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Coupled Fixed Point Theorems in Partially Ordered Metric Space

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Abstract: - There is several generalization of Banach contraction principle. Recently Bhaskaran and Lakshmikantham generalized this result and prove coupled fixed point theorems in ordered metric space. In this present work, we proof some coupled fixed point theorems in ordered metric space.

Key words: - Ordered Metric Space, Fixed point, Coupled Fixed point, mixed monotone property.

Introduction

The Banach contraction principle is one of the pivotal results of analysis. It is widely considered as the source if metric fixed point theory. Also, its significance lies in its vast applicability in a number of branches of mathematics. Generalization of the above principle has been a heavily investigated branch of research.

The fixed points of mappings in ordered metric space are of great use in many mathematical problems in applied and pure mathematics. The first result in this direction was obtained by Ran and Reurings [1], in this study, the authors present some applications of their obtained results of matrix equations. In [2], Nieto and Lopez extended the result of Ran and Reurings [3],for non decreasing mappings and applied their result to get a unique solution for a first order differential equation. While Agrawal et al.[4] and O'Regan and Petrutel [5] studied some results for generalized contractions in ordered metric spaces. Bhaskar and Lakshmikantham [6] introduced the notion of a coupled fixed point of mapping F from $X \times X$ into X. They established some coupled fixed point results and applied there results to the study of existence and uniqueness of solution for a periodic boundary value problem. Lakshmikantham and Ciric [7] introduced the concept of coupled coincidence point and proved coupled coincidence and coupled common fixed point results for mappings F from $X \times X$ into X and g from X into X satisfying non linear contraction in ordered metric space.

In this paper, we drive new coupled fixed point theorems for mapping having the mixed monotone property in partially ordered metric space.

1. Preliminaries

We recall the definitions and results that will be needed in the sequel.

Definition 2.1 A partially ordered set is a set P and a binary relation $\leq$, denoted by $(X, \leq)$ such that for all $a, b, c \in P$

(i) $a \leq a$, (reflexivity)

(ii) $a \leq b$ and $b \leq c$ implies $a \leq c$, (transitivity)

(iii) $a \leq b$ and $b \leq a$ implies $a = b$. (anti − symmetry)

Definition 2.2 A sequence $\{x_n\}$ in a metric space (X, d) is said to be convergent to a point $x \in X$, denoted by $\lim_{n \to \infty} x_n = x$, if $\lim_{n \to \infty} d(x_n, x) = 0.$

Definition 2.3 A sequence $\{x_n\}$ in a metric space (X, d) is said to be Cauchy sequence if if $\lim_{n \to \infty} d(x_n, x_m) = 0.$ for all $n, m > t.$

Definition 2.4 A metric space is said to be complete if every Cauchy sequence in X is convergent.

Definition 2.5 Let $(X, <)$ be a partially ordered set and $F : X \times X \to X$. The mapping F is said to has the mixed monotone property if $F(x, y)$ is non – decreasing in x and is monotone non-increasing in y, that is, for any $x, y \in X$,

$x_1, x_2 \in X,\ x_1 \leq x_2 \ \Rightarrow\ F(x_1, y) \leq F(x_2, y)$

and

$y_1, y_2 \in X,\ y_1 \leq y_2 \ \Rightarrow\ F(x, y_1) \geq F(x, y_2)$

Definition 2.6 An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if

$x = F(x, y)$ and $y = F(y, x)$

Theorem 2.7 Let $(X, \leq)$ be a partially ordered set and suppose there exists a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \to X$ be a continuous mapping having the mixed monotone property on X. assume that there exists a $\alpha \in [0,1)$ with

$$d\big(F(x, y), F(u, v)\big) \leq \frac{\alpha}{2}[\, d(x, y) + d(u, v)]$$

For all $x \geq u$ and $y \leq v$. if there exist two elements $x_0, y_0 \in X$ with

$x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$

then there exist $x, y \in X$ such that $x = F(x, y)$ and $y = F(y, x)$.

2. Main Results

Theorem 3.1 Let $(X, \leq)$ be a partially ordered set and suppose there exists a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \to X$ be a continuous mapping having the mixed monotone property on X. assume that there exists a $\alpha \in [0,1)$ with

$$d\big(F(x,y),F(u,v)\big) \le \alpha \max\left\{\frac{d(x,F(x,y)).d(u,F(u,v))}{d(x,u)}, \frac{d(u,F(x,y)).d(x,F(u,v))}{d(x,u)}, d(x,u)\right\} \qquad (3.1.1)$$

For all $x \ge u$ and $y \le v$. if there exist two elements $x_0, y_0 \in X$ with $x_0 \le F(x_0,y_0)$ and $y_0 \ge F(y_0,x_0)$, then there exist $x, y \in X$ such that $x = F(x,y)$ and $y = F(y,x)$.

Proof

Let $x_0, y_0 \in X$ with

$$x_0 \le F(x_0,y_0) \quad \& \quad y_0 \ge F(y_0,x_0) \qquad (3.1.2)$$

Define the sequence $\{x_n\}$ and $\{y_n\}$ in X such that,

$$x_{n+1} = F(x_n,y_n) \quad \& \quad y_{n+1} = F(y_n,x_n) \qquad (3.1.3)$$

For all $n = 0, 1, 2, \dots \dots \dots$

We claim that $\{x_n\}$ is monotone non decreasing and $\{y_n\}$ monotone non increasing i.e,

$$x_n \le x_{n+1} \text{ and } y_n \ge y_{n+1} \text{ for all } n = 0, 1, 2, \dots \dots \qquad (3.1.4)$$

From $(3.1.2)$ and $(3.1.3)$ we have

$x_0 \le F(x_0,y_0)$, $y_0 \ge F(y_0,x_0)$ And $x_1 = F(x_0,y_0)$, $y_1 = F(y_0,x_0)$

Thus $x_0 \le x_1$, $y_0 \ge y_1$ i.e equation $(3.1.4)$ true for some $n = 0$.

Now suppose that equation $(3.1.4)$ hold for some n.

i.e. , $x_n \le x_{n+1}$ and $y_n \ge y_{n+1}$

We shall prove that the equation $(3.1.4)$ is true for $n+1$

Now $x_n \leq x_{n+1}$ and $y_n \geq y_{n+1}$ then by mixed monotone property of F, we have

$$x_{n+2} = F(x_{n+1}, y_{n+1}) \geq F(y_n, x_{n+1}) \geq F(x_n, y_n) = x_{n+1}$$

and

$$y_{n+2} = F(y_{n+1}, x_{n+1}) \leq F(y_n, x_{n+1}) \leq F(y_n, x_n) = y_{n+1}$$

Thus by the mathematical induction principle equation $(3.1.4)$ holds for all n in N.

So $x_0 \leq x_1 \leq x_2 \leq \ldots\ldots\ldots \leq x_n \leq x_{n+1} \leq \cdots\ldots\ldots$

and

$$y_0 \geq y_1 \geq y_2 \geq \ldots\ldots\ldots \geq y_n \geq y_{n+1} \geq \ldots\ldots$$

Since $x_{n-1} \leq x_n$ and $y_{n-1} \geq y_n$, from $(3.1.1)$ we have,

$$d\big(F(x_n, y_n), F(x_{n-1}, y_{n-1})\big) \leq \alpha \max\left\{ \begin{array}{c} \dfrac{d(x_n, F(x_n, y_n)).d(x_{n-1}, F(x_{n-1}, y_{n-1}))}{d(x_n, x_{n-1})}, \\ \dfrac{d(x_{n-1}, F(x_n, y_n)).d(x_n, F(x_{n-1}, y_{n-1}))}{d(x_n, x_{n-1})}, d(x_n, x_{n-1}) \end{array} \right\}$$

$$d(x_{n+1}, x_n) \leq \alpha \max\{d(x_n, x_{n+1}), 0, d(x_n, x_{n-1})\}$$

If we take $\max$ is equal to $d(x_n, x_{n+1})$,

$d(x_n, x_{n+1}) \leq \alpha\, d(x_n, x_{n+1})$, which contradiction of the hypothesis,

This implies, $d(x_n, x_{n+1}) < \alpha . d(x_n, x_{n-1})$ $(3.1.5)$

Similarly since $y_{n-1} \geq y_n$ and $x_{n-1} \leq x_n$ and from $(3.1.1)$ we have

$$d(y_n, y_{n+1}) \leq \alpha\, d(y_n, y_{n-1}) \tag{3.1.6}$$

By adding $(3.1.5)$ and $(3.1.6)$ we get,

$$d(x_n, x_{n+1}) + d(y_n, y_{n+1}) \leq \alpha\, d(x_n, x_{n-1}) + \alpha\, d(y_n, y_{n-1})$$

$$d(x_n, x_{n+1}) + d(y_n, y_{n+1}) \leq \alpha \left(d(x_n, x_{n-1}) + d(y_n, y_{n-1}) \right)$$

let us denote $d(x_n, x_{n+1}) + d(y_n, y_{n+1})$ by d_n then

$$d_n \leq \alpha\, d_{n-1}$$

Similarly it can be proved that $d_{n-1} \leq \alpha\, d_{n-2}$

Therefore $d_n \leq \alpha^2\, d_{n-2}$

By repeating we get, $d_n \leq \alpha\, d_{n-1} \leq \alpha^2 d_{n-2} \leq \dots \dots \leq \alpha^n d_0$

This implies that,

$$\lim_{n\to\infty} d_n = 0$$

Thus $\lim\limits_{n\to\infty} d(x_{n+1}, x_n) = \lim\limits_{n\to\infty} d(y_{n+1}, y_n) = 0$

For each $m > n$ we have

$$d(x_n, x_m) \leq d(x_n, x_{n+1}) - d(x_{n+1}, x_{n+2}) + \dots \dots + d(x_{m-1}, x_m)$$

and

$$d(y_n,y_m) \le d(y_n,y_{n+1}) + d(y_{n+1},y_{n+2}) - \dots \dots + d(y_{m-1},y_m)$$

By adding these, we get

$$d(x_n,x_m) + d(y_n,y_m) \le \frac{\alpha^n}{1-\alpha} c_0$$

This implies that,

$$\lim_{n,m\to\infty}\left(d(x_n,x_m) + d(y_n,y_m)\right) = 0$$

Therefore $\{x_n\}$ and $\{y_n\}$ are Cauchy sequence in X. since X is a complete metric space, there exist

$x, y \in X$ such that $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} y_n = y$.

Thus by taking limit as $n \to \infty$ in (3.1.3) we get,

$$x = \lim_{n\to\infty} x_n = \lim_{n\to\infty} F(x_{n-1},y_{n-1}) = F \lim_{n\to\infty}(x_{n-1},y_{n-1}) = F(x,y)$$

and

$$y = \lim_{n\to\infty} y_n = \lim_{n\to\infty} F(y_{n-1},x_{n-1}) = F \lim_{n\to\infty}(y_{n-1},x_{n-1}) = F(y,x)$$

Therefore $x = F(x,y)$ and $y = F(y,x)$

Thus F has a coupled fixed point in X.

Theorem 3.2

Let $(X, \leq)$ be a partially ordered set and suppose there exists a metric d on X such that (X, d) is a complete metric space. Let $F : X \times X \to X$ be a continuous mapping having the mixed monotone property on X. assume that there exists a $\alpha \in [0,1)$ with

$$d(F(x,y), F(u,v)) \leq \alpha \max\{d(u, F(x,y)), d(x, F(u,v))\} \qquad (3.2.1)$$

For all $x \geq u$ and $y \leq v$. if there exist two elements $x_0, y_0 \in X$ with $x_0 \leq F(x_0, y_0)$ and $y_0 \geq F(y_0, x_0)$, then there exist $x, y \in X$ such that $x = F(x,y)$ and $y = F(y,x)$.

Proof

Let $x_0, y_0 \in X$ with

$$x_0 \leq F(x_0, y_0) \quad \text{and} \quad y_0 \geq F(y_0, x_0) \qquad (3.2.2)$$

Define the sequence $\{x_n\}$ and $\{y_n\}$ in X such that,

$$x_{n+1} = F(x_n, y_n) \quad \text{and} \quad y_{n+1} = F(y_n, x_n) \qquad (3.2.3)$$

For all $n = 0, 1, 2, \ldots \ldots \ldots$

We claim that $\{x_n\}$ is monotone non decreasing and $\{y_n\}$ monotone non increasing i.e,

$$x_n \leq x_{n+1} \quad \text{and} \quad y_n \geq y_{n+1} \quad \text{for all } n - 0, 1, 2, \ldots \ldots \qquad (3.2.4)$$

From $(3.2.2)$ and $(3.2.3)$ we have

$$x_0 \leq F(x_0, y_0), y_0 \geq F(y_0, x_0) \quad \text{and} \quad x_1 = F(x_0, y_0), \; y_1 = F(y_0, x_0)$$

Thus $x_0 \leq x_1$, $y_0 \geq y_1$ i.e equation $(3.2.4)$ true for some $n = 0$.

Now suppose that equation $(3.2.4)$ and hold for some n.

i.e. , $x_n \le x_{n+1}$ and $y_n \ge y_{n+1}$

We shall prove that the equation $(3.2.4)$ is true for $n+1$

Now $x_n \le x_{n+1}$ and $y_n \ge y_{n+1}$ then by mixed monotone property of F, we have

$$x_{n+2} = F(x_{n+1}, y_{n+1}) \ge F(y_n, x_{n+1}) \ge F(x_n, y_n) = x_{n+1}$$

and

$$y_{n+2} = F(y_{n+1}, x_{n+1}) \le F(y_n, x_{n+1}) \le F(y_n, x_n) = y_{n+1}$$

Thus by the mathematical induction principle equation $(3.2.4)$ holds for all n in N.

So $x_0 \le x_1 \le x_2 \le \ldots\ldots\ldots\ldots \le x_n \le x_{n+1} \le \cdots\ldots\ldots$

and

$$y_0 \ge y_1 \ge y_2 \ge \ldots\ldots\ldots\ldots \ge y_n \ge y_{n+1} \ge \ldots\ldots\ldots$$

Since $x_{n-1} \le x_n$ and $y_{n-1} \ge y_n$, from $(3.2.1)$ we have,

$$d(F(x_n, y_n), F(x_{n-1}, y_{n-1})) \le \alpha \max[d(x_{n-1}, F(x_n, y_n)), d(x_n, F(x_{n-1}, y_{n-1}))]$$

$$d(x_{n+1}, x_n) \le \alpha \max\{d(x_{n-1}, x_{n+1}), 0\}$$

This implies, $d(x_n, x_{n+1}) \le \frac{\alpha}{1-\alpha} d(x_n, x_{n-1})$ $(3.1.5)$

Similarly since $y_{n-1} \geq y_n$ and $x_{n-1} \leq x_n$ and from $(3.1.1)$ we have

$$d(y_n, y_{n+1}) \leq \frac{\alpha}{1-\alpha}\, d(y_n, y_{n-1}) \qquad (3.1.6)$$

By adding $(3.1.5)$ and $(3.1.6)$ we get,

$$d(x_n, x_{n+1}) + d(y_n, y_{n+1}) \leq \alpha\, d(x_n, x_{n-1}) + \alpha\, d(y_n, y_{n-1})$$

$$d(x_n, x_{n+1}) + d(y_n, y_{n+1}) \leq \alpha \left(d(x_n, x_{n-1}) + d(y_n, y_{n-1}) \right)$$

Let us denote $h = \frac{\alpha}{1-\alpha}$ and $d(x_n, x_{n+1}) + d(y_n, y_{n+1})$ by d_n then

$$d_n \leq \alpha\, d_{n-1}$$

Similarly it can be proved that $d_{n-1} \leq h\, d_{n-2}$

Therefore $d_n \leq h^2\, d_{n-2}$

By repeating we get, $d_n \leq h\, d_{n-1} \leq h^2 d_{n-2} \leq \dots \dots \leq h^n d_0$

This implies that,

$$\lim_{n\to\infty} d_n = 0$$

Thus $\lim\limits_{n\to\infty} d(x_{n+1}, x_n) = \lim\limits_{n\to\infty} d(y_{n+1}, y_n) = 0$

For each $m \geq n$ we have

$$d(x_n, x_m) \leq d(x_n, x_{n+1}) - d(x_{n+1}, x_{n+2}) + \dots \dots + d(x_{m-1}, x_m)$$

and

$$d(y_n, y_m) \leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) - \dots \dots + d(y_{m-1}, y_m)$$

By adding these, we get

$$d(x_n, x_m) + d(y_n, y_m) \leq \frac{h^n}{1-h} \delta_0$$

This implies that,

$$\lim_{n,m \to \infty} \left(d(x_n, x_m) + d(y_n, y_m) \right) = 0$$

Therefore $\{x_n\}$ and $\{y_n\}$ are Cauchy sequence in X. since X is a complete metric space, there exist

$x, y \in X$ such that $\lim_{n \to \infty} x_n = x$ and $\lim_{n \to \infty} y_n = y$.

Thus by taking limit as $n \to \infty$ in (3.1.3) we get,

$$x = \lim_{n \to \infty} x_n = \lim_{n \to \infty} F(x_{n-1}, y_{n-1}) = F \lim_{n \to \infty} (x_{n-1}, y_{n-1}) = F(x, y)$$

and

$$y = \lim_{n \to \infty} y_n = \lim_{n \to \infty} F(y_{n-1}, x_{n-1}) = F \lim_{n \to \infty} (y_{n-1}, x_{n-1}) = F(y, x)$$

Therefore $x = F(x, y)$ and $y = F(y, x)$

Thus F has a coupled fixed point in X.

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