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New Proposed Length-Biased Weighted Exponential and Rayleigh Distribution with Application

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Abstract
The concept of length-biased distribution can be employed in development of proper models for lifetime data. Length-biased distribution is a special case of the more general form known as weighted distribution. In this paper we introduce a new class of length-biased of weighted exponential and Rayleigh distributions(LBW$_1$E$_1$D), (LBWRD).This paper surveys some of the possible uses of Length - biased distribution We study the some of its statistical properties with application of these new distribution**.**

Keywords: length- biased weighted Rayleigh distribution, length- biased weighted exponential distribution, maximum likelihood estimation.

1. Introduction

Fisher (1934) [1] introduced the concept of weighted distributions Rao (1965) [2]developed this concept in general terms by in connection with modeling statistical data where the usual practi ce of using standard distributions were not found to be appropriate, in which various situations that can be modeled by weighted distributions, where non-observe ability of some events or damage caused to the original observation resulting in a reduced value, or in a sampling procedure which gives unequal chances to the units in the original That means the recorded observations cannot be considered as a original random sample.

A mathematical definition of the weighted distribution

Let $(\boldsymbol{\delta}, \pounds, P)$ be a probability space, $X: \delta \rightarrow H$ be a random variable (rv) where $H = (a, b)$ be an interval on real line with $0 < a, b$ with $a < b$ can be finite or infinite. When the distribution function(df) F(t) of X is absolutely continuous(discrete) with probability density function (pdf) (probability mass function), f(x). And w(x) be a non-negative weight function satisfying

$\mu_w = \mathrm{E}(w(X)) < \infty$, then the rv X_w having pdf

$$\boldsymbol{f_w(x) = \frac{w(x)f(x)}{E[w(x)]}, a < x < b} \quad \textbf{(1)}$$

Where $\boldsymbol{E[w(x)] = \int_{-\infty}^{\infty} w(x)f(x)\, dx}$, $-\infty < \boldsymbol{x} < \infty$

is said to have weighted distribution

One of the basic problems when one use weighted distributions as a tool in the selection of suitable models for observed data is the choice of the weight function that fits the data. Depending upon the choice of weight function w(x), we have different weighted models. For example,

1) Length-Biased Distribution

When the weight function depends on the lengths of units of interest (i.e. $w(x) = x$), the resulting distribution is called length-biased. In this case, the pdf of a length-biased rv X_L is defined as

$$f_w(x) = \frac{x\, f(x)}{\mu}, a < x < b \quad \textbf{(2)}$$

Where $\mu = E[X] = \int_{-\infty}^{\infty} x f(x)\, dx$, $-\infty < x < \infty$

The statistical interpretation of length-biased distributions was originally identified by Cox (1962) [3] in the context of renewal theory.

Length-biased distributions are a special case of the more general form known as weighted distributions.

Fisher [1934] introduced these distributions to model ascertainment bias and formalized in a unifying theory by Rao [1965]. These distributions arise in practice when observations from a sample are recorded with unequal probability and provide a unifying approach for the problems where the observations fall in the non-experimental.

Patil and Rao (1978) [4] examed some general models leading to weighted distributions with weight functions not necessarily bounded by unity and studied length biased (size biased) sampling with applications to wildlife populations and human families.

Khattree, 1989[6];Oluyede and George, 2002[5] presented the characteristics of many length biased distributions, with comparisons for weighted and length biased distributions.

In this paper we introduce a new class of length-biased of weighted exponential and Rayleigh distributions(LBW_1E_1D), (LBWRD).
With study the some of its statistical properties**.**

The probability density function (pdf) of the one parameter exponential distribution(ED) is given by

$$f(x;\lambda) = \lambda\, e^{-\lambda\, x} \qquad (3)$$

And the pdf of Rayleigh distribution(RD) is given by

$$g(x;\beta;\theta) = \frac{2\,x\,\beta}{\theta}\; e^{\frac{-\beta\, x^2}{\theta}} \qquad (4)$$

And the weighted function used are as follows:

$$w_1(x) = e^{n\,x}, n = 1,2,\ldots \qquad (5)$$

$$w_2(x) = \frac{n\,x}{\theta}\,, n = 1,2,\ldots \qquad (6)$$

Such that $w_1(x)$ using for the one-parameter exponential distribution

And $w_2(x)$ using for the two parameter Rayleigh distribution

2. MATERIALS AND METHODS

This section, we derived the shape of the pdf, cdf for the LBW_1E_1D and LBWRD distribution and consider its some sub-models.

Definition 1: Let X be a nonnegative r.v. with pdf f(x) and assuming that $E(X) <\infty$. Then the pdf $f_L(x)$ for a length biased distribution of X can be obtained by

$$f_L\,(x) = \frac{w_*(x)\, w_i(x)\ \ f(x)}{\mu_w = E[w_*(x)\, w_i(x)\,]} \quad , a\ < x\ < b \qquad (7)$$

assuming that $\mu_w = \int_0^\infty w_*(x)\ \ w_i(x)\ \ f(x)\, dx\ < \infty\, , i = 1,2$
Provided that $w_*(x) = x$

Theorem 1: If X~ LBW_1E_1D, then the pdf and cdf of X are given respectively as :

$$f_L(x;\lambda) = x\,(\lambda - n)^{\,2}\, e^{-x\,(\lambda - n)}\,,\ \lambda > n \qquad (8)$$

$$F_L(x;\lambda) = \ \ 1 - e^{-x(\lambda - n)}(1 - x\,n + x\,\lambda) \qquad (9)$$

And if X~ LBWRD(β;θ), then the pdf and cdf of X are given respectively as :

$$g_L(x;\beta\,;\theta) = \frac{2\,x^3\beta^2\,e^{\frac{-\beta x^2}{\theta}}}{\theta^2}\ , \qquad \frac{\beta}{\theta} > 0 \tag{10}$$

$$G_L(x;\beta,\theta) = 1 - \frac{e^{\frac{-\beta x^2}{\theta}}(x^2\beta + \theta)}{\theta} \tag{11}$$

Proof:

When we Substitute (3) and (5) in (7), then the pdf of LBW_1E_1D r.v. can be obtained as:

$$f_L(x;\lambda) = \frac{x\,e^{n\,x}\,\lambda\,e^{-\lambda\,x}}{E[w_*(x)\,w_i(x)\,]}$$

where $E[w_*(x)w_1(x)] = \int_0^\infty x\,\lambda\,e^{-x(\lambda-n)}\,dx = \dfrac{\lambda}{(\lambda - n)^2}$

$$\therefore\ f_L(x;\lambda) = \frac{x\,\lambda\,e^{-x\,(\lambda-n)}}{\int_0^\infty x\,\lambda\,e^{-x\,(\lambda-n)}\,dx} = x\,(\lambda-n)^{\,2}\,e^{-x\,(\lambda-n)}$$

And the cdf of LBW_1E_1D r.v., denoted as $F_L(x)$

$$F_L(x;\lambda) = \int_0^x t\,(\lambda-n)^2\,e^{-t\,(\lambda-n)}\,dt = 1 - e^{-x(\lambda-n)}(1 - x\,n + x\,\lambda)$$

Now if X~ LBWRD (β;θ), then the pdf and cdf of X can be obtained as :

Substitute (4) and (6) in (7), then the pdf of LBWRD r.v.

$$g_L(x;\beta\,;\theta) = \frac{\frac{nx}{\theta}\ \frac{2\,x^2\,\beta\ e^{\frac{-\beta x^2}{\theta}}}{\theta}}{E[w_*(x)w_2(x)]} = \frac{\frac{nx}{\theta}\ \frac{2\,x^2\,\beta\ e^{\frac{-\beta x^2}{\theta}}}{\theta}}{\int_0^\infty \frac{2n\,x^3\,\beta\ e^{\frac{-\beta x^2}{\theta}}}{\theta^2}\,dx} = \frac{n}{\beta}$$

$$\therefore\ g_L(x;\beta\,;\theta) = \frac{2\,x^3\beta^2\ e^{\frac{-\beta x^2}{\theta}}}{\theta^2}$$

And the cdf of LBWRD r.v., denoted as $G_L(x)$

$$G_L(x;\beta,\theta) = \int_0^x \frac{2\,t^3\beta^2\ e^{\frac{-\beta t^2}{\theta}}}{\theta^2}\ dt = 1 - \frac{e^{\frac{-\beta x^2}{\theta}}(x^2\beta + \theta)}{\theta}$$

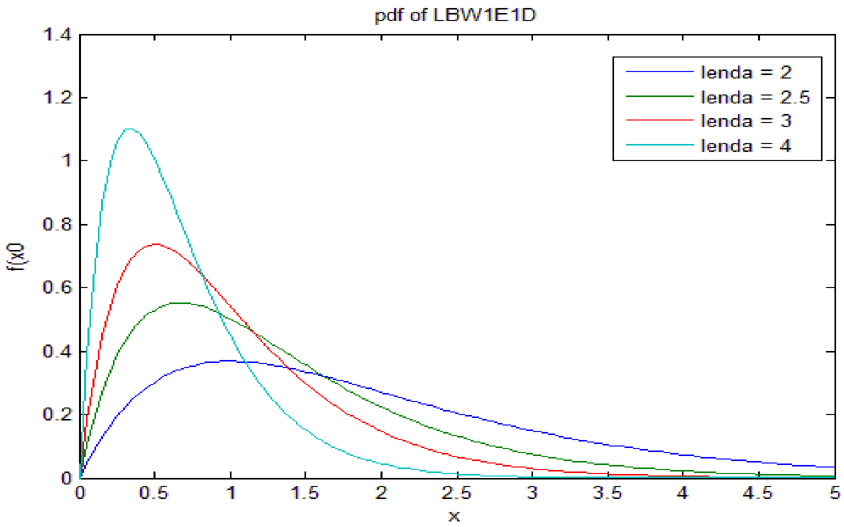
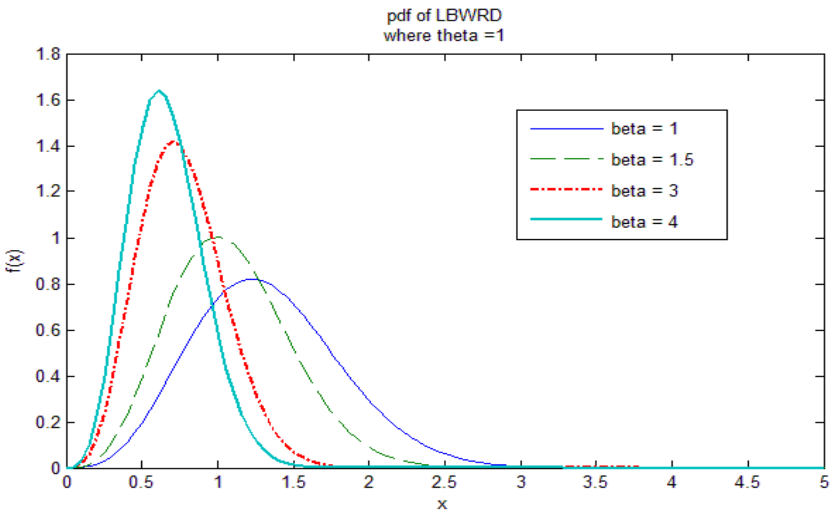
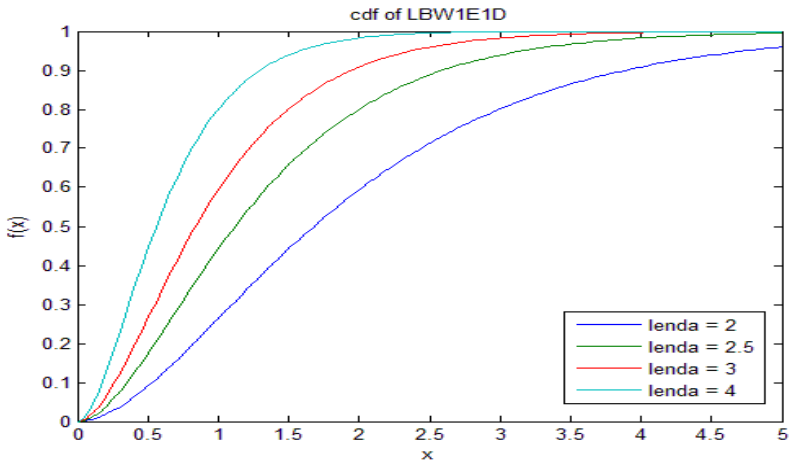


Figure (1) The probability density function of (LBW$_1$E$_1$D) for λ=(2,2.5,3,4) and *n*=1



θ= 1 , β=(1, 1.5 ,3,4)

Figure(1) shows us the plots of the probability density function of (LBW$_1$E$_1$D). Note that its shapes like the weibull distribution, and it has the height peak at λ= 4. Figure (2) shows us the plots of the probability density function, and the height peak at β=4.



Figure(3) : plot cdf of LBW_1E_1D where n =1

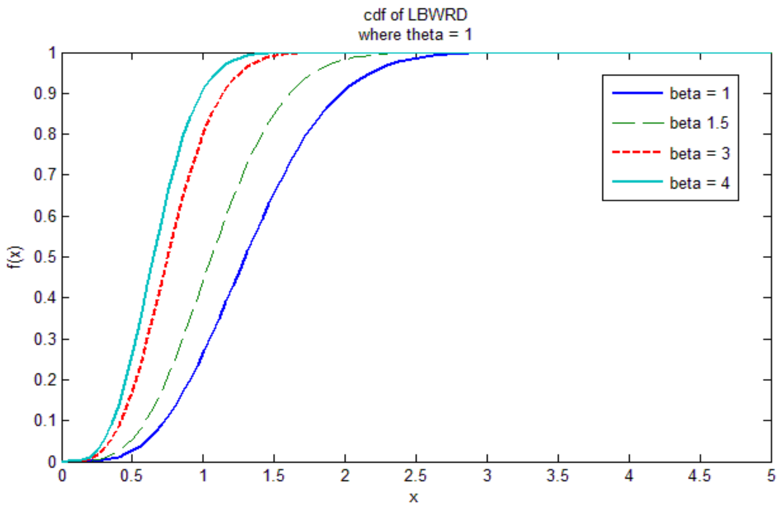


Figure (4): cdf of LBWRD

3. Mode of (LBW_1E_1D) and (LBWRD)

3.a. Mode of (LBW_1E_1D)

The first and second derivatives of (8) with respect to x are obtained as follows:

1) Taking logarithm of $f_L(x;\lambda)$ in (8) we get

$$\log[f_L(x;\lambda)] = \log x + 2\log(\lambda - n) - x(\lambda - n)$$

2) Derivation for the x gives

$$\frac{d \log f_L(x;\lambda)}{dx} = \frac{1}{x} - (\lambda - n)$$

3) Equating the above derivative to 0 leads to

$$\frac{1}{x} - (\lambda - n) = 0 \quad \Longrightarrow \quad x = \frac{1}{\lambda - n} \qquad (12)$$

4) The second derivation is as follows $\frac{d^2 \log f_L(x;\lambda)}{dx^2} = -\frac{1}{2x^2}$

3.b. Mode of (LBWRD)

By taking the logarithm of eq. (10), and derived relative to X we get

$$\frac{d\log g_L(x;\beta;\theta)}{dx} = \frac{6}{x} - \frac{2\,x\,\beta}{\theta}$$

Equating the above derivative to 0 leads to

$$\frac{6}{x} - \frac{2\,x\,\beta}{\theta} = 0 \Longrightarrow x^2 = \frac{3\,\theta}{\beta} \;\; \Longrightarrow x = \pm\sqrt{3\,\theta/\beta}. \qquad (13)$$

We take positive $x = \sqrt{3\,\theta/\beta}$.

And the second derivation is as follows $\frac{d^2 \log g_L(x;\lambda;\theta)}{dx^2} = -\frac{1}{x^2}$ (14)

λ	Mode x
2	1
3	0.5
4	0.3
5	0.25
6	0.2

Table(1): Mode of LBW_1E_1D

θ	β	Mode x
1	1	1,732
	2	1,224
	3	1
1	1	1.732
2		2.449
3		3

Table(2): Mode of LBWRD

4. Properties of (LBW_1E_1D) and (LBWRD)

Theorem 3: If X~ LBW_1E_1D ,then its *r*th moment about the origin and *r*th moment about the mean are given respectively as:

(a) $\mu'_r = E(x^r) = \frac{1}{(\lambda-n)^r}\Gamma(2+r)$, $\lambda > n$, $r = 1,2,\ldots$ (15)

(b) $E(x-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(-\mu)^j \frac{\Gamma(r-j+2)}{(\lambda-n)^{r-j}}$, $\lambda > n$, $r = 1,2,..$ (16)

And its mean , variance and coefficient of variation , skewness and kurtosis are as follows respectively:

$$\boldsymbol{Mean} = \mu'_1 = \frac{2}{\lambda-n} \qquad (17)$$

$$\boldsymbol{varince} = \frac{2}{(\lambda-n)^2} \qquad (18)$$

$$\boldsymbol{C.V} = 0.707 \qquad (19)$$

$$\boldsymbol{C.S} = 1.414 \qquad (20)$$

$$\boldsymbol{C.K} = 6 \qquad (21)$$

Proof (a):

$$E(x^r) = \int_0^\infty x^{r+1} \ (\lambda - n)^{\,2}\, e^{-x\,(\lambda - n)} \ dx$$

Let $u = x\,(\lambda - 1) \ \Rightarrow \ x = \frac{u}{\lambda - 1} \ \Rightarrow \ dx = \frac{du}{\lambda - 1}$

$$E(x^r) = \int_0^\infty \frac{(u/(\lambda - n)\,)^{r+1}(\lambda - n)^{\,2}\, e^{-u} du}{(\lambda - n)} = \frac{1}{(\,\lambda - n)^{\,r}}\,\Gamma\,(2 + r)$$

Proof (b):

$$Let \ \ (x - \mu) = \sum_{j=0}^{r} \binom{r}{j} (-\,\mu)^j \, x^{r-j}$$

$$E(x - \mu)^r = \sum_{j=0}^{r} \binom{r}{j} (-\,\mu)^j \int_0^\infty x^{r-j+1}\,(\lambda - n)^2 e^{-x\,(\lambda - n)} \ dx$$

By using transformation

$$u = x\,(\lambda - n) \ \Rightarrow \ x = \frac{u}{\lambda - n} \ \Rightarrow \ dx = \frac{du}{\lambda - n}$$

$$E(x - \mu)^r = \sum_{j=0}^{r} \binom{r}{j} (-\,\mu)^j \int_0^\infty \left(\frac{u}{\lambda - n}\right)^{r-j+1} (\lambda - n)^2 e^{-u} \ \frac{du}{(\lambda - n)}$$

$$\therefore \ E(x - \mu)^r = \sum_{j=0}^{r} \binom{r}{j} (-\,\mu)^j \ \frac{\Gamma\,(r - j + 2)}{(\,\lambda - n)^{\,r-j}}$$

Now we can use (15) to find the mean and the variance as follows:

If $r = 1$, then the $\quad \boldsymbol{Mean} = \mu_1' = E(x) = \frac{2}{\lambda - n}$

$$\boldsymbol{variance} = E(X)^2 - \mu^2 = \frac{2}{(\lambda - n)^2}$$

Or using (16) to find the variance and the other measures as follows:

$$E(X - \mu)^2 = \sum_{j=0}^{2} \binom{2}{j} (-\,\mu)^j \ \frac{\Gamma\,(2 - j + 2)}{(\,\lambda - n)^{\,2-j}} = \frac{6}{(\,\lambda - n)^{\,2}} - \frac{8}{(\,\lambda - n)^2} + \frac{4}{(\lambda - n\,)^2} = \frac{2}{(\lambda - n)^2}$$

And

$$\boldsymbol{C.V} = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\frac{2}{(\lambda - n)^2}}}{\frac{2}{\lambda - n}} = \ 0.707$$

$$\boldsymbol{C.S} = \frac{E(x - \mu)^3}{(\sigma^2)^{3/2}} = \frac{\frac{4}{(\lambda - n)^3}}{\left(\frac{2}{(\lambda - n)^2}\right)^{3/2}} = 1.414$$

$$\boldsymbol{C.K} = \frac{E(x-\mu)^4}{(\sigma^2)^2} = \frac{\frac{24}{(\lambda-n)^4}}{\left(\frac{2}{(\lambda-n)^2}\right)^2} = 6$$

Theorem 4: Let X~ LBWRD(θ ;β) ,then its rth moment about the origin and rth moment about the mean

(a) $\mu_r' = E(x^r) = \frac{\theta^{r/2}\Gamma\left(\frac{r-j+2}{2}\right)}{\beta^{r/2}}$ $,\frac{\beta}{\theta} > 0\,, r > -4$ (22)

(b) $E(x-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(-\mu)^j \frac{\beta^{\frac{j-r}{2}}}{\theta^{\frac{j-r}{2}}}\Gamma\left(\frac{r-j+2}{2}\right)$ $, r-j > 4\,, \frac{\beta}{\theta} > 0$ (23)

So its mean , variance and coefficient of variation , skewness and kurtosis, respectively are as follows:

$$\boldsymbol{Mean} = \frac{3\sqrt{\pi}\sqrt{\theta}}{4\sqrt{\beta}} \quad (24)$$

$$\boldsymbol{varince} = \frac{32\,\theta - 9\,\pi\,\theta}{16\,\beta} \quad (25)$$

$$\boldsymbol{C.V} = \frac{\sqrt{\frac{32\,\theta-9\,\pi\,\theta}{16\,\beta}}}{\frac{3\sqrt{\pi}\sqrt{\theta}}{4\sqrt{\beta}}} = 0.363 \quad (26)$$

$$\boldsymbol{C.S} = \frac{\frac{81\,\theta^{3/2}\,\pi^{3/2}}{64\,\beta^{3/2}} - \frac{21\,\theta^{3/2}\sqrt{\pi}}{8\,\beta^{3/2}}}{\left(\frac{32\,\theta-9\,\pi\,\theta}{16\,\beta}\right)^{3/2}} = 21.152 \quad (27)$$

$$\boldsymbol{C.K} = \frac{E(x\mu)^4}{(\sigma^2)^2} = \frac{\left(\frac{9\,\theta^2\pi}{8\,\beta^2} - \frac{243\,\theta^2\,\pi^2}{256\,\beta^2}\right)}{\left(\frac{32\,\theta-9\,\pi\,\theta}{16\,\beta}\right)^2} = 43{,}04 \quad (28)$$

Proof (a): $E(x^r) = \int_0^\infty \frac{2\,x^{3+r}\,\beta^2\, e^{\frac{-\beta x^2}{\theta}}}{\theta^2}\, dx$

Let $u = x\sqrt{\frac{\beta}{\theta}} \implies x = \frac{u}{\sqrt{\frac{\beta}{\theta}}} \implies dx = \frac{du}{\sqrt{\frac{\beta}{\theta}}}$

$$\mu_r' = E(x^r) = \int_0^\infty 2\left(\frac{u}{\sqrt{\beta/\theta}}\right)^{3+r} \frac{\beta^2\, e^{-u^2}\, du}{\theta^2\sqrt{\beta/\theta}}$$

$$\therefore\ \mu_r' = E(x^r) = \frac{2\,\beta^2}{\theta^2(\sqrt{\beta/\theta}\,)^{4+r}}\int_0^\infty u^{3+r}\, e^{-u^2}\, du$$

$$= \frac{\theta^{r/2}\,\Gamma\left(\frac{4+r}{2}\right)}{\beta^{r/2}}$$

Proof (b):

$$Let\ (x-\mu) = \sum_{j=0}^{r}\binom{r}{j}(-\mu)^j\, x^{r-j}$$

$$E(x-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(-\mu)^j \int_0^\infty 2\,x^{r-j+3}\,\frac{\beta^2}{\theta^2} e^{\frac{-\beta x^2}{\theta}}\, dx$$

By using transformation

$$u = x\sqrt{\frac{\beta}{\theta}} \quad \Rightarrow x = \frac{u}{\sqrt{\frac{\beta}{\theta}}} \quad \Rightarrow dx = \frac{du}{\sqrt{\frac{\beta}{\theta}}}$$

$$E(x-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(-\mu)^j \frac{\beta^2}{\theta^2}\int_0^\infty 2\left(\frac{u\sqrt{\theta}}{\sqrt{\beta}}\right)^{r-j+3} e^{-u^2} \frac{du\sqrt{\theta}}{\sqrt{\beta}}$$

$$E(x-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(-\mu)^j \frac{\beta^{\frac{j-r}{2}}}{\theta^{\frac{j-r}{2}}}\int_0^\infty 2\,u^{r-j+3}\,e^{-u^2}\,du$$

$$\therefore E(x-\mu)^r = \sum_{j=0}^{r}\binom{r}{j}(-\mu)^j \frac{\beta^{\frac{j-r}{2}}}{\theta^{\frac{j-r}{2}}}\,\Gamma\left(\frac{r-j+2}{2}\right)$$

Now the mean, variance using (22), again the variance and other can be measurements proved using (23) as in **(a)**.

Theorem 5: If X~LBW$_1$E$_1$D, then its moment generating function of X is

$$M_{x(f_L)}(t) = \frac{(n-\lambda)^2}{(n-\lambda+t)^2}\ ,n-\lambda+t<0 \qquad (29)$$

Proof:

$$M_{x(f_L)}(t) = E(e^{tx}) = \int_0^\infty x\,(\lambda-1)^2\,e^{-x[\,(\lambda-1)-t\,]}\,dx$$

By using transformation

$$u = x\,[(\lambda-n)-t] \quad \Rightarrow \quad x = \frac{u}{(\lambda-n)-t} \quad \Rightarrow dx = \frac{du}{(\lambda-n)-t}$$

$$\therefore M_{x(f_L)}(t) = E(e^{tx}) = \frac{(n-\lambda)^2}{(n-\lambda+t)^2}$$

Theorem 6: Let X have of length-Biased weighted Rayleigh distribution **(LBW RD**) ,Then the moment generating function of X, say $M_x(t)$, is

$$M_{x(g_L)}(t) = \sum_{i=0}^{\infty}\frac{t^i\,\Gamma\,(4+i/2)\,\theta^{\frac{i}{2}}}{i!\ \beta^{\frac{i}{2}}} \qquad (30)$$

Proof:

$$M_{x(g_L)}(t) = E(e^{tx}) = \int_0^\infty e^{tx} f(x)\,dx$$

$$= \int_0^{\infty} \left(1 + tx + \frac{t^2x^2}{2!} + \cdots + \frac{t^n x^n}{n!} + \cdots\right) f(x)\ d\,x$$

$$= \sum_{i=0}^{\infty} \frac{t^i E(x^i)}{i!} = \sum_{i=0}^{\infty} \frac{t^i\, \Gamma\,(4+i/2)\, \theta^{\frac{i}{2}}}{i!\ \beta^{\frac{i}{2}}}$$

5. Reliability Analysis :

a) Reliability Analysis of LBW$_1$E$_1$D :

1) Reliability Function :

$$R_{f_L}(x;\lambda) = 1 - F_L(x;\lambda)$$

$$= e^{-x(\lambda-n)}(1 - x\,n + x\,\lambda) \qquad (31)$$

Table (3) contains the values of survival function (31). Looking at this table we can see that the survival probability of the distribution decreases with increase in the value of λ and n=1 , for a holding x .Further, from the table we can see that; for fixed λ ; the survival probability decreases with increase in x.

Table (3) : Reliability function of LBW$_1$E$_1$D

x \ λ	1.5	2	2.5	3	3.5
0.1	0.998791	0.995321	0.989814	0.982477	0.973501
0.2	0.995321	0.982477	0.963064	0.938448	0.909796
0.3	0.989814	0.963064	0.924561	0.878099	0.826641
0.4	0.982477	0.938448	0.878099	0.808792	0.735759
0.5	0.973501	0.909796	0.826641	0.735759	0.644636

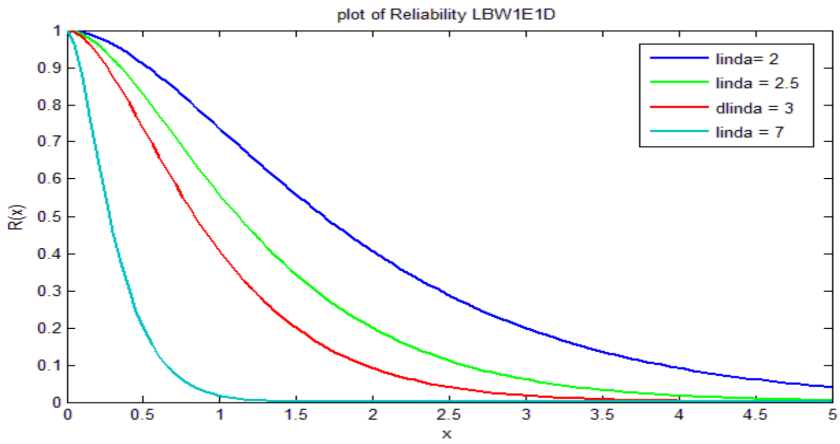


Figure (5) : Reliability of LBW$_1$E$_1$D

Figure (5): illustrates that the reliability behavior of weighted exponential distribution as the value of the parameter λ= (2,2.5,3,7) and n=1 , It is clear that all curves intersect at the point of 1, as x → ∞ then the survival function approaches to zero.

2) Hazard functions

It is an important quantity characterizing life phenomenon. It can be loosely interpreted as the conditional probability of failure, given it has survived to Timex the hazard rate function for of a length-Biased weighted exponential distribution(LBW$_1$E$_1$D), random variable is given by

$$h_{f_L}(x;\lambda) = \frac{f_L\,(x;\lambda)}{R_{f_L}(x;\lambda)} = \frac{x\,(\lambda-n)^{\,2}\,e^{-x\,(\lambda-n)}}{e^{-x(\lambda-n)}(1-nx+x\,\lambda)} \qquad (32)$$

The figures below shows the hazard function of LBW$_1$E$_1$D

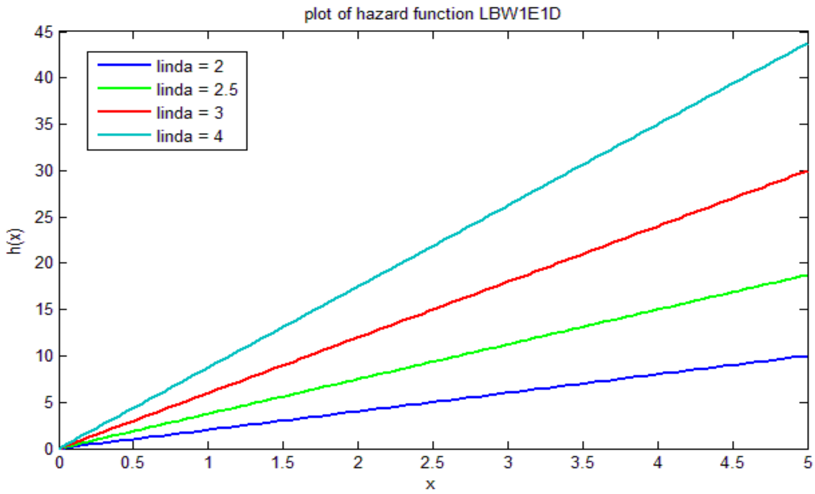


Figure (6) indicates the behavior of function hazard. It is clear that increasing Linda lead to increased failure rate more.

3) Reverse hazard functions

The reverse hazard function of LBW$_1$E$_1$D is given by

$$\varphi_{f_L}(x;\lambda) = \frac{f_L\,(x;\lambda)}{F_L\,(x;\lambda)} = \frac{x\,(\lambda-n)^{\,2}\,e^{-x\,(\lambda-n)}}{1-e^{-x(\lambda-n)}(1-nx+x\,\lambda)} \qquad (33)$$

The figure below shows the reverse hazard functions of LBW$_1$E$_1$D

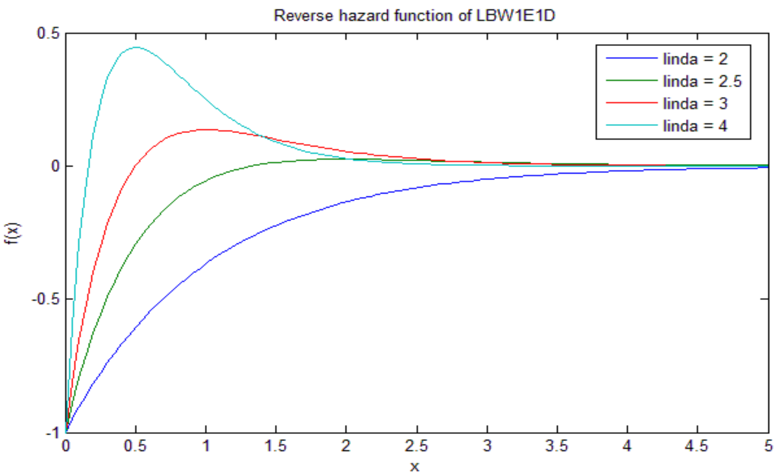


Figure (7): Reverse hazard function of LBW$_1$E$_1$D

Figure (7) indicates the behavior of Reverse function hazard for λ=(2,2.5,3,4) and n=1 .

b) Reliability Analysis of LBWRD:

1) Reliability functions:

$$R_{g_L}(x;\theta;\beta) = 1 - G_L(x;\theta;\beta) = \frac{e^{\frac{-\beta x^2}{\theta}}(x^2\beta+\theta)}{\theta} \qquad (34)$$

Table (4) contains the values of survival function (34). Looking at this table we can see that the survival probability of the distribution decreases with increase in the value of β for a holding x and θ at a fixed level. Further, from the table we can see that; for fixed θ and β; the survival probability decreases with increase in x.

θ=1					
x \ β	0.1	0.2	0.3	0.4	0.5
1	0.993521	0.982477	0.963064	0.938448	0.909796
2	0.938448	0.808792	0.662627	0.524931	0.406006
3	0.772482	0.462837	0.28466	0.12568	0.061099
4	0.524931	0.171201	0.04773	0.01229	0.003019
5	0.287297	0.040427	0.00470	0.00049	0.00005

Table (4): Survival function of LBWRD

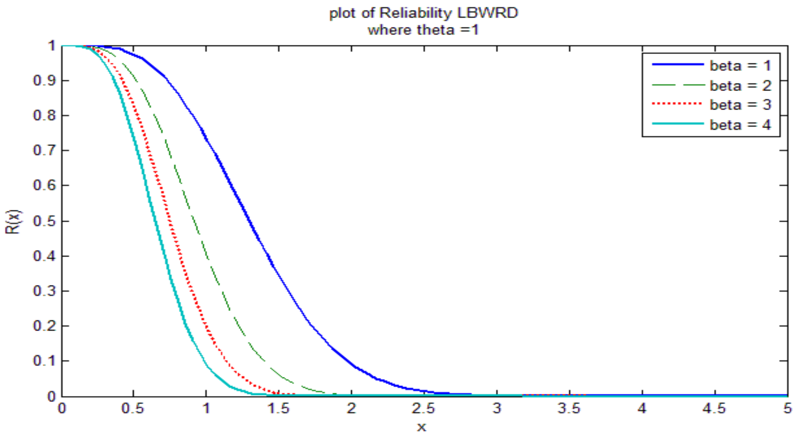


Figure (8) : Reliability of LBWRD

Figure (8) shows the behavior of the reliability function when θ=1 and β(=1,2,3,4). It is clear that all curves intersect at a point 1, and then the reliability function of approachs to zero as x →∞

2) Hazard functions

The hazard function of LBWRD is given by

$$h_{g_L}(x;\theta;\beta) = \frac{g_L(x;\theta;\beta)}{R_{g_L}(x;\theta;\beta)} = \frac{\frac{2x^3\beta^2 e^{\frac{-\beta x^2}{\theta}}}{\theta^2}}{\frac{e^{\frac{-\beta x^2}{\theta}}(x^2\beta+\theta)}{\theta}} \qquad (35)$$

The figure below shows the hazard function of LBWRD

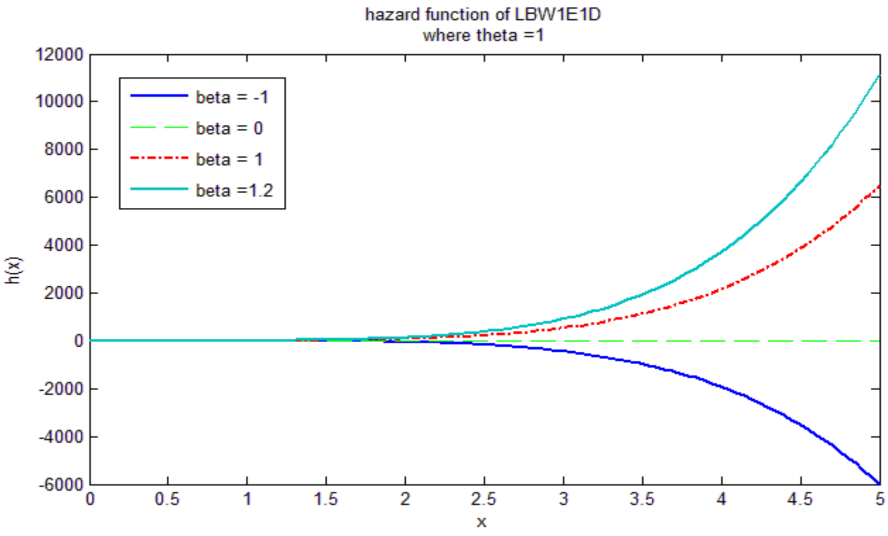


Figure (9): Hazard function of LBWRD

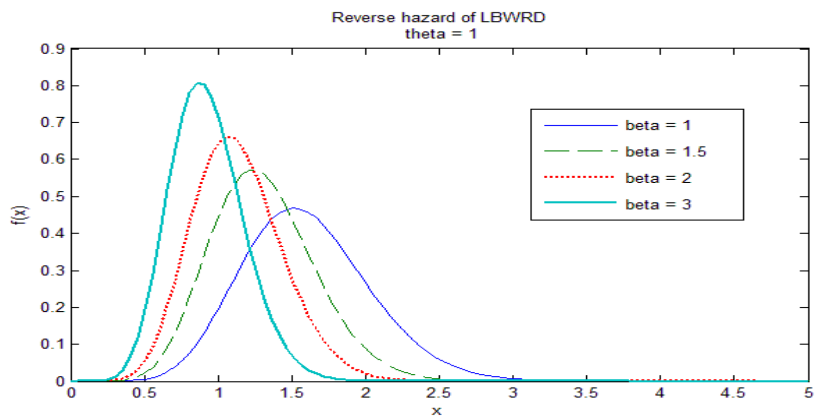
Figure (9): indicates to behavior of function hazard on θ=1 and β(= -1,0, 1 ,1.2) ,It is clear that the increase in value of the parameter β leads to increased failure rate more.

3) Reverse hazard functions

the Reverse hazard functions of LBWRD is given by

$$\varphi_{g_L}(x;\lambda) = \frac{g_L\ (x;\theta;\beta)}{G_L\ (x;\theta;\beta)} == \frac{\dfrac{2\,x^3\beta^2\ e^{\frac{-\beta x^2}{\theta}}}{\theta^2}}{1 - \dfrac{e^{\frac{-\beta x^2}{\theta}}(x^2\beta + \theta)}{\theta}} \qquad (36)$$

The figure below shows the Reverse hazard functions of LBWRD



Figure(10): indicates to behavior of Reverse hazard function of LBWRD

6. METHODES OF ESTIMATION

a. Maximum Likelihood Estimator of LBW_1E_1D

This is the one of the most important methods of appreciation aims to make possible a function at the end of maximizes. All that is done to write down the likelihood function L(x; λ), and then find the value $\hat{\lambda}$ of λ which maximizes L(x; λ).

The log-likelihood function based on the random sample $x_1, x_2, \dots, x_m$ of LBW_1E_1D, using the weight function ($w_1(x;\lambda)$) is given by:

$$L(x_1, \dots\dots, x_m, \lambda) = \prod_{i=1}^{m} x_i \ (\lambda - n)^{2m} \, e^{-\sum_{i=1}^{m} x_i(\lambda-n)} \tag{37}$$

$$\log L\,(x_1, \dots\dots, x_m, \lambda) = 2m \log(\lambda - n) + \sum_{i=1}^{m} \log x_i - \sum_{i=1}^{m} x_i(\lambda - n)$$

Then by take the partial derivative of log $L(x_1, \dots\dots, x_n, \lambda)$ we get

$$\frac{\partial logL(x_1,\dots\dots,x_m,\lambda)}{\partial\lambda} = \frac{2m}{(\lambda-n)} - \sum_{i=1}^{m} x_i$$

The solving of the equations $\frac{\partial logL(x_1,\dots\dots,x_m,\lambda)}{\partial\lambda} = 0$, yields the maximum likelihood

$$\hat{\lambda}_{mle} = \frac{2m}{\sum_{i=1}^{m} x_i} + n \tag{38}$$

2.2.5.1. Maximum Likelihood Estimator of LBWRD

We find the value $\hat{\beta} and\ \hat{\theta}$ of β , θ,respectively, which maximizes $L\,(x;\theta;\beta)$ The log-likelihood function based on the random sample $x_1, x_2, \dots\dots, x_m$ of LBWRD, using the weight function ($w_1(x;\beta;\theta)$) is given by:

$$L = \prod_{i=1}^{n} f_L(x\,;\beta\,;\,\theta\,)$$

$$L(x_1, \dots\dots, x_m, \beta;\theta) = \prod_{i=1}^{m} x^{\,3m} \beta^{2m}\, \theta^{-2m} \; e^{-\sum_{i=1}^{m}\frac{x_i^2\beta}{\theta}} \tag{39}$$

$$\log L\,(x_1, \dots\dots, x_m, \beta;\theta) = 6m \log x + 2m\, log\, \beta - 2m \log\theta - \sum_{i=1}^{m} \frac{x_i^2 \beta}{\theta}$$

Then by take the partial derivative of log $L(x_1, \dots\dots, x_n, \beta;\theta)$ we get

$$\frac{\partial logL(x_1,\dots\dots,x_m,\beta;\theta)}{\partial\beta} = \frac{2m}{\beta} - \frac{\sum_{i=1}^{m} x_i^2}{\theta}$$

The solving of the equations $\frac{\partial logL(x_1,\dots\dots,x_m,\beta;\theta)}{\partial\beta} = 0$, yields the maximum likelihood

$$\hat{\beta}_{mle} = \frac{\sum_{i=1}^{m} x_i^2}{2\,n\,\theta} \tag{40}$$

Now we find $\hat{\theta}_{mle}$

$$\frac{\partial logL(x_1, \dots\dots, x_m, \beta;\theta)}{\partial\theta} = \frac{-2m}{\theta} + \frac{\beta \sum_{i=1}^{m} x_i^2}{\theta^2}$$

The solving of the equations $\frac{\partial logL(x_1,\dots\dots,x_m,\beta;\theta)}{\partial\theta} = 0$, yields the maximum likelihood

$$\hat{\theta}_{mle} = \frac{\hat{\beta}_{mle} \sum_{i=1}^{m} x_i^2}{2\,n} \tag{41}$$

APPLICATION

The chi-square test of goodness of fit is applied to data are times when failure to the operating in months to fifty organ of protection adopted by the General Company for Electronic Industries in production[10] ,and tabulated data in the frequency table and extracted the probabilities for each cell and the expected frequencies by formula chi - Square**.**

$$\chi^2 = \sum_{i=1}^{k} \frac{(O_i - E_i)^2}{E_i}$$

Table 5: Fitting of LBWRD for data protection devices for the General Company for Electronic Industries

Class Interval	Observed Frequency(O_i)	Expected Frequency(E_i)	$(O_i - E_i)^2/E_i$
0.9 – 1.8	8	6.5	0.346
1.8 – 2.7	16	15	0.066
2.7 -3.6	14	15	0.066
3.6 – 4.5	5	9	1.777
4.5 – 5.4	4	3	0.333
5.4 – 6.3	2	1	1.000
6.3 – 7.2	1	0.12	6.453
Total	50	50	10.04

$\beta = 0.25\,, \theta = 1.12\,,\ \chi^2 = 10.04$

The following hypotheses are tested

H_0 : Data is distributed length biased weighted Rayleigh distribution (two parameters)
H_1 : Data is not length biased weighted Rayleigh distribution (two parameters)

The Statistical Decision

Since $\Pr\,(\mathrm{Q_{Calculated}} < \mathrm{Q_{tabel}}; \mathrm{H_0}) = 0.05$, where $\mathrm{Q_{tabel}} = 12.6$

So we accept H_0

CONCLUSION

In this article, the LBWRD, LBW_1E_1D are introduced. And new distribution LBWRD is turns out to be quite flexible for modeling the quality of the data protection devices for the Electronic Industries . An application to the real data showed that this new model is a flexible alternative to other well-known models, that might be of use for practitioners in the applied sciences (the industrial).

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