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Common Fixed Point Theorems for Countable Faintly Compatible Mappings in Fuzzy Metric Space

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Abstract

In the present paper, we prove some fixed point theorems for countable faintly compatible mappings in fuzzy metric space. Our results extend and generalized the result of S. Manro and A. Tomar [Faintly compatible maps and existence of common fixed points in fuzzy metric space, Anal. of Fuzzy Math. and Informatics, 10 (2014), 1-8].

Keywords: Fuzzy Metric Spaces, non-compatible mappings, faintly compatible mappings and reciprocally continuous mappings.

1. Introduction

One of the weaker forms of the commuting mappings is weak compatibility. Many researchers use this concept to prove the existence of unique common fixed point in fuzzy metric space. Al-Thagafi and Shahzad [2] introduced the concept of (owc) occasionally weakly compatible and weaken the concept of nontrivial weakly compatible maps. Pant and Pant [8], redefined it as conditionally commuting maps. Pant and Bisht [7] introduced the concept of conditional compatible maps. Faintly compatible maps introduced by Bisht and Shahzad [3], as an improvement of conditionally compatible maps. This gives the existence of a common fixed point or multiple fixed point or coincidence points under contractive and non-contractive conditions.

S. Manro and A. Tomar [4] recently proved existence of common fixed points in fuzzy metric space using faintly compatible maps. The aim of this paper is to extend and generalize their result and prove the existence of common fixed point for countable faintly compatible maps in fuzzy metric space.

2. Preliminaries

In this section, we recall some definitions and useful results which are already in the literature.

Definition 2.1[9]: A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is continuous t- norm if $*$ satisfies the following conditions:

(i) $*$ is commutative and associative; (ii) $*$ is continuous; (iii) $a * 1 = a\ \forall\ a\in[0,1]$; (iv) $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d\ \forall a, b, c, d\in[0,1]$.

Example of continuous t-norm 2.2[9]: $a * b = \min\{a, b\}$, minimum t-norm.

George and Veeramani modified the nothing of fuzzy metric space of Kramosil and Michalek as follows:

Definition 2.3: The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions: $\forall x, y, z\in X$, $t, s>0$;

(1) $M(x, y, t) > 0$;

(2) $M(x, y, t) = 1$ iff $x = y$;

(3) $M(x, y, t) = M(y, x, t)$;

(4) $M(x, y, t)*M(y, z, s) \leq M(x, z, t + s)$;

(5) $M(x, y, \cdot): [0,\infty) \to [0, 1]$ is continuous.

Definition 2.4: A pair of self-maps (A, S) on a fuzzy metric space (X, M, *) is said to be

(a) Non-compatible: if (A, S) is not compatible, i.e., if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n = x$, for some $x \in X$, and $\lim_{n\to\infty}M(ASx_n, SAx_n, t) \neq 1$ or non-existent $\forall\ t > 0$.

(b) Conditionally compatible [7]: if whenever the set of sequences $\{x_n\}$ satisfying $\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n$, is non-empty, there exists a sequence $\{z_n\}$ in X such that $\lim_{n\to\infty}Az_n = \lim_{n\to\infty}Sz_n = t$, for some $t\in X$ and $\lim_{n\to\infty}M(ASz_n, SAz_n, t) = 1$ for all $t > 0$.

(c) Reciprocally continuous [6]: if $\lim_{n\to\infty}ASx_n= Ax$, $\lim_{n\to\infty}SAx_n = Sx$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n = x$, for some $x \in X$.

(d) Faintly compatible [3]: if (A, S) is conditionally compatible and A and S commute on a non-empty subset of the set of coincidence points, whenever the set of coincidence points is nonempty.

(e) Satisfy the property (E.A.) [1]: if there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n = x$, for some $x \in X$.

(f) Sub Sequentially continuous [10]: iff there exists a sequence $\{x_n\}$ in X such that $\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n = x$, $x\in X$ and satisfy $\lim_{n\to\infty}ASx_n= Ax$ and $\lim_{n\to\infty}SAx_n = Sx$.

Note that, compatibility, non-compatibility and faint compatibility are independent concepts. Faintly compatibility is applicable for mappings that satisfy contractive and non contractive conditions.

Lemma 2.5[5]: Let (X, M, *) be a fuzzy metric space and for all $x, y \in X$, $t > 0$ and if there exists a constant $k\in(0,1)$ such that $M(x, y, kt) \geq M(x, y, t)$ then $x = y$.

Now, we prove fixed point theorems for pair of faintly compatible mappings.

3. Main Result:

Theorem 3.1: Let (A, S) and (B, T) be non-compatible, reciprocally continuous faintly compatible pair of self mappings of a fuzzy metric spaces (X, M, *) satisfying the following condition:

(3.1.1) $M(Ax, By, kt) \geq \min\{M(Sx, Ty, t), M(Sx, By, t), M(Ty, By, t)\}$;

$\forall\ x, y\in X$ & for some $k\in(0,1)$ & $t>0$. Then A, B, S and T have a unique common fixed point in X.

Proof: Non-compatibility of (A, S) and (B, T) implies that there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty}Ax_n = \lim_{n\to\infty}Sx_n = t_1$ for some $t_1\in X$, and $M(ASx_n, SAx_n, t)\neq1$ or nonexistent $\forall\ t > 0$; Also $\lim_{n\to\infty}Bx_n = \lim_{n\to\infty}Tx_n = t_2$ for some $t_2\in X$, and $M(BTx_n, TBx_n, t)\neq1$ or nonexistent $\forall\ t > 0$.

Since pairs (A, S) and (B, T) are faintly compatible therefore conditionally compatibility of

(A, S) and (B, T) implies that there exist sequences $\{z_n\}$ and $\{z_n'\}$ in X satisfying

$\lim_{n\to\infty}Az_n = \lim_{n\to\infty}Sz_n = u$ for some $u\in X$, such that $M(ASz_n, SAz_n, t)=1$;

Also $\lim_{n\to\infty}Bz_n' = \lim_{n\to\infty}Tz_n' = v$ for some $v\in X$, such that $M(BTz_n', TBz_n', t)=1$.

As the pairs (A, S) and (B, T) are reciprocally continuous, we get

$\lim_{n\to\infty}ASz_n = Au$, $\lim_{n\to\infty}SAz_n = Su$ and so $Au = Su$;

Also $\lim_{n\to\infty}BTz_n' = Bv$, $\lim_{n\to\infty}TBz_n' = Tv$ and so $Bv = Tv$.

Since pairs (A, S) and (B, T) are faintly compatible, we get

ASu = SAu & so AAu=ASu=SAu=SSu; and Also BTv=TBv & so BBv=BTv=TBv=TTv.

Now we show that Au=Bv, AAu= Au and BBv=Bv.

By taking x=u and y=v in (3.1.1)

$M(Au, Bv, kt) \geq \min\{M(Su, Tv, t), M(Su, Bv, t), M(Tv, Bv, t)\};$

$M(Au, Bv, kt) \geq \min\{M(Au, Bv, t), M(Au, Bv, t), M(Bv, Bv, t)\};$

$M(Au, Bv, kt) \geq \min\{M(Au, Bv, t), M(Au, Bv, t), 1\};$

$M(Au, Bv, kt) \geq M(Au, Bv, t)$, by lemma (2.5), we have Au=Bv.

Taking x=Au and y=v in (3.1.1),

$M(AAu, Bv, kt) \geq \min\{M(SAu, Tv, t), M(SAu, Bv, t), M(Tv, Bv, t)\};$

$M(AAu, Bv, kt) \geq \min\{M(AAu, Bv, t), M(AAu, Bv, t), M(Bv, Bv, t)\};$

$M(AAu, Bv, kt) \geq \min\{M(AAu, Bv, t), M(AAu, Bv, t), 1\};$

$M(AAu, Bv, kt) \geq M(AAu, Bv, t)$, by lemma (2.5), we have AAu=Bv. Therefore AAu=Bv=Au.

Taking x=u and y=Bv in (3.1.1),

$M(Au, BBv, kt) \geq \min\{M(Su, TBv, t), M(Su, Bbv, t), M(TBv, BBv, t)\};$

$M(Au, BBv, kt) \geq \min\{M(Au, BBv, t), M(Au, BBv, t), M(BBv, BBv, t)\};$

$M(Au, BBv, kt) \geq \min\{M(Au, BBv, t), M(Au, BBv, t), 1\};$

$M(Au, BBv, kt) \geq M(Au, BBv, t)$, by lemma (2.5), we have Au=BBv. Therefore BBv=Au=Bv.

Now, we have AAu=SAu=Au, Au= BBv=BAu and Au= BBv=TBv=TAu, since Bv=Au.

Hence AAu=SAu=BAu=TAu=Au, i.e. Au is a common fixed point of A, B, S and T.

The uniqueness follows from (3.1.1).This completes the proof of the theorem.

Now we are generalizing our result for six mappings and prove the following theorem.

Theorem 3.2: Let (A, SP) and (B, TQ) be non-compatible, reciprocally continuous faintly compatible pair of self mappings of a fuzzy metric spaces (X, M, *) satisfying the following condition:

(3.2.1) Pair (A, P), (S, P), (B, Q), (T, Q) are commuting;

(3.2.2) $M(Ax, By, kt) \geq \min\{M(SPx, TQy, t), M(SPx, By, t), M(TQy, By, t)\};$

$\forall x, y \in X$ and for some $k \in (0, 1)$ & $t > 0$. Then A, B, S, T, P and Q have a unique common fixed point in X.

Proof: Non-compatibility of (A, SP) and (B, TQ) implies that there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} (SP)x_n = t_1$ for some $t_1 \in X$, and $M(A(SP)x_n, (SP)Ax_n, t) \neq 1$ or nonexistent $\forall t > 0$; Also

$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} (TQ)x_n = t_2$ for some $t_2 \in X$, and $M(B(TQ)x_n, (TQ)Bx_n, t) \neq 1$ or nonexistent $\forall t > 0$.

Since pairs (A, SP) and (B, TQ) are faintly compatible therefore conditionally compatibility of (A, SP) and (B, TQ) implies that there exist sequences $\{z_n\}$ and $\{z_n'\}$ in X satisfying

$\lim_{n\to\infty} Az_n = \lim_{n\to\infty} (SP)z_n = u$ for some $u \in X$, such that $M(A(SP)z_n, (SP)Az_n, t)=1$;

Also $\lim_{n\to\infty} Bz_n' = \lim_{n\to\infty} (TQ)z_n' = v$ for some $v \in X$, such that $M(B(TQ)z_n', (TQ)Bz_n', t)=1$.

As the pairs (A, SP) and (B, TQ) are reciprocally continuous, we get

$\lim_{n\to\infty} A(SP)z_n = Au$, $\lim_{n\to\infty} (SP)Az_n = (SP)u$

and so Au = (SP)u i.e. (u is coincidence point of A and (SP));

Also $\lim_{n\to\infty} B(TQ)z_n' = Bv$, $\lim_{n\to\infty} (TQ)Bz_n' = (TQ)v$

and so Bv = (TQ)v i.e. (v is coincidence point of B and (TQ)).

Since pairs (A, SP) and (B, TQ) are faintly compatible, we get

A(SP)u=(SP)Au & so AAu=A(SP)u=(SP)Au=(SP)(SP)u;

and Also B(TQ)v=(TQ)Bv & so BBv=B(TQ)v=(TQ)Bv=(TQ)(TQ)v.

Now, we show that Au=Bv, AAu= Au, BBv=Bv, PAu=Au and QAu=Au.

By taking x=u and y=v in (3.2.2),

$M(Au, Bv, kt) \geq \min\{M((SP)u, (TQ)v, t), M((SP)u, Bv, t), M((TQ)v, Bv, t)\};$

$M(Au, Bv, kt) \geq \min\{M(Au, Bv, t), M(Au, Bv, t), M(Bv, Bv, t)\};$

$M(Au, Bv, kt) \geq \min\{M(Au, Bv, t), M(Au, Bv, t), 1\};$

$M(Au, Bv, kt) \geq M(Au, Bv, t)$, by lemma (2.5), we have Au=Bv.

By taking x=Au and y=v in (3.2.2),

$M(AAu, Bv, kt) \geq \min\{M((SP)Au, (TQ)v, t), M((SP)Au, Bv, t), M((TQ)v, Bv, t)\};$

$M(AAu, Bv, kt) \geq \min\{M(AAu, Bv, t), M(AAu, Bv, t), M(Bv, Bv, t)\};$

$M(AAu, Bv, kt) \geq \min\{M(AAu, Bv, t), M(AAu, Bv, t), 1\};$

$M(AAu, Bv, kt) \geq M(AAu, Bv, t)$, by lemma (2.5), we have AAu=Bv. Therefore AAu=Bv=Au.

By taking x=u and y=Bv in (3.2.2),

$M(Au, BBv, kt) \geq \min\{M((SP)u, (TQ)Bv, t), M((SP)u, BBv, t), M((TQ)Bv, BBv, t)\};$

$M(Au, BBv, kt) \geq \min\{M(Au, BBv, t), M(Au, BBv, t), M(BBv, BBv, t)\};$

$M(Au, BBv, kt) \geq \min\{M(Au, BBv, t), M(Au, BBv, t), 1\};$

$M(Au, BBv, kt) \geq M(Au, BBv, t)$, by lemma (2.5), we have Au=BBv. Therefore BBv=Au=Bv.

Now we have AAu=(SP)Au=Au, Au=BBv=BAu and Au= BBv=(TQ)Bv=(TQ)Au, since Bv=Au.

Hence AAu=(SP)Au=BAu=(TQ)Au=Au and Au is a common coincidence point of A, B, SP and TQ.

By taking x=PAu and y=Au in (3.2.2),

$M(APAu, BAu, kt) \geq \min\{M((SP)PAu, (TQ)Au, t), M((SP)PAu, BAu, t), M((TQ)Au, BAu, t)\};$

Since (A, P) and (S, P) are commuting, therefore,

$M(PAAu, BAu, kt) \geq \min\{M(PSPAu, (TQ)Au, t), M(PSPAu, BAu, t), M((TQ)Au, BAu, t)\};$

$M(PAu, Au, kt) \geq \min\{M(PAu, Au, t), M(PAu, Au, t), M(Au, Au, t)\};$

$M(PAu, Au, kt) \geq \min\{M(PAu, Au, t), M(PAu, Au, t), 1\}$;

$M(PAu, Au, kt) \geq M(PAu, Au, t)$, by lemma (2.5), we have PAu= Au.

By taking x=Au and y=QAu in (3.2.2),

$M(AAu, BQAu, kt) \geq \min\{M((SP)Au, (TQ)QAu, t), M((SP)Au, BQAu, t), M((TQ)QAu, BQAu, t)\}$;

Since (B, Q) and (T, Q) are commuting, therefore

$M(Au, QAu, kt) \geq \min\{M(Au, QAu, t), M(Au, QAu, t), M(QAu, QAu, t)\}$;

$M(Au, QAu, kt) \geq \min\{M(Au, QAu, t), M(Au, QAu, t), 1\}$;

$M(Au, QAu, kt) \geq M(Au, QAu, t)$, by lemma (2.5), we have Au=QAu.

Therefore AAu=(SP)Au=BAu=(TQ)Au=Au $\Rightarrow$ AAu=SPAu=SAu and BAu=TQAu=TAu.

Hence AAu=BAu=SAu=TAu=PAu=QAu=Au, i.e. Au is a common fixed point of A, B, S, T, P and Q in X.

The uniqueness follows from (3.2.2).This completes the proof of the theorem.

Now we generalize our result for countable mappings.

Theorem 3.3: Let (X, M, *) be a fuzzy metric spaces with continuous t-norm and S, T, P, Q and A_i $\forall i \in$ **W** (Set of whole numbers) are self mappings on X, such that:

(3.3.1) (A_0, SP) and (A_i, TQ) be non-compatible, reciprocally continuous faintly compatible, $\forall i \in$ **W-{0};**

(3.3.2) Pair (A_0, P), (S, P), (A_i, Q), (T, Q) are commuting;

(3.3.3) for each $i \in$ **W-{0}**, $M(A_0x, A_iy, kt) \geq \min\{M(Sx, TQy, t), M(SPx, A_iy, t), M(TQy, A_iy, t)\}$;

$\forall$ x, y$\in$X and for some k$\in$(0, 1) & t > 0. Then S, T, P, Q and A_i $\forall i \in$ **W** have a unique common fixed point in X.

Proof: The proof of this theorem follows from theorem 3.2.

4. Remark

1. Our result is generalization and extension of S. Manro and A. Tomar [4] in the sense of countable faintly compatible mappings.
2. If we replace the condition of non-compatibility of pairs of mappings by Property (E. A.) then our theorems 3.1, 3.2 and 3.3 will be true.
3. If we replace reciprocally continuity by sub sequentially continuity of pairs of mappings then our theorems 3.1, 3.2 and 3.3 will be true.
4. If we replace inequality (3.1.1) by following rational inequality:

   (4.1.1) $M(Ax, By, kt) \geq \min\left\{M(Sx, Ty, t), \frac{cM(Sx,By,t)+dM(Sx,Ty,t)}{cM(By,Ty,t)+d}, \frac{eM(By,Ty,t)+fM(Sx,Ax,t)}{e+f}\right\}$;

   where c, d, e, f≥0 with c&d and e&f cannot be simultaneously 0.

   Then our results will be more improved and more generalized.

5. If (3.1.1) replaced by $M(Ax, By, t) > \min\{M(Sx, Ty, t), M(Sx, By, t), M(Ty, By, t)\}$ with Sx≠Ty≠By, $\forall$x, y$\in$X & t > 0, then our theorems are also true with strict contractive condition.
6. If (3.1.1) replaced by $M(Ax, By, t) \neq \min\{M(Sx, Ty, t), M(Sx, By, t), M(Ty, By, t)\}$ with right hand side ≠ 1, $\forall$x, y$\in$X & t > 0, then our theorems are also true with Lipschitz type condition.
7. These results can be proved using implicit relation.

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