ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.4, No.7, 2014

Effects of Flow Parameters on Flow Variables of a Newtonian Fluid through a Cylindrical Collapsible Tube

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Abstract

The dynamics of fluid flow in collapsible tube are vital in understanding the behavior and analysis of flow phenomenon in veins, arteries, airways, urethra, etc. These fluid-conveying vessels in human body are highly flexible and collapsible, and accommodate deformation that result to a highly noncircular cross sectional area. This research aimed at determining the effects of various flow parameters on the flow variables of a Newtonian fluid flowing through a cylindrical collapsible tube. The tube is considered collapsible in the transverse direction, taken to be perpendicular to the main flow direction. The mathematical model in this study can be employed to simulate biological fluid flows such as blood flow in the arteries and veins. The equations governing the flow i.e. the continuity equation, the momentum equation and the tube law have been discussed. The results are presented in form of graphs. The results show that the flow parameters considered are directly proportional to both the cross sectional area and internal pressure and inversely proportional to the flow velocity

Keywords: collapsible tube, flow parameters, flow variables, transverse direction

1. Introduction

Matter is said to be a fluid if it undergoes continuous deformation when some external force is applied. A fluid is said to be Newtonian if it obeys the Newton's law of viscosity which states that the shear stress is proportional to the velocity gradient. For a Newtonian fluid, viscosity does not change with the rate of flow.

A tube is said to be collapsible if the tube has sufficiently flexible walls such that it can elastically accommodate deformation to a highly noncircular cross section when the external pressure exceeds the internal pressure. When fluid flows through the collapsible tube, various complicated phenomena are revealed. Such conditions occur rarely in industrial applications but are very common in biological studies such as blood flow in veins and air flow in lungs. Vessel collapse is seen in the veins, such as in the veins of a hand raised above the level of the heart or in the jugular vein when a person is standing upright. In the arteries, collapse occurs when high external pressures are applied, such as when an artery is compressed by a sphygmomanometer cuff during blood pressure measurement. Similarly, in the industry collapse may be experienced during cementing operations, trapped fluid expansion, or well evacuation, among many others. Most oilfield tubulars also experience collapse.

1.1 Literature Review

Flow through collapsible tubes has been extensively studied in the laboratory. In experimental studies a flexible tubing is employed. Edward (1972), explained that these experiments are based upon the assumption that the difference between the flexible tubing and veins are quantitative in nature rather than qualitative. Bertram (1986) did an experimental study on collapsible and elastic tubes and observed that when the external pressure exceeded the internal fluid pressure the tube buckled non-axisymmetrically, leading to a nonlinear relation between pressure-drop and flow rate.

Brian (2003) formulated a mathematical model for a collapsible tube and developed a computational methodology to determine the best-fit values for parameters that describe constitutive behavior of the tube. He noted that the tube stiffness affected the cross sectional area of the tube and had no large influence on the internal pressure. Heil and Jensen (2003) explained that nonaxisymmetrically collapsed vessels readily develop flow-induced, self excited oscillations. Hazel and Heil (2003) investigated the steady flow through thin-walled elastic tubes for a finite Reynolds number. Results indicated that nonaxisymmetric buckling of the tube contributes to nonlinear pressure-flow relations that can exhibit flow limitation through purely viscous mechanisms. Makinde (2005) described the fluid dynamics of a collapsible tube using a mathematical model. He noted that fluid axial velocity generally decreases with an increase in tube contraction due to the strong influence of the negative transmural pressure owing to marked reduction of rigidity. Marzo *et al*. (2005) studied three-dimensional collapse of a steady flow through finite-length elastic tubes numerically. Odejide *et al*. (2008) examined an incompressible viscous fluid flow and heat transfer in a collapsible tube. It was also noted that increase in Reynolds number led to an increase in the fluid temperature with maximum magnitude at the pipe center and minimum at the wall. The fluid velocity profile was noted to be parabolic in nature. Andrew *et al*. (2008) described the role of venous valves in pressure shielding. A one-dimensional mathematical model of a collapsible tube, with the facility to introduce valves at any position, was used. They observed that a valve decreased the dynamic pressures applied to a vein when gravity is applied by a considerable amount. Liu *et al*. (2009) explained that the wall stiffness is dominated by the axial tension. Eleuterio (2013) fomulated a one-dimensional time-dependent non-linear mathematical model for physiological fluid flow in collapsible tubes with discontinuous material properties. He observed that although the solution algorithm dealt with idealised cases, it is uniquely well-suited for assessing the performance of numerical methods intended for simulating more general situations.

1.1.1 The Continuity Equation

The law of conservation of mass states that the rate of increase of mass within a controlled volume is equal to the net rate of influx through the controlled surface. The equation can be written as:

$$\frac{\partial \rho}{\partial t}+\vec{\nabla}\left(\rho \vec{u}\right)=0 \tag{1}$$

For incompressible fluid flow, ρ is a constant and hence for steady, incompressible fluid flow the equation of continuity is given as follows:

$$\frac{\partial (Au)}{\partial x}=\frac{\partial Q}{\partial x}=0 \tag{2}$$

1.1.2 The Momentum Equation

The equation of conservation of momentum is derived from Newton's second law of motion, which states that the time rate change of momentum of a body matter is equal to the net external forces applied to the body. The momentum equation can be expressed as;

$$\frac{du}{dt}+u\frac{du}{dx}=-\frac{1}{\rho}\frac{dp}{dx}-Ru+g \tag{3}$$

where R>0 is a friction factor and g is the gravitational acceleration when the tube is held vertically.

The equation of conservation of momentum for steady laminar fluid flow is given by;

$$\rho u\frac{\partial u}{\partial x}=-\frac{\partial p}{\partial x}-f_L\frac{\mu u}{D_e}\left(\frac{s}{A}\right) \tag{4}$$

where *s* the peripheral length and f_l is the skin-friction coefficient for laminar flow.

1.1.3 Tube law

The tube law relates the transmural pressure (internal pressure –external pressure) to the cross-sectional area of an ellastic tube. It is given by;

$$p - p_E = \phi\left(\frac{A}{A_0}\right) \tag{5}$$

Putting into consideration the longitudinal tension in the tube law yields;

$$p - p_E = \phi\left(\frac{A}{A_o}\right) - T\frac{\partial^2 A}{\partial x^2} \tag{6}$$

Where $\phi\left(\frac{A}{A_0}\right) = K_{PE}(\alpha^n - \alpha^{-\frac{3}{2}})$, $\alpha = \frac{A}{A_0}$ and K_{PE} is the combined stiffness, which represents the overall stiffness of the tube, whether collapsed or distended.

1.1.4 Final set of Equations

Making the substitution Q=Au, equation (4) becomes;

$$\rho\frac{Q}{A}\frac{\partial\frac{Q}{A}}{\partial x} = -\frac{\partial p}{\partial x} - f_l\frac{\mu}{D_e}\frac{Q}{A}\frac{s}{A} \tag{7}$$

Which reduces to;

$$\frac{\partial p}{\partial x} = \rho\frac{Q}{A^3}^2\frac{\partial A}{\partial x} - \rho f_l\frac{\mu}{D_e}\frac{Q}{A}\frac{s}{A} \tag{8}$$

Thus the final set of equations for a laminar steady fluid flow through a collapsible tube becomes;

$$\frac{\partial p}{\partial x} = \rho\frac{Q}{A^3}^2\frac{\partial A}{\partial x} - \rho f_l\frac{\mu}{D_e}\frac{Q}{A}\frac{s}{A} \tag{9}$$

$$p - p_E = K_{PE}(\alpha^n - \alpha^{-\frac{3}{2}}) - T\frac{\partial^2 A}{\partial x^2} \tag{10}$$

2. Method of solution

The equations governing the flow problem were written in finite difference form and then reorganized and written in matrix form.

The governing equations describing the steady, incompressible laminar fluid flow through a cylindrical collapsible tube, in finite difference form are given as:

$$p_i = p_{i-1} + \frac{\rho Q^2(A_i - A_{i-1})}{A_{im}(i)^3} - \Delta x f_l\frac{\mu}{D_e}\frac{Q}{A_{im}(i)}\frac{s_i}{A_{im}(i)} \tag{11}$$

subject to the boundary condition P(0) = P_o where P_o is the inlet pressure at the tube.
and

$$T\frac{A_{i+1}+A_{i-1}}{\Delta x^2}+A_i\left[\frac{-2T}{\Delta x^2}+K_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right)\right]= \tag{12}$$

$$p_E-p_i+K_{PE}\left[\left(\frac{c}{A_0}\right)^{10}-\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]-K_{PE}\left[10\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right]$$

Subject to the boundary conditions: A(0) = A_0 , A(L) = A_0 where A_0 is the area at the inlet.

The above system with the boundary conditions becomes the matrix system:-

$$B\,\vec{A}=\vec{T}$$

The tridiagonal matrix B is given by : :

$$\begin{pmatrix} k_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}-\frac{2T}{\Delta x^2}\right) & \frac{T}{\Delta x^2} & \cdots \\ \frac{T}{\Delta x^2} & k_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}-\frac{2T}{\Delta x^2}\right) & \frac{T}{\Delta x^2} \\ \frac{T}{\Delta x^2} & k_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}-\frac{2T}{\Delta x^2}\right) & \frac{T}{\Delta x^2} \\ \cdots & \frac{T}{\Delta x^2} & k_{PE}\left(\frac{10}{c}\left(\frac{c}{A_0}\right)^{10}+\frac{3}{2c}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}-\frac{2T}{\Delta x^2}\right) \end{pmatrix}$$

Vector T is given by:

$$\vec{T} = \begin{pmatrix} -p_2 + p_E + k_{PE}\left(\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - k_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - \frac{T}{\Delta x^2}A_0 \\ -p_3 + p_E + k_{PE}\left(\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - k_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] \\ \cdot \\ \cdot \\ \cdot \\ -p_{N-2} + p_E + k_{PE}\left(\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - k_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] \\ -p_{N-1} + p_E + k_{PE}\left(\left(\frac{c}{A_0}\right)^{10} - \left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right) - k_{PE}\left[10\left(\frac{c}{A_0}\right)^{10} + \frac{3}{2}\left(\frac{c}{A_0}\right)^{-\frac{3}{2}}\right] - \frac{T}{\Delta x^2}A_0 \end{pmatrix}$$

2.1 Results and discussion

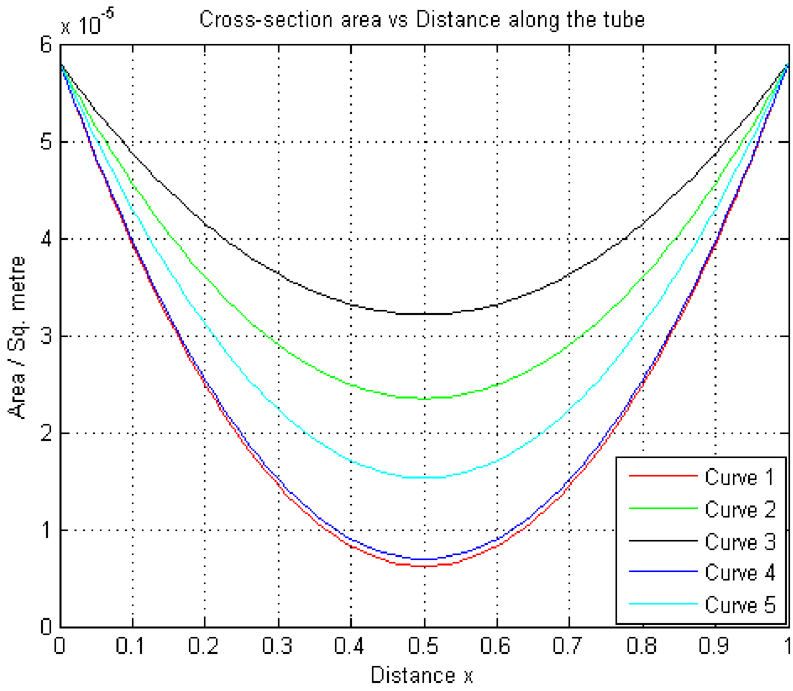
The table below shows varying values used for longitudinal tension and tube stiffness.

Table 1: Longitudinal tension and tube stiffness

CURVES	TUBE STIFFNESS	LONGITUDINAL TENSION
Curve 1	1.21×10^{-5}	4.0×10^{2}
Curve 2	1.21×10^{-5}	6.0×10^{2}
Curve 3	1.21×10^{-5}	8.0×10^{2}
Curve 4	1.21×10^{-1}	4.0×10^{2}
Curve 5	1.21×10^{-2}	4.0×10^{2}

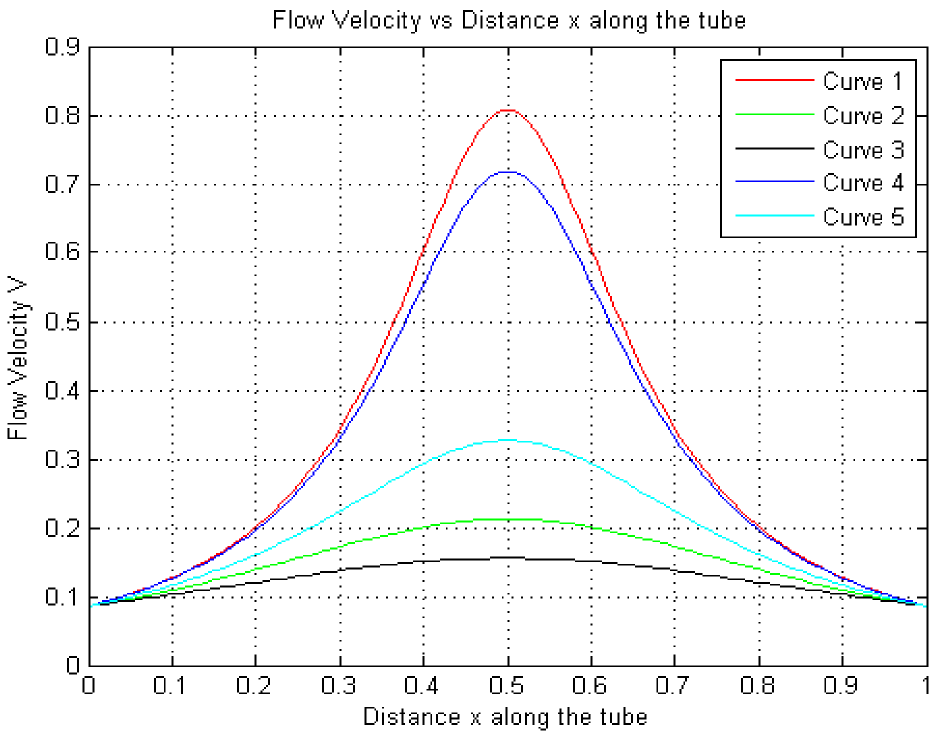
These values have been used to plot the graphs of cross sectional area versus distance with varying tube stiffness and longitudinal tension, flow velocity versus distance with varying tube stiffness and longitudinal tension and the graph of internal pressure versus distance with varying tube stiffness and longitudinal tension. The effect of varying the longitudinal tension with tube stiffness constant is shown by Curves 1, 2 and 3. The effect of varying tube stiffness with longitudinal tension constant is shown by Curves 1, 4 and 5.

Effect of Varying longitudinal Tension and tube stiffness on the cross sectional area.



Graph 1: Cross sectional area versus distance for $Q=5\times10^{-6}$, $\rho=1.0\times10^{3}$
$Pe=4.00\times10^{3}$ $r=4.3\times10^{-3}$ $l=0.2$

Effect of Varying longitudinal Tension and tube stiffness on the flow velocity.

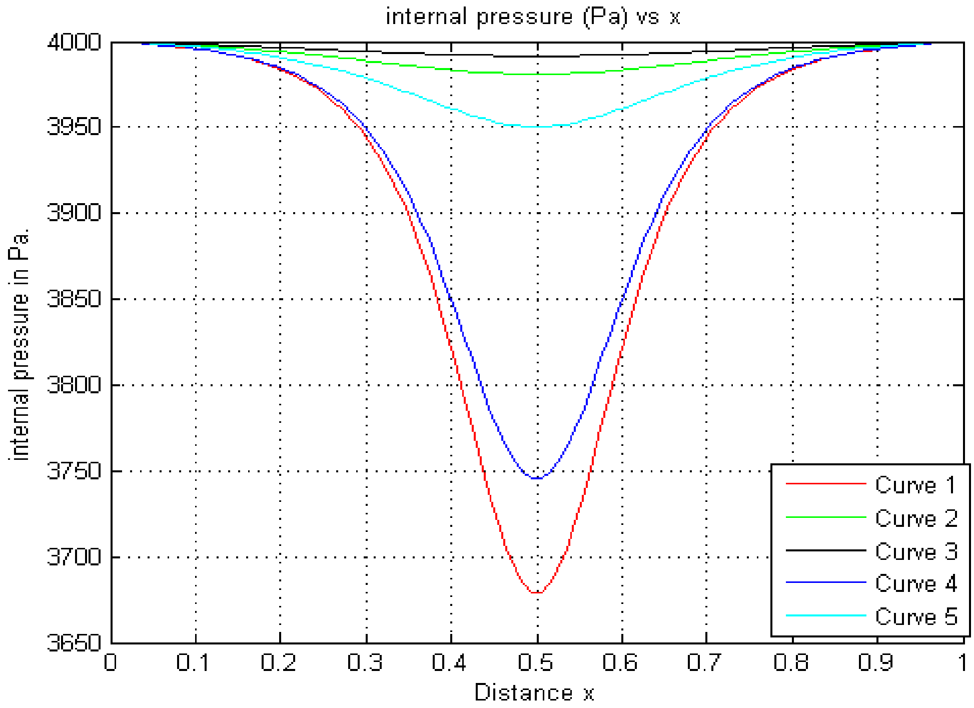


Graph 2: Flow velocity area versus distance for $Q=5\times10^{-6}$, $\rho=1.0\times10^{3}$

$Pe=4.00\times 10^{3}$ $r=4.3\times 10^{-3}$ $l=0.2$

Results show that both the longitudinal tension and the tube stiffness are inversely proportional to the flow velocity. As the longitudinal tension increases, the flow velocity decreases. Similarly, as the tube stiffness increases, the flow velocity decreases.

Effect of Varying longitudinal Tension and tube stiffness on the internal pressure



Graph 3:Internal pressure versus distance for $Q=5\times 10^{-6}$, $\rho=1.0\times 10^{3}$
$Pe=4.00\times 10^{3}$ $r=4.3\times 10^{-3}$ $l=0.2$

It is observed that as the longitudinal tension increases the internal pressure increases and that as the tube stiffness increases the internal pressure increases.

2.2 Conclusion

The objective of this study was to determine the effect of flow parameters on flow variables of a Newtonian fluid through a cylindrical collapsible tube. The flow parameters that were considered are tube stiffness and longitudinal tension. The flow variables in consideration are; cross sectional area of a collapsible tube, flow velocity and internal pressure.

Longitudinal tension describes how tight the tube is pulled out when attached at the edges. Tube's stiffness describes the tube's capability of resistance to elastic deformation in response to an applied force. The values for the flow variables i.e. cross sectional area, flow velocity and internal pressure are taken at the point where the collapse of the tube is mostly felt, that being midway along the length of the tube.

As fluid flows through a collapsible tube there is collision between molecules hence a decrease in kinetic energy. Pressure energy is converted into kinetic energy to maintain the flow velocity since the cross sectional area is the same.This leads to a decrease in internal pressure. Since the external pressure remains constant, it exceeds the internal pressure causing the tube to collapse. The collapse leads to a decrease in cross sectional area and consequently the flow velocity increases in order to maintain a constant flow rate. This is as shown in graph 1 and 2.

An increase in velocity of the fluid leads to an increase in the collision between the molecules hence greater loss in kinetic energy. This causes the internal pressure to decrease even more, hence the observation in graph 3. In addition, according to Bernoulli principle, an increase in fluid velocity leads to a decrease in pressure.

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