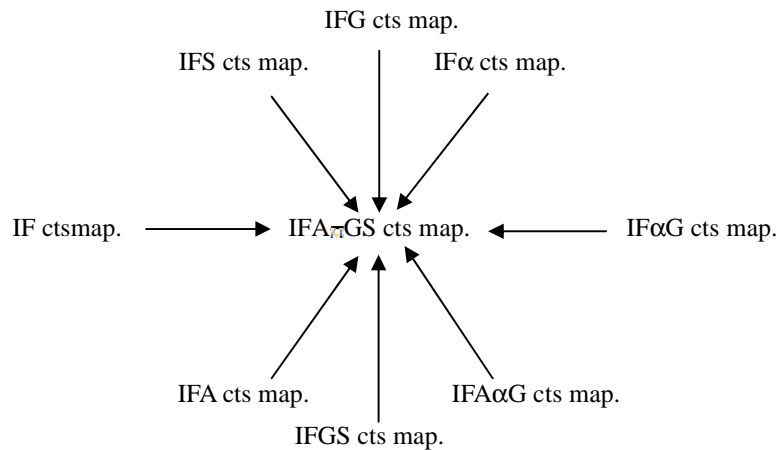
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Figure -1

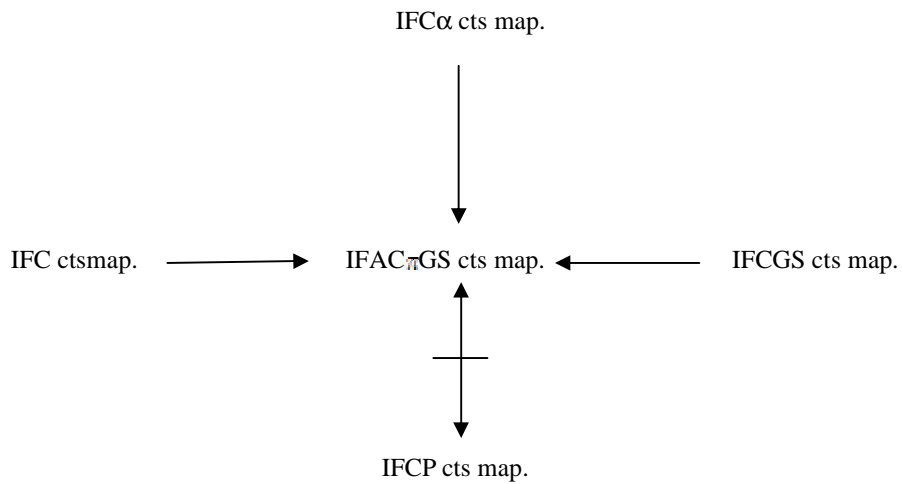
The relations between various types of intuitionistic fuzzy continuity are given in the following diagram. In this diagram ‘cts map.’ means continuous mapping.



In this diagram by “A \longrightarrow B” we mean A implies B but not conversely.

Figure -2

The relations between various types of intuitionistic fuzzy contra continuity are given in the following diagram. In this diagram 'cts map.' means continuous mapping.



In this diagram by "A \longrightarrow B" we mean A implies B but not conversely and

"A \longleftrightarrow B" means A and B are independent of each other.

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