Effect of heat transfer on the peristaltic transport of MHD with couple-stress fluid through a porous medium with slip effect

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Abstract

This paper concern with the Peristaltic transport of MHD Newtonian fluid in asymmetric two dimensional channel with couple –stress through a porous medium under the influence of heat transfer analysis. For the formulation of the problem long wave length and low Reynold number assumption is taken into account. An exact solution is presented for velocity field and the temperature field through solving a non-homogenies partial differential equation that described the flow field. Effects of different physical parameters, reflecting couple-stress parameter, permeability parameter, slip parameter, Hartman number, constant heat radiation factor and Grashof number, as well as amplitude ration on pumping characteristics and frictional force, stream lines pattern and trapping of peristaltic flow pattern studied with particular emphasis. This study are discussed through graphs.

Keywords: Peristaltic Transport, Couple-Stress, Magnetic Field, Newtonian Fluid, Porous Medium, Reynolds Number, Heat transfer.

1 Introduction

The investigation of peristaltic pumping is a form of fluid transport which occurs in a biological systems. This mechanisms of fluid transport has received considerable attention in recent times in engineering as well as in physiological sciences. The physiological phenomena, like urine transport from kidney to bladder through the ureter, movement of chime in the gastrointestinal tract, the movement of spermatozoa in the ducts afferents of the male reproductive tract and the ovum in the female fallopian tube, the locomotion of some warms, transport of lymph in the lymphatic vessels and vasomotor of small blood vessels such as arterioles, venules and capillaries involves the peristaltic motion. In addition peristaltic pumping occurs in many practical applications involving biomechanical systems. Also, finger and roller pumping are frequently used for pumps corrosive or very pure materials so as to prevent direct contact of the fluid with the pumps internal surfaces. Moreover, by using the principle of peristalsis, some biomechanical instruments, e.g., heart-lung machine have been fabricated. The results of some investigations according to different flow geometries are studied in (Pozrikidis,1987,Eytan,1999,Hayat, et.al. 2003,Haroun,2007,Herwig,1986). The effects of magnetic field on the peristaltic mechanisms is important in connection with certain problems of the movement of the conductive physiological fluids, e.g, the blood pump machines. A number of researchers have been discussed the effects of magnetic field on the peristaltic flow (Mekheimer,2003,Hayat, et.al.2007,Hayat, et.al. 22006, Hayat, et.al . 2005). Flow through a porous medium has been of considerable interest in recent years, number of workers employing Darcy's law. (Rapit et al,1982, and Varshney,1979) have solved problems of the flow of a viscous fluid through a porous medium boundary by vertical surface. (Mekheimer and Al-Arabi, 2003), studied nonlinear peristaltic transport of MHD flow through a porous medium also(Mekheimer,2003) studied nonlinear peristaltic transport through a porous medium in an inclined planar channel. (Mekheimer and Abd Elmaboud,2008) investigated the peristaltic flow through a porous medium in an annulus. Some recent studies(Srinivas,et.al. 2009, ,Kothandopani,2008) take the effect of wall properties, heat transfer and magnetic field. The study of couple stress fluid is very usefull in understanding various physical problems because it possesses the mechanism to describe rheological complex fluids such as liquid crystals and human blood. By couple stress fluid, we mean a fluid whose particles size are taken into account, a special case of non-Newtonion fluids. Some of recent studies on peristaltic transport of couple stress fluid have been done by Srivastava(1986), Elshehawey and Mekheimer (1994),Elsoud et al. (1998), Elshehehawey and El-Sebaei(2001) and Ali et al.(2007). Some of the studies on couple stress fluid just mentioned considered the blood as a couple stress fluid and they were carried out using no slip condition, although in real systems there is always a certain amount on slip. However, the interaction of peristalsis with heat transfer has not received much attention. The thermo dynamical aspects of blood may not be important when blood is inside the body but they become significant when it is drawn out of the body. Keeping in view the significance of heat transfer in blood flow, (Victor and Shah,1976) studied the thermo dynamical aspects of blood flowing in a tube treating blood as Casson fluid.(Agrawal,1982) analyzed the heat transfer to pulsatile flow of a conducting fluid through a porous channel in the presence of magnetic field. All the above investigation on peristaltic transport have been done taking into account the classical no slip boundary condition. However, in several applications, the flow pattern corresponds to a slip flow and the fluid present a loss of adhesion at the wetted wall making the fluid slide along the wall. Beaver and Joseph interface between a porous medium and fluid layer in an experimental study and proposed a slip boundary condition at the interface. Flows with slip would be useful for problems in chemical engineering, for example flows through pipe in which chemical reactions occur at the walls, two-phase flows in porous slider bearings. (Saffman,1971) proposed an improved slip boundary condition. (Terrill,1984) discussed the laminar flow a through a porous pipe with slip and(Hummady and Abdulhadi,2014) discussed Influence of MHD on peristaltic flow of couple-stress fluid through a porous medium with slip effect . In view of the above discussion, in this paper, effect of heat transfer for a Newtonian fluid with couple stress in MHD field through a porous medium with slip condition under the assumptions of long wave length and low Reynolds number. The closed form solutions of velocity field and magnetic field are obtained. The influence of various pertinent parameters on the flow characteristics, this study are discussed through graphs.

2 Mathematical Formulation and Analysis

Consider the two dimensional flow of a Newtonian incompressible viscous fluid in a uniform channel with heat transfer. The fluid is electrically conducting in the presence of uniform magnetic field B_o, which is applied in the transverse direction to the flow. The electric field is taken to be zero. The magnetic Reynold number is taken to be very small and the induced magnetic field neglected. We select rectangular coordinate system in a such a way that $\bar{X}$ -axis lies along the center line of the channel and $\bar{Y}$ –axis normal to it. Since we are considering uniform channel therefor the upper wall is maintained at temperature T_1 and due to symmetry at the center of the channel the change of the temperature is taken to be zero. We assume an infinite wave train travelling with velocity c along the walls. The geometry of the wall surface is presented in Figure1.

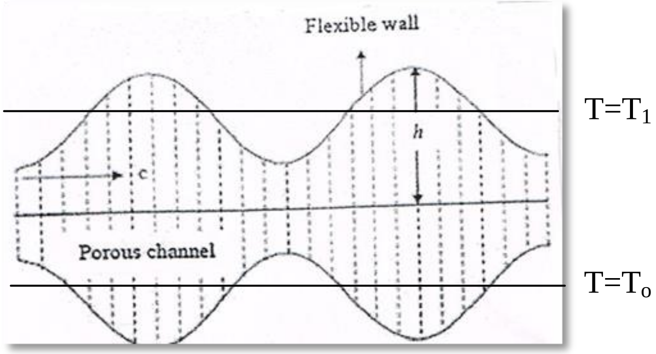

Figure1.Geometry of the problem

$$\bar{h}(\bar{X},\bar{t}) = a + b\sin\frac{2\pi}{\lambda}(\bar{x} - c\,\bar{t}) \quad (1)$$

where a, b, λ , x, c, and t are the half width of the channel, amplitude, wavelength, axial coordinate, wave velocity, and time respectively. The equation of motion for incompressible couple- stress fluid in porous medium in the present of magnetic field are :

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \quad (2)$$

$$\rho\left(\frac{\partial\bar{u}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{u}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{u}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{x}} + \mu\nabla^2\bar{u} - \mu_1\nabla^4\bar{u} - \mu\frac{\bar{u}}{\bar{k}} - \sigma\beta_o^2\,\bar{u} \quad (3)$$

$$\rho\left(\frac{\partial\bar{v}}{\partial\bar{t}} + \bar{u}\frac{\partial\bar{v}}{\partial\bar{x}} + \bar{v}\frac{\partial\bar{v}}{\partial\bar{y}}\right) = -\frac{\partial\bar{p}}{\partial\bar{y}} + \mu\nabla^2\bar{v} - \mu_1\nabla^4\bar{v} - \mu\frac{\bar{v}}{\bar{k}} \quad (4)$$

$$\rho\, c_p\left(\frac{\partial}{\partial\bar{t}} + \bar{u}\frac{\partial}{\partial\bar{x}} + \bar{v}\frac{\partial}{\partial\bar{y}}\right)\bar{T} = k\left(\frac{\partial^2}{\partial\bar{t}^2} + \frac{\partial^2}{\partial\bar{x}^2} + \frac{\partial^2}{\partial\bar{y}^2}\right)\bar{T} + Q_o \quad (5)$$

where the $\rho, \bar{u}$,$\bar{u}, \bar{v}$, $\bar{y}$, $\bar{p}, \mu, \mu_1$, $\bar{k}$, β_o ,c_p, k, $\bar{T}$, Q_o are the fluid density ,axial velocity , transverse velocity, transverse coordinate , pressure, viscosity , material constant associated with couple-stress, permeability parameter, magnetic field, specific heat at constant pressure, thermal conductivity, temperature and constant heat addition/absorption, respectively. And

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \quad \nabla^4 = \nabla^2(\nabla^2)$$

Introducing the following dimensionless parameters

$$x = \frac{\bar{x}}{\lambda}, y = \frac{\bar{y}}{a}, u = \frac{\bar{u}}{c}, \ v = \frac{\bar{v}}{c\delta}, \phi = \frac{a}{b}, h = \frac{\bar{h}}{a} = 1 + \phi\sin(2\pi x)$$

$$P = \frac{\bar{p}a^2}{\mu c\lambda}, k^* = \frac{\bar{k}}{a^2}, \quad Re = \frac{\rho c a}{\mu}, \quad \delta = \frac{a}{\lambda}, \alpha = a\sqrt{\frac{\mu}{\mu 1}}, Ha = \sqrt{\frac{\sigma}{\mu}}\, a\, B_o$$

$$\theta = \frac{(\bar{T}-\bar{T}_0)}{(\bar{T}_1-\bar{T}_0)},\ B = \frac{Q_0 a^2}{k(\bar{T}_1-\bar{T}_0)},\ G_r = \frac{p\alpha g a^2(\bar{T}_1-\bar{T}_0)}{\mu c},\ p_r = \frac{\mu c_p}{k}, \quad (6)$$

where δ, ε, ϕ, Re, α, Ha, θ, B, G_r, p_r, are the wave number, ratio of the width of channels, amplitude ratio, Reynolds number, couple – stress parameter, Hartman parameter, temperature distribution, constant heat radiation, Grashof number, Prandtle number, respectively. Using the set of non- dimensional variables and parameters (5) in Eqs. (2-5) applying the long wavelength and low Reynolds number approximation we get.

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (7)$$

$$\frac{\partial p}{\partial x} = \frac{\partial^2 u}{\partial y^2} - \frac{1}{\alpha^2}\frac{\partial^4 u}{\partial y^4} - \frac{u}{k} - \mathrm{Ha}\, u + G_r\theta \quad (8)$$

$$\frac{\partial p}{\partial y} = 0 \quad (9)$$

$$\frac{\partial^2 \theta}{\partial y^2} + B = 0 \quad (10)$$

The associated boundary conditions are

Slip condition : $u = -\beta\frac{\partial u}{\partial y}$ at $y=h$ (11)

Where β is the slip parameter,

Regularity condition: $\frac{\partial u}{\partial y} = 0$ at $y = 0$ (12)

Vanishing of couple stresses, $\frac{\partial^2 u}{\partial y^2} = 0$ at $y = h$, $\frac{\partial^3 u}{\partial y^3} = 0$ at $y = 0$. (13)

The heat transfer condition, $\frac{\partial \theta}{\partial y} = 0$, at $y = 0$, $\theta = 1$ at $y = h$ (14)

The solution of Eq.(10) subject to the associated boundary conditions (14) is found of the form

$$\theta = 1 - \frac{B}{2}(y^2 - h^2) \quad (15)$$

Substituted in Eq.(8) and the solution of Eq.(8) subject to the associated boundary condition (11-13) is found of the form

$$u = -\left(\frac{\left(\frac{\partial p}{\partial x} - G_r\right)}{\alpha^2 N} + \frac{B G_r}{\alpha^2 N^2}\right)\left[1 + f\left\{\frac{m_2^2 \cosh(m_1 y)}{\cosh(m_1 h)} - \frac{m_1^2 \cosh(m_2 y)}{\cosh(m_2 h)}\right\}\right] + \frac{G_{rB}}{2\alpha^2 N}(h^2 - y^2) \quad (16)$$

where

$$m_1 = \sqrt{\frac{\alpha^2 + \sqrt{\alpha^2 - 4N}}{2}} \quad (17)$$

$$m_2 = \sqrt{\frac{\alpha^2 - \sqrt{\alpha^2 - 4N}}{2}} \tag{18}$$

$$N = \frac{1}{k} + (Ha)^2 , \tag{19}$$

$$f = \frac{1}{m_1^2 - m_2^2} \left\{1 + \frac{\beta f_1}{m_1^2 - m_2^2} \right\} , \tag{20}$$

$$f_1 = \frac{m_2^2}{m_1} \tanh(m_1 h) - \frac{m_1^2}{m_2} \tanh(m_2 h) + \frac{G_r B h^3}{3\alpha^2 N} \tag{21}$$

2.1 Volume Flow Rate

In laboratory frame $(\bar{x}, \bar{y})$ the flow is unsteady. However if observed in a coordinate moving at the wave speed c (wave frame) $(\bar{X}, \bar{Y})$it can be treated as steady. The coordinate frame are related in the following

$$\bar{X} = \bar{x} - ct , \ \bar{Y} = \bar{y} , \bar{U} = \bar{u} - c, \ \bar{V} = \bar{v}, \tag{22}$$

Where $(\bar{U}, \bar{V})$ and $(\bar{u}, \bar{v})$ are the velocity components in the wave and fixed frames, respectively.

The dimensional volume flow rate in the laboratory frame is

$$Q_1 = \int_0^{\bar{h}(\bar{x}, \bar{y})} u(\bar{x}, \bar{y}, \bar{t}) \ d\bar{y} \tag{23}$$

Where $\bar{h}$ is a function of x , t. Eq.(23) in the wave frame can be expressed as

$$q_1 = \int_0^{\bar{h}(x)} \bar{U} (\bar{X}, \bar{Y}) d\bar{Y} \tag{24}$$

In which $\bar{h}$ is a function at $\bar{x}$ only . By using Eqs. (22), (23) and (24) we have

$$Q_1 = q_1 + ch \tag{25}$$

Averaging volume flow rate along a period T, we have

$$\bar{Q}_1 = \frac{1}{T} \int_0^T Q_1 dt. = q_1 + ca \tag{26}$$

Now, introducing the dimensionless mean flow Q in the laboratory frame

$$\bar{Q} = \frac{Q_1}{ca} , q = \frac{q_1}{cq} \tag{27}$$

Eq.(26) can be written as

$$\bar{Q} = q + 1 = Q + 1 - h \tag{28}$$

Here the dimensionless from at Eq.(18) has been used. The dimensionless form at Eq.(17) is

$$Q = \int_0^h u dy \tag{29}$$

Invoking Eq.(12) into Eq.(24) and then integrating one can write

$$Q = -\left(\frac{\left(\frac{\partial p}{\partial x} - G_r\right)}{\alpha^2 N} + \frac{B G_r}{\alpha^2 N^2} \right)(h + f * f1) \tag{30}$$

Now, the substitution of Eq.(26) into(28) gives

$$\frac{\partial p}{\partial x} = - \alpha^2 N \left\{ \frac{\bar{Q} - 1 + h}{h + f * f1} - \frac{G_r B}{N} + G_r \right\} \tag{31}$$

The pressure difference (Δp) and frictional force (F), respectively, across the one wavelength, are given by

$$\Delta p = \int_0^1 \frac{\partial p}{\partial x}\, dx, \tag{32}$$

$$F = \int_0^1 h\left(-\frac{\partial p}{\partial x}\right) dx. \tag{33}$$

From Eqs.(15), (29) ,and using transformation of Eq.(21),the stream function in wave form $U=\frac{\partial \varphi}{\partial y}$ and $V=-\frac{\partial \varphi}{\partial x}$ is obtained as

$$\varphi(x,y) = \frac{\bar{\vartheta}-1+h}{h+f(x)f_1(x)}\left[y + f(x)\left\{\frac{{m_2}^2 \sinh(m_1 y)}{m_1 \cosh(m1h)} - \frac{{m_1}^2 \sinh(m_2 y)}{m_2 \cosh(m_2 h)}\right\}\right] + \left(\frac{G_r B}{2\alpha^2 N} - 1\right) y - \left(\frac{G_r B}{6\alpha^2} y^3\right).$$
(34)

2.2 Mechanical Efficiency

Mechanical efficiency is the ratio of the average rate per wavelength at which work is done by the moving fluid against a pressure head and the average rate at which the walls do work on the fluid. It is derived as

$$E = -\frac{\bar{Q}\,\Delta P}{\phi\, I}, \tag{35}$$

Where

$$I = \int_0^1 \frac{\partial P}{\partial x} \sin(2\pi x)\, dx. \tag{36}$$

3 Numerical Results and Discussion

In this section , the numerical and computational result are discussed through the graphical illustration. Mathematica software is used to find out numerical results and illustration. The salient feature of uniform peristaltic flow of couple-stress fluids through the porous medium in magnetic felid with effect heat transfer are discussed through Figures.(2,3,4) and (5). Based on Eq.29 Fig.(2a-g) is drawn between pressure difference across one wavelength and averaged flow rate. The variation of the volumetric flow rate of peristaltic waves with pressure gradient for different values of the couple –stress parameter, with Hartman parameter ,grashof parameter, constant of heat parameter, permeability parameter, slip parameter, and amplitude ratio are studied through these figures. These figures demonstrate that here is a linear relation between pressure and average flow rate. On the basis of the values of pressure gradient, different regions examined in this study, the region for $\Delta p > 0$ is entire pumping region, the region for $\Delta p = 0$ is free pumping zone and the for $\Delta p < 0$ is co-pumping region. In figure(2) we can see that form in Fig.(2a) with increases the grashof parameter, the volumetric flow rate gradually increasing in the entire pumping region , free pumping region and co-pumping region. (2b) with increases the constant heat parameter, the volumetric flow rate gradually decreasing in the entire pumping region , free pumping region and co-pumping region. Fig.(2c) with increases the Hartman parameter, the volumetric flow rate gradually increasing in the entire pumping region and in free pumping region with co-pumping region the volumetric gradually decreasing . (2d) shows that, with increases the couple –stress parameter, the volumetric flow rate gradually reduce in the entire pumping and free pumping but the volumetric flow rate decreasing in co-pumping. Fig.(2e) we see that, with increases the permeability parameter, the volumetric flow rate can be gradually reduced in the pumping region and the free pumping region but decreasing in co-pumping region. Fig.(2f) shows that, with the rise in the magnitude of slip parameter, the volumetric flow rate identical with the same distance between of them in the pumping region and in free pumping and co-pumping. Fig.(2g) shows that, when the magnitude of amplitude ratio decreases, the volumetric flow rate decreasing in the pumping region and free pumping region but in co-pumping region the volumetric flow rate increasing. Frictional force (F) in the case of couple-stress fluid with magnetic felid is calculated over one wave period in the term of averaged volume flow rate. Fig.(3a-g) is illustrated to show the variation of frictional force with averaged flow rate for different values pertinent parameters. It can be seen that the effect of increasing the flow rate is to enhance the frictional force. In fig. Fig.(3a) shows that, with the rise in the grashof parameter, the volumetric flow rate increases with the same distance between of them in the pumping region.(3b) shows that, with the rise in the constant heat parameter, the volumetric flow rate decreases with the same distance between of them in the pumping region and free pumping with co-pumping. (3c) we can see that with increasing the magnitude of Hartman parameter, identical the volumetric flow rate, as well as,Fig.(3d-f) shows that frictional force enhances with rise in couple- stress parameter, permeability parameter, slip parameter, identical the volumetric flow rate. Fig.(3g), with increases the magnitude of amplitude ratio parameter the magnitude of fractional force decreasing for $Q1<0.3$ and increasing for $Q1>0.3$. In Fig. (4a-g) we draw graphs between mechanical efficiency E and the averaged flow rate to study the variations of mechanical efficiency for different maximum value and decreases to zero. It is found that the efficiency increases with increasing the magnitude of grashof parameter, constant heat parameter, and identical with increases Hartman parameter, couple- stress parameter , permeability parameter and slip parameter, whereas it decreases with increasing the magnitude of amplitude ratio. The streamline on the center line in the wave frame reference are found to split in order to enclosed a bolus of fluid particles circulating along closed streamline under certain conditions. This phenomenon is referred to as trapping, which is a characteristic of peristaltic motion. Since this bolus appears to be trapped by the wave, the bolus moves with the same speed as that of the wave. Fig.(5) drawn for streamline patterns. The impacts of Grashof parameter, constant heat parameter, couple- stress parameter, Hartman parameter, permeability parameter and slip parameter on trapping are discussed through these figures. It is important to observe that the size of trapping bolus reduces when the magnitude of said parameters (G_r, B, Ha, α, k and β) increases.

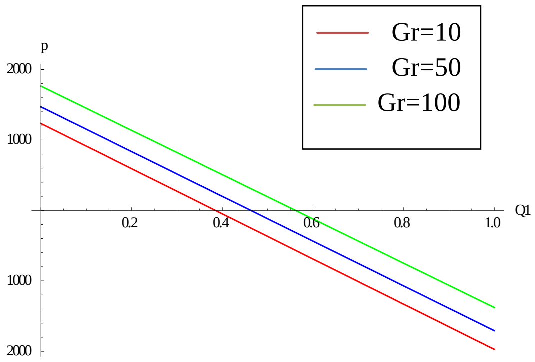

Fig.2a. Pressure versus averaged flow rate for difference value of Gr at Ha=2 ,α=10,β=0.1,k= 0.1 , Φ =0.3, B=5.

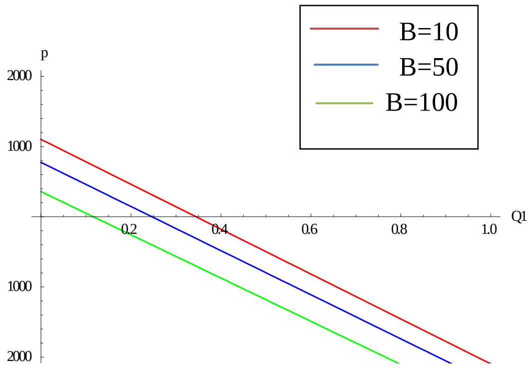

Fig.2b. Pressure versus averaged flow rate for difference value of B at Ha=2 ,α=10,β=0.1,k= 0.1 , Φ =0.3, Gr=10.

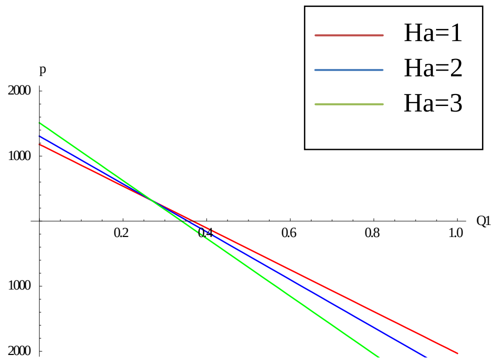

Fig.2c. Pressure versus averaged flow rate for difference value of Ha at α=10,β=0.1,k= 0.1 , Φ =0.3, Gr=10, B= 5.

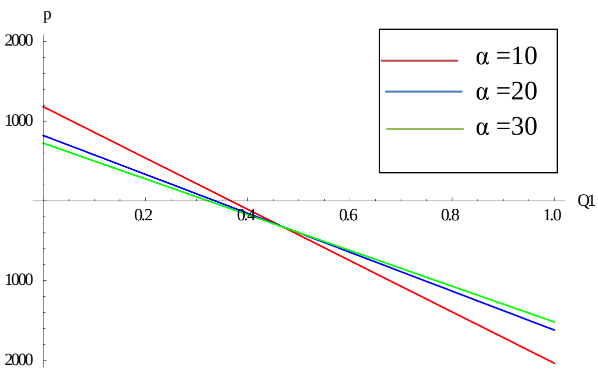

Fig.2d. Pressure versus averaged flow rate for difference value of α at Ha=2,β=0.1,k= 0.1 , Φ =0.3, Gr=10, B= 5.

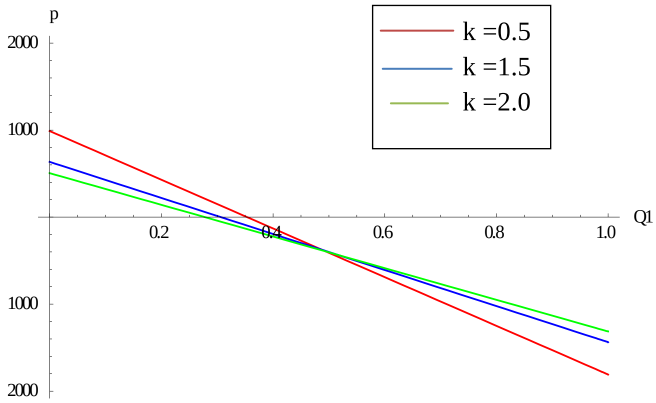

Fig.2e. Pressure versus averaged flow rate for difference value of K at Ha=2 , α=10,β=0.1, Φ=0.3, Gr=10, B= 5.

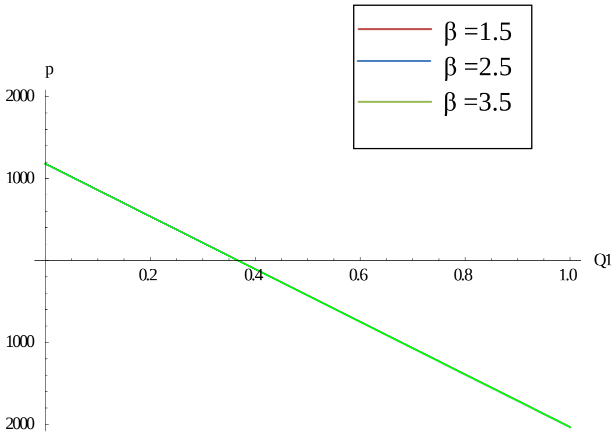

Fig.2f. Pressure versus averaged flow rate for difference value of β at Ha=2 , α=10, k =0.1, Φ=0.3, Gr=10, B= 5.

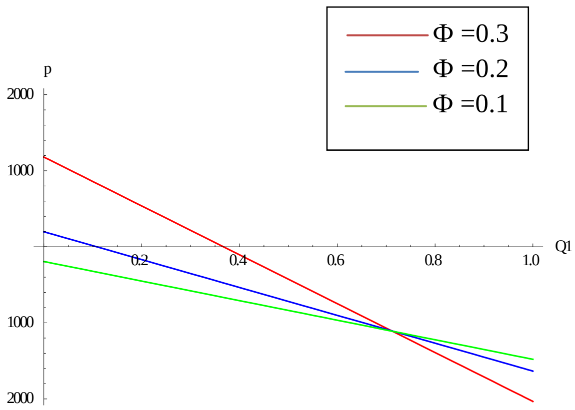

Fig.2g. Pressure versus averaged flow rate for difference value of Φ at Ha=2 , α=10, k =0.1, β=0.1 Gr=10, B= 5.

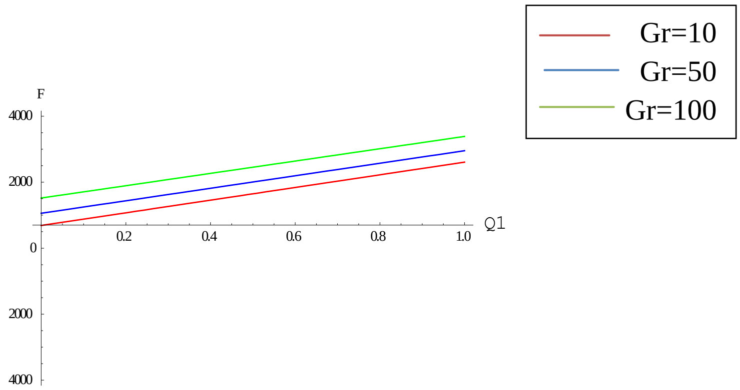

Fig.3a. Frictional force versus averaged flow rate for various value of Gr at Ha=2, k=0.1, β =0.1 , α=10 , Φ=0.3 , B=5

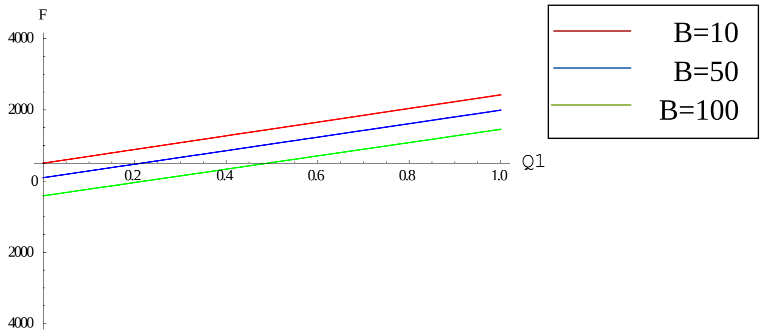

Fig.3b. Frictional force versus averaged flow rate for various value of B at Ha=2, k=0.1, β =0.1 , α=10 , Φ=0.3 , Gr=10

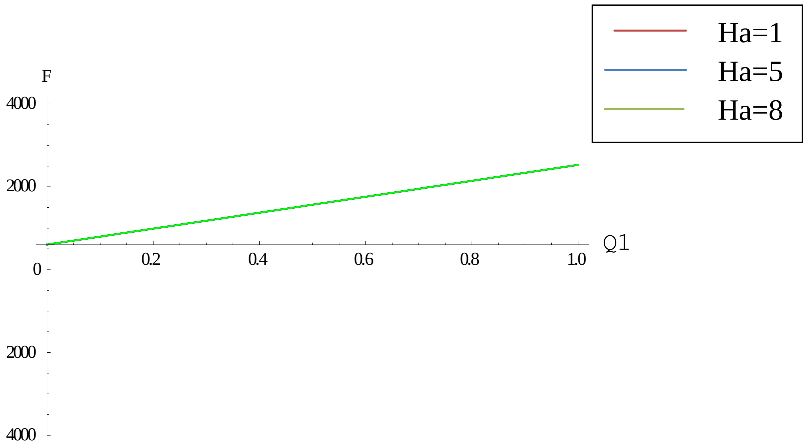

Fig.3c. Frictional force versus averaged flow rate for various value of Ha at k=0.1, β =0.1 , α=10 , Φ=0.3 ,B=5, Gr=10.

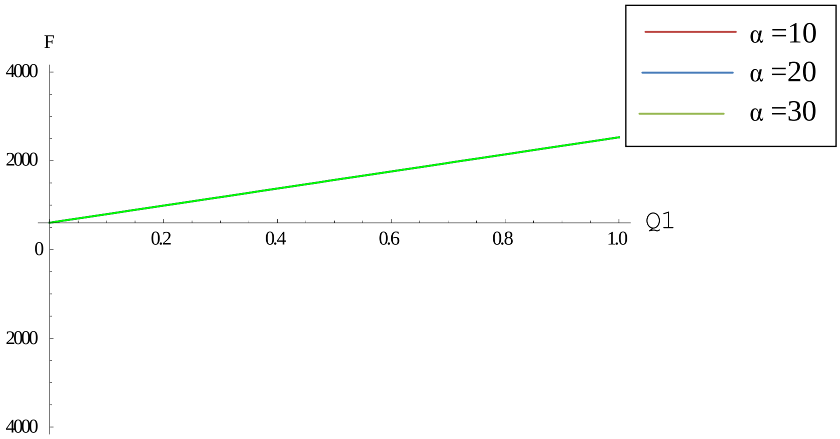

Fig.3d. Frictional force versus averaged flow rate for various value of α at Ha=2, k=0.1, β =0.1 , Φ=0.3, B=5, Gr= 10.

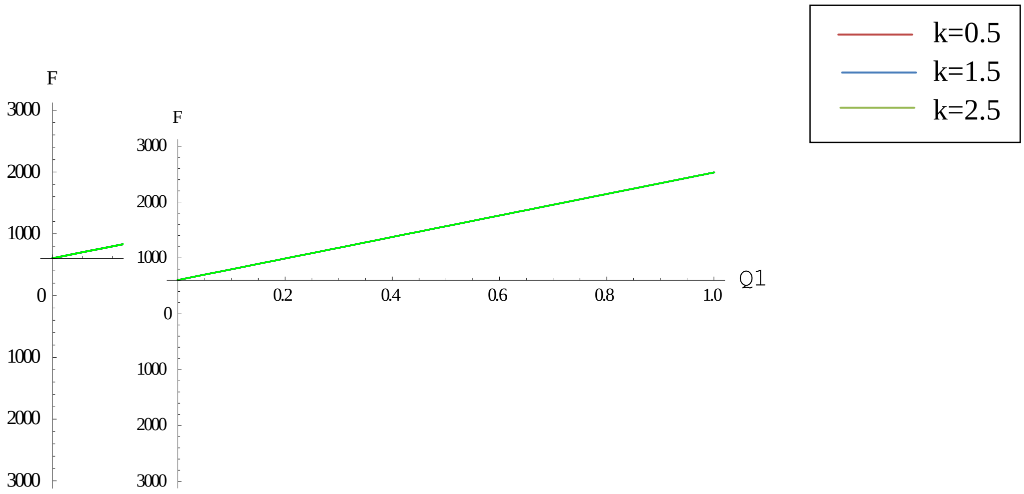

Fig.3e. Frictional force versus averaged flow rate for various value of k at Ha=2, β =0.1 , α=10 , Φ=0.3,B=5, Gr=10

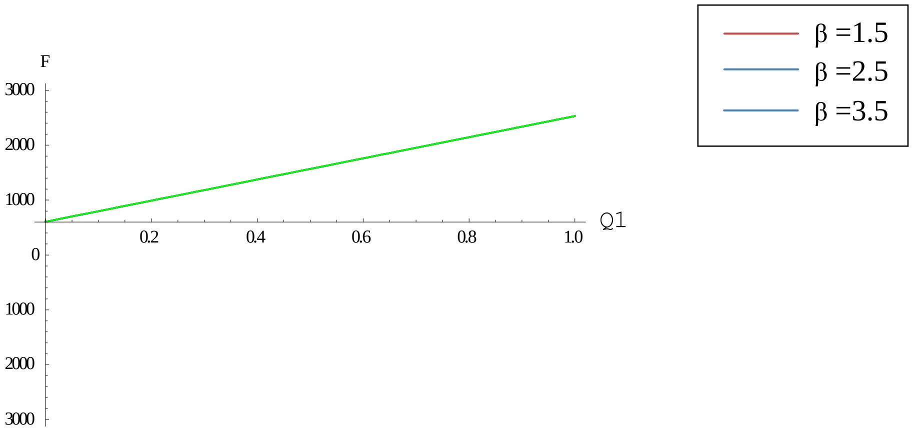

Fig.3f. Frictional force versus averaged flow rate for various value of β at Ha=2, k=0.1, α=10 , Φ=0.3, B=5, Gr=10

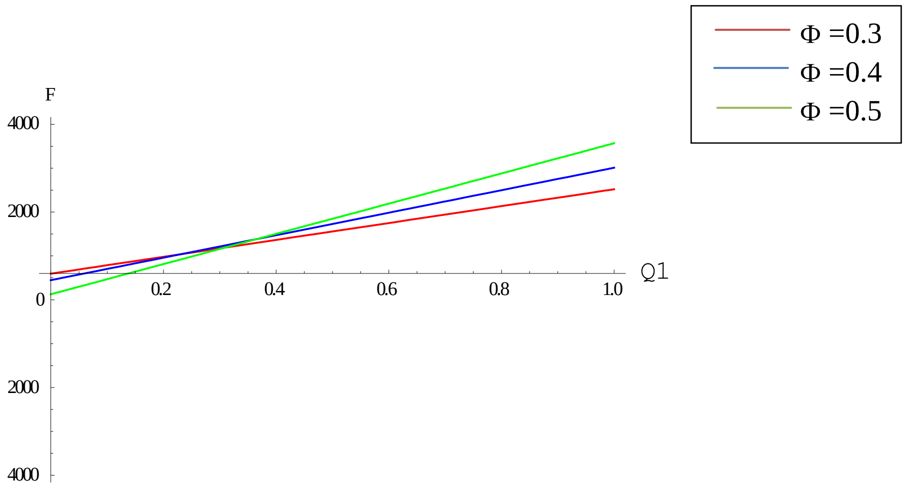

Fig.3g. Frictional force versus averaged flow rate for various value of Φ at Ha=2, k=0.1, β =0.1 , α=10, B=5, Gr=10

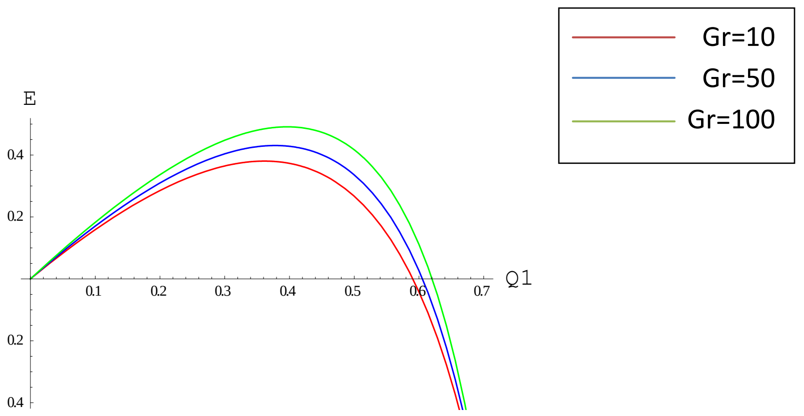

Figure 4a. Mechanical Efficiency versus averaged flow rate for various value of Gr at Ha=2,k=0.1, β =0. 1, α=10 , Φ=0.5 , B=5

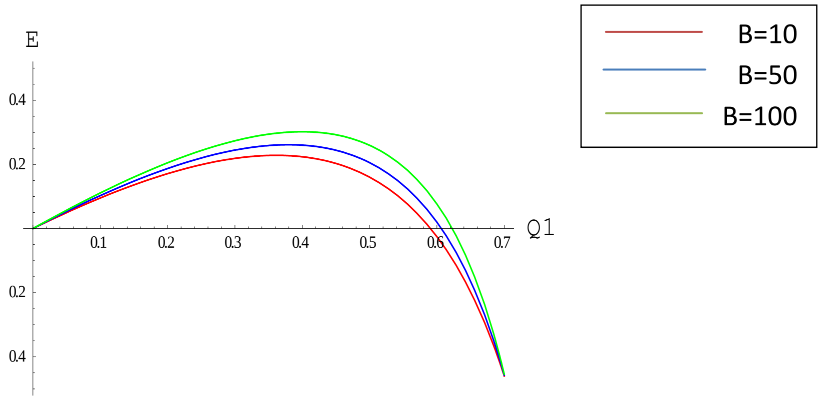

Figure 4b. Mechanical Efficiency versus averaged flow rate for various value of B at Ha=2, k=0.1, β =0. 1, α=10 , Φ=0.5, Gr=10

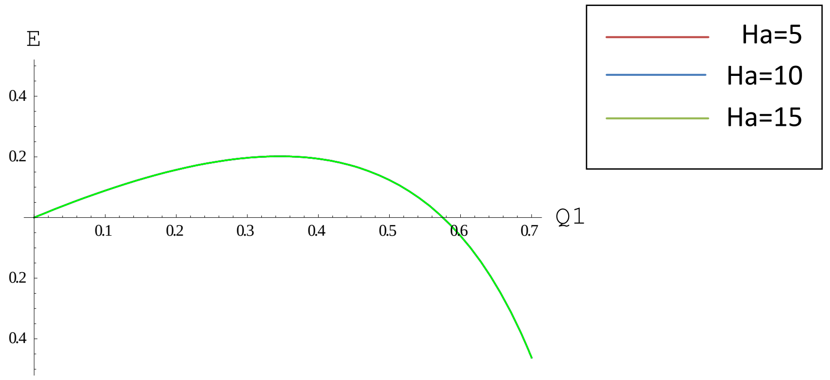

Figure 4c. Mechanical Efficiency versus averaged flow rate for various value of Ha at k=0.1, β =0. 1, α=10 , Φ=0.5 , Gr=10, B=5

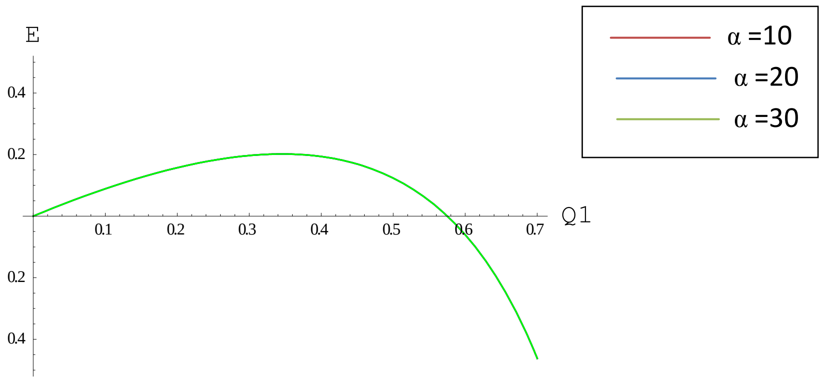

Figure.4d. Mechanical Efficiency versus averaged flow rate for various value of α at Ha=2 , k=0.1, β =0. 1, Φ=0.5 , Gr=10 , B=5

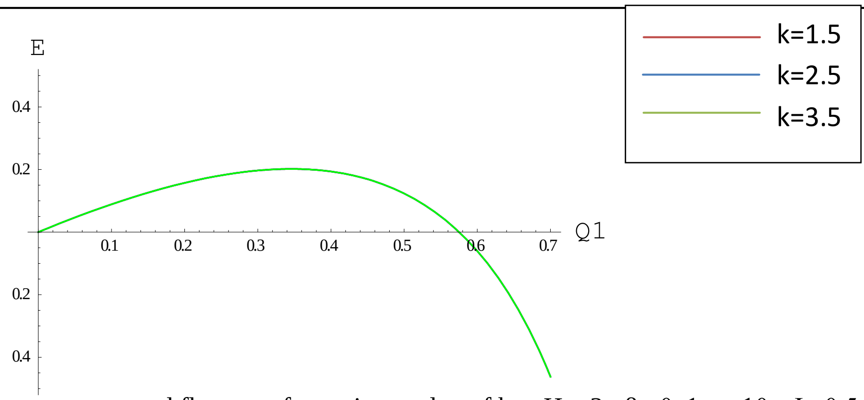

Figure 4e. Mechanical Efficiency versus averaged flow rate for various value of k at Ha=3, β =0. 1, α=10 , Φ=0.5 , Gr=10 , B=5

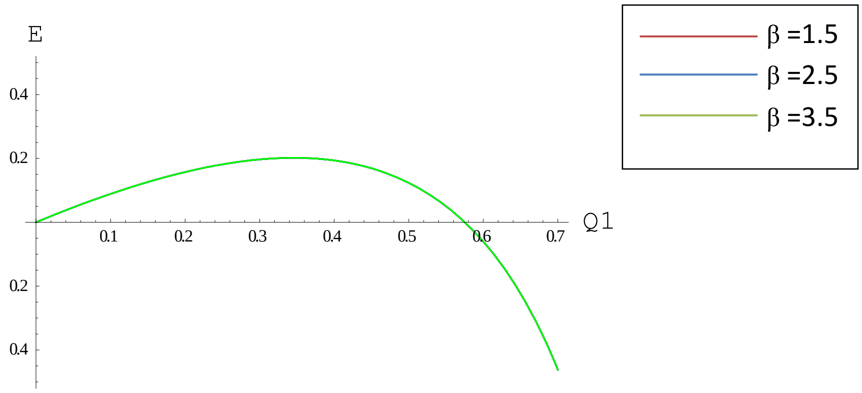

Figure 4g. Mechanical Efficiency versus averaged flow rate for various value of β at Ha=2, k=0.1, α=10 , Φ=0.5 , Gr=10, B=5

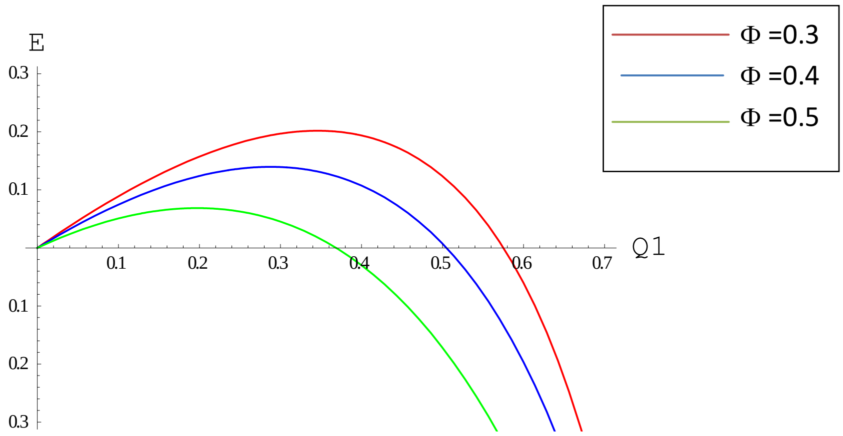

Figure 4f. Mechanical Efficiency versus averaged flow rate for various value of Φ at Ha=2, k=0.1, β =0. 1, α=10 , Gr=10 , B=5

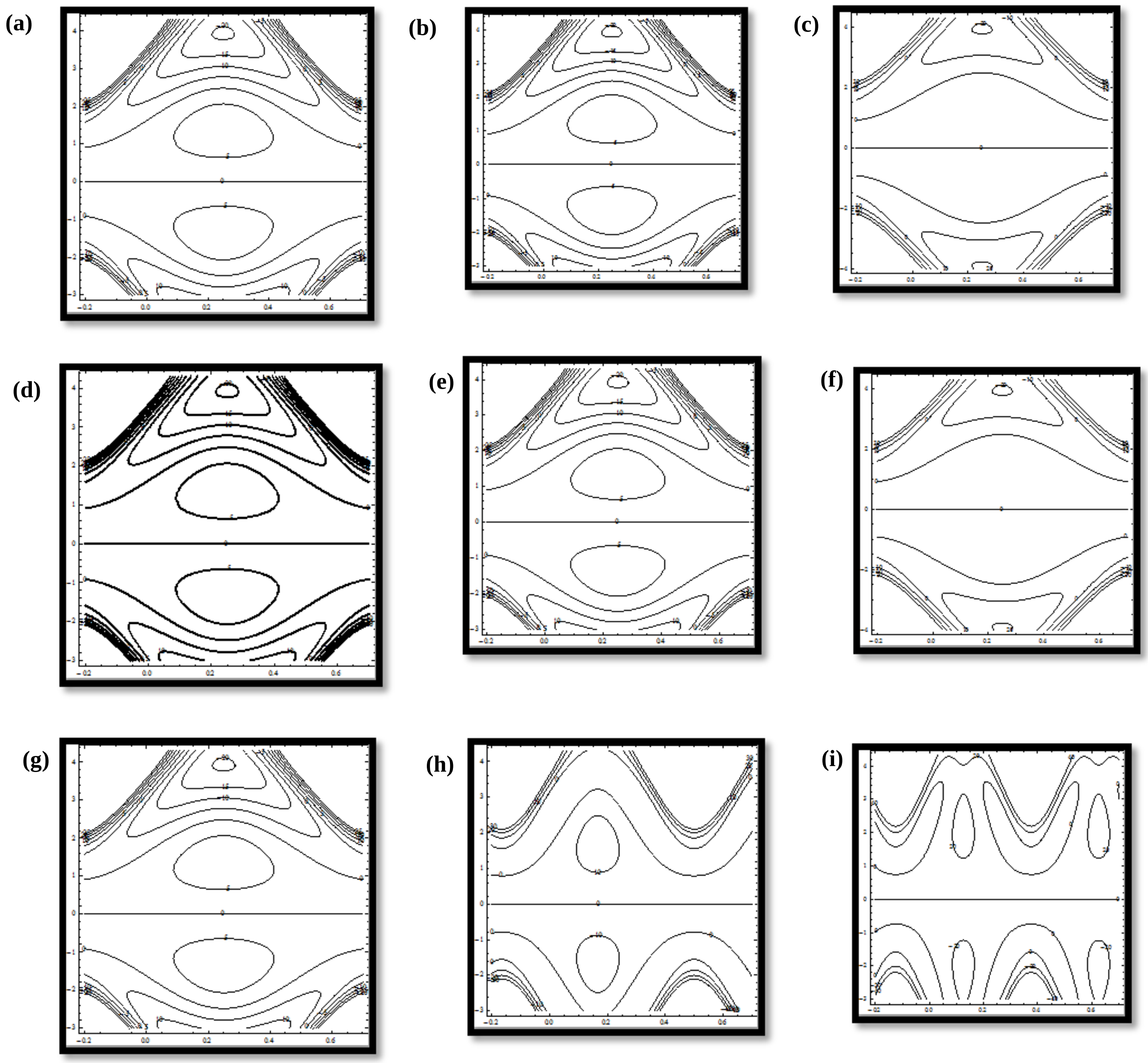

Figure 5. Streamline in the wave frame(axial coordinate. transverse coordinate in Q1 = .95 & Φ = 0.3

at **a**: Gr=5,B=5 ,Ha = 2, k = .1 , β = .1, α = 1.0 , **b**:Gr=15,B=5,Ha = 2, k = .1, β = .1, α = 1.0, **c**:Gr=15, B=5,Ha = 2, k = .1, β = .1, α = 1.0, **d** :B=5, Gr=10 ,Ha = 2, k = .1, β = .1, α = 1.0 , **e** :B=10,Gr=5,Ha = 2, k = .1, β = .1, α = 1.0 , **f**: B=15, Gr=10,Ha =2, k = .1, β = .1, α = 2..0, **g** : Ha =3, Gr=10, B=5, k = .1, β = .1, α =1.0 , **h** : Ha = 4, Gr=10, B=5,k = .1, β = .1, α = 1.0, **i** : Ha = 8,Gr=10, B=5, k = .1, β = .1, α =1 .0.

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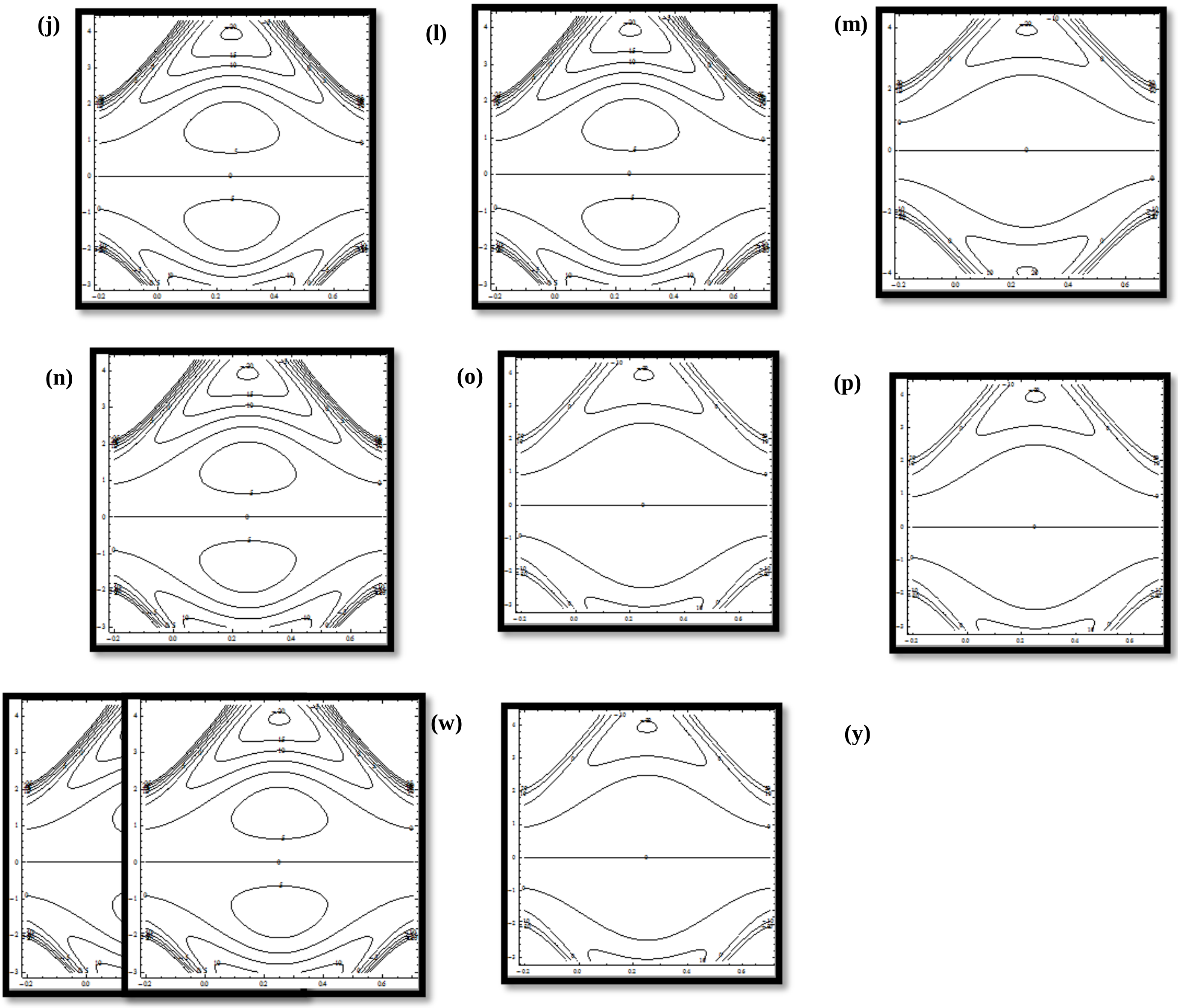

Figure 5. Streamline in the wave frame(axial coordinate. transverse coordinate in Q1 = .95, Φ = 0.5 at

j :Gr=10, B=5,Ha =2, k =0.1, β = .1, α = 1.5, **l** :Ha = 2, Gr=10, B=5, k = .1, β = .1, α = 2.5, **m**: Gr=10, B=5,Ha = 2, k = .1, β = .1, α =3.5. **n** : Ha = 2, ,Gr=10, B=5, k = 1.5, β = 1., α =1.0, **o :**Ha = 2,Gr=10, B=5, k = 2.5, β = .1, α= 1.0 , **p** :Ha= 2,Gr=10,B=5, k =3.5, β = .1, α =1, **w** :Ha =2,Gr=10, B=5, k = .1, β = 1 .5, α = 1.0, ,**y:** Ha=2, Gr=10, B=5,k=0.1,β =2.5, α = 1.0, **z**: Ha=2, Gr=10, B=5,k=0.1,β =3.5, α = 1.0

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