Modelling of the Spread of Rabies with Pre-Exposure Vaccination of Humans

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Abstract

Apart from the quest to save human lives, it is also of paramount interest to curtail the spread of rabies in human population. The control of rabies in animals is usually handled either through culling or vaccination of the animal population. Consequently, the possibility of transmitting this disease or virus to healthy animals is drastically reduced. Radies is deadly, and its treatment or control in infected animals is often done successfully during the incubation period before the occurence of symptoms. Thus, the need for pre-exposure vaccination cannot be over emphasized. This paper developed a mathematical model, from that of Zhang *et al*. which was based on the transmission dynamics and control of rabies in China, to explore the need for pre-exposure vaccination of humans and its impact on the spread of rabies from dogs to humans. The study employed an SEIRS model which consists of both dog and human population.

Keywords: Rabies, Pre-exposure vaccination, Mathematical model, Dogs, Humans

1. Introduction

Rabies, also known as *hydrophobia* (fear of water) is an acute, viral disease that attacks all warm blooded animals including man. It attacks the central nervous system of the host animal leading to paralysis and death (Caron & Barczewske 1997; Jackson & Wunner 2003). According to Adeyemi *et al*. (2005), rabies becomes fatal to both animals and humans once the symptoms of the disease develop.

Rabies is caused by a virus present in the saliva of the infected animal and is spread when an infected animal bites another. Raccoons have been singled out as the major carrier of this virus, while others include dogs, cats, skunks, foxes, bats, groundhogs, farm livestock (e.g. cows, sheep, goats, horses and pigs) etc.; humans are also susceptible to the disease (Caron & Barczewske 1997). According to Wiraningsih *et al*. (2010) and Zhang *et al*. (2011), humans are most often infected through the bite or scratch of an infected dog or cat. This implies that the disease is transmitted to both domestic animals and humans through bites exposed to infected saliva.

Research by scholars, (Zhang *et al*. 2011; Ogun *et al*. 2010), revealed that rabies is one of the major public health problems, especially in the developing countries. This is because dog bites are common and the regular routine of vaccination of dogs is very low with none or little mass campaign against rabies. Zhang *et al*. state that there had been dramatic increase in cases of human rabies in China, partially due to poor understanding of the transmission dynamics of rabies and lack of effective control measures of the disease (Zhang *et al*. 2011).

According to Adeyemi *et al*. (2005), routine prophylactic vaccination of dogs has been accepted for effective control of urban rabies world-wide basically because dogs play major role in the transmission of rabies to humans. Rabies can be successfully treated, but only during the incubation period before symptoms occur. It is most often fatal when vaccine is not administered. The incubation period in humans varies from three weeks to 120 days, with an average of about four to six weeks (Evans & Pritchard 2001; Redmond 2007).

Caron & Barczewske (1997) assert that, permanent brain damage had been the outcome in few known cases in which people have actually survived rabies without seeking for medical attention. They further state that humans were not often vaccinated for rabies unless they come regularly in close contact with wild animals. In the same light, Rabies Management Guidelines (2005) and Jackson & Wunner (2003) revealed that few humans were vaccinated occassionally against rabies; they include: Laboratory workers, Veterinarians, Veterinary technicians, Animals control officers, Care explorers, etc.

Consequently, this study intends to explore the impact of pre-exposure vaccination of humans in the dynamics of the spread of rabies through the application of mathematical model.

2. Methods

In the quest for control measures for the spread of rabies in China, Zhang *et al*. in 2011 developed a deterministics model that studied the transmission dynsmics and control of rabies in China in order to explore effective control and preventive measures (Zhang *et al*. 2011). This paper however, intends to slightly modify their model for the spread of rabies in China by including pre-exposure vaccination of humans in the model.

2.1 The Mathematical Model

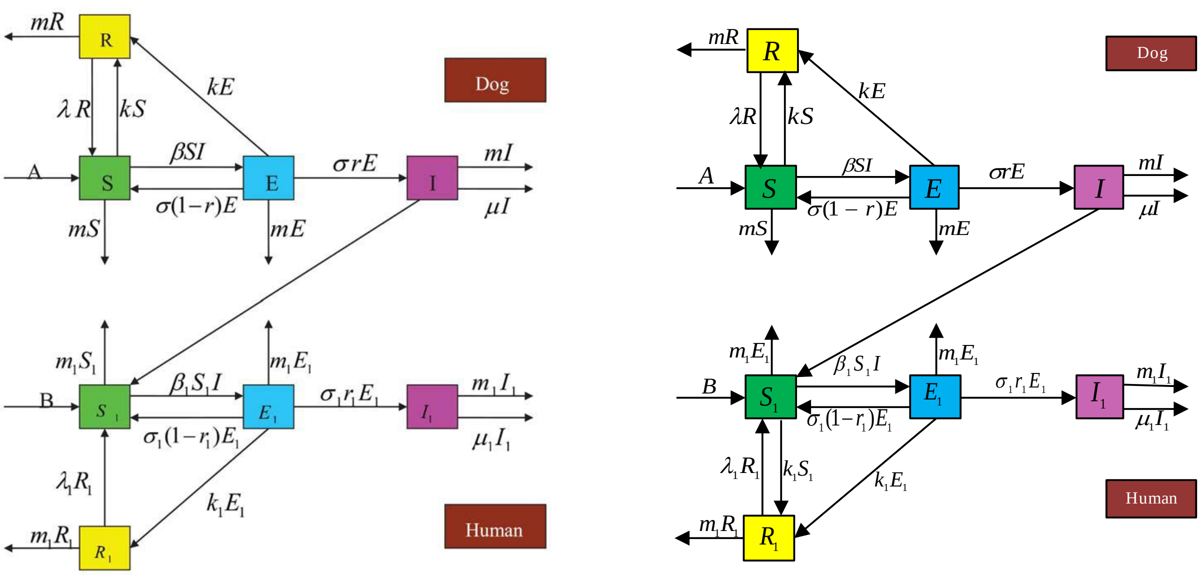
The SEIRS model consists of four classes which include: (i) the susceptible class, with population density S; (ii) the exposed class, with population density E; (iii) the infectious class, with population density I (the infectious class is sometime referred to as the rabid class where the infected becomes infectious after the incubation period has elapsed); and lastly, (iv) the recovered class, with population density R.

2.2 Definition of Parameters

Table I: Definition of parameters

Parameters		Definition
Dog	Human	
A	B	A is the annual number of newborn pupies while B is human annual birth
λ	λ_1	Loss rate of vaccination immunity
i	i_1	Incubation period of the infected
$\sigma = \frac{1}{i}$	$\sigma_1 = \frac{1}{i_1}$	The reciprocal of the incubation period
r	r_1	Risk of clinical outcome of the exposed
m	m_1	Natural mortality rate
k	k_1	Vaccination rate
μ	μ_1	Disease-related death rate
β	β_1	Dog-to-dog and dog-to-human transmission rate respectively

This paper adopts the parameters in Zhang *et al*. (2011) model. They are stated in Table I above.



(a) (b)

Figure 1: A flow diagram of the transmission of rabies among dogs and from dogs to humans:
(a) Zhang *et al*.'s model (b) modified model

$$\left.\begin{aligned}
\frac{dS}{dt} &= A+\lambda R+\sigma(1-r)E-\beta SI-(m+k)S\\
\frac{dE}{dt} &= \beta SI-(m+\sigma+k)E\\
\frac{dI}{dt} &= \sigma rE-(m+\mu)I\\
\frac{dR}{dt} &= k(S+E)-(m+\lambda)R\\
\frac{dS_1}{dt} &= B+\lambda_1 R_1+\sigma_1(1-r_1)E_1-\beta_1 S_1 I-m_1 S_1\\
\frac{dE_1}{dt} &= \beta_1 S_1 I-(m_1+\sigma_1+k_1)E_1\\
\frac{dI_1}{dt} &= \sigma_1 r_1 E_1-(m_1+\mu_1)I_1\\
\frac{dR_1}{dt} &= k_1 E_1-(m_1+\lambda_1)R_1
\end{aligned}\right\} \quad \ldots \quad (i)$$

$$\left.\begin{aligned}
\frac{dS}{dt} &= A+\lambda R+\sigma(1-r)E-\beta SI-(m+k)S\\
\frac{dE}{dt} &= \beta SI-(m+\sigma+k)E\\
\frac{dI}{dt} &= \sigma rE-(m+\mu)I\\
\frac{dR}{dt} &= k(S+E)-(m+\lambda)R\\
\frac{dS_1}{dt} &= B+\lambda_1 R_1+\sigma_1(1-r_1)E_1-\beta_1 S_1 I-(m_1+k_1)S_1\\
\frac{dE_1}{dt} &= \beta_1 S_1 I-(m_1+\sigma_1+k_1)E_1\\
\frac{dI_1}{dt} &= \sigma_1 r_1 E_1-(m_1+\mu_1)I_1\\
\frac{dR_1}{dt} &= k_1(S_1+E_1)-(m_1+\lambda_1)R_1
\end{aligned}\right\} \quad \ldots \quad (ii)$$

Figure 1(a) gives the graphical representation of Zhang *et al*. (2011) model from which they derived the system of equations (i). The modified model gives the flowchart shown in Figure 1(b) and the system of equations (ii). In the modified model, the section for humans shows the inclusion of pre-exposure vaccination of some human population. This tends to evaluate the importance of pre-exposure vaccination of humans as a control measure in the dynamics of the spread of rabies, especially from dogs to humans.

2.3 Equilibrium State

The equilibrium state of (ii) gave two steady states, the disease-free state $(S^0,0,0,R^0,S_1^0,0,0,R_1^0)$ and the endemic state $(S^*,E^*,I^*,R^*,S_1^*,E_1^*,I_1^*,R_1^*)$

where

$$S^0=\frac{A(m+\lambda)}{m(m+k+\lambda)},\qquad R^0=\frac{kA}{m(m+k+\lambda)},\qquad S_1^0=\frac{B(m_1+\lambda_1)}{m_1(m_1+k_1+\lambda_1)}\quad \text{and}\quad R_1^0=\frac{k_1 B}{m_1(m_1+k_1+\lambda_1)}$$

and for the endemic state we have

$$S^*=\frac{(m+\mu)(m+\sigma+k)}{\beta\sigma r},\qquad E^*=\frac{(m+\mu)I^*}{\sigma r},\qquad I^*=\frac{A-mN^*}{\mu},\qquad R^*=\frac{k(N^*-I^*)}{m+k+\lambda}$$

$N^*=S^*+E^*+I^*+R^*$ (total dog population)

$$\Rightarrow\quad I^*=\frac{\beta\sigma r(m+\lambda)A-m(m+k+\lambda)(m+\mu)(m+\sigma+k)}{\beta(m+\mu)[(m+\sigma r)(m+\lambda)+mk]}$$

$$S_1^*=\frac{B(m_1+\lambda_1)-[(m_1+\lambda_1)(m_1+\sigma_1 r_1)+m_1k_1]E_1^*}{m_1(m_1+\lambda_1+k_1)},$$

$$E_1^*=\frac{\beta_1(m_1+\lambda_1)BI^*}{m_1(m_1+\lambda_1+k_1)(m_1+\sigma_1+k_1)+\beta_1[(m_1+\lambda_1)(m_1+\sigma_1 r_1)+m_1k_1]I^*},\qquad I_1^*=\frac{\sigma_1 r_1 E_1^*}{m_1+\mu_1},$$

$$R_1^*=\frac{k_1(N_1^*-I_1^*)}{m_1+\lambda_1+k_1}$$

$N_1^*=S_1^*+E_1^*+I_1^*+R_1^*$ (total human population)

The basic reproduction number is

$$R_0=\frac{\beta\sigma rS^0}{(m+\mu)(m+\sigma+k)}$$

3. Results

The numerical solutions of the systems were obtained using MAPLE application with the parameter values stated in Table II.

Table II: Parameter values

Definition	Parameters					
	Dog			Human		
	Parameter	Value	Unit	Parameter	Value	Unit
A is the annual number of newborn puppies while B is human annual birth	A	3×10^6	$year^{-1}$	B	1.54×10^7	$year^{-1}$
Loss rate of vaccination immunity	λ	1	$year^{-1}$	λ_1	1	$year^{-1}$
The reciprocal of the incubation period	σ	6	$year^{-1}$	σ_1	6	$year^{-1}$
Risk of clinical outcome of the exposed	r	0.4	$year^{-1}$	r_1	0.4	$year^{-1}$
Natural mortality rate	m	0.08	$year^{-1}$	m_1	0.0066	$year^{-1}$
Vaccination rate	k	0.09	$year^{-1}$	k_1	0.54	$year^{-1}$
Disease-related death rate	μ	1	$year^{-1}$	μ_1	1	$year^{-1}$
Dog-to-dog and dog-to-human transmission rate respectively	β	1.58×10^{-7}	$year^{-1}$	β_1	2.29×10^{-12}	$year^{-1}$

Source: Adopted from Zhang *et al*. 2011.

From the analysis of the trend of rabies cases in China (within the period of fifty years), the following results were obtained:

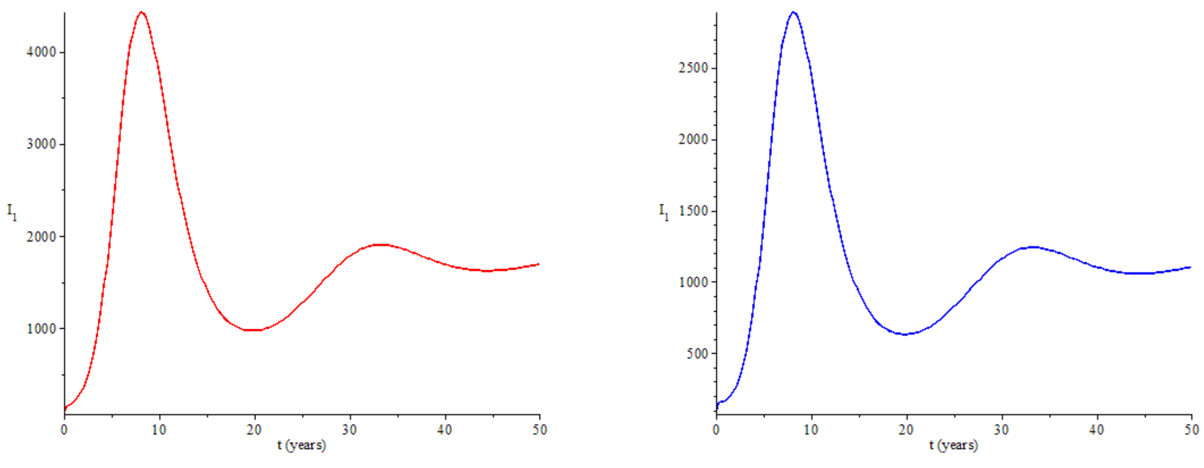


Figure 2: The trend of human rabies cases in China for the period of fifty years:
(a) Zhang *et al*.'s model (b) modified model

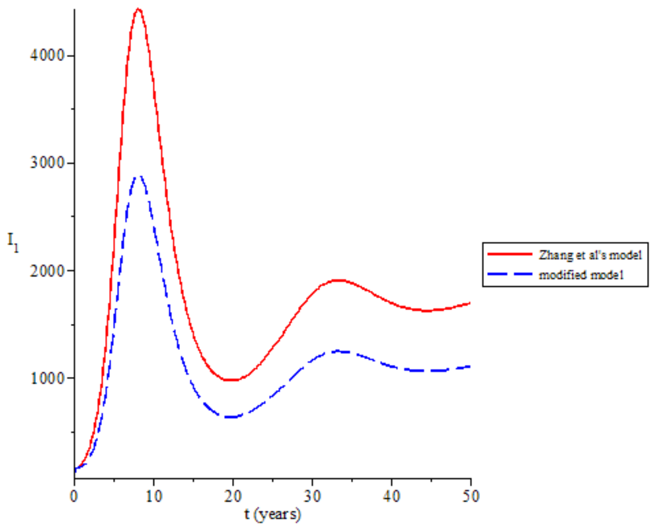


Figure 3: The trend of human rabies cases in China for the period of fifty years: Comparing both models

From the results obtained, Figures 2(a) and (b) show the trend of human rabies cases in China for the period of fifty years using the Zhang *et al*.'s model and the modified model (with pre-exposure vaccination of humans) respectively. The parameter values used were as defined in Table II. The initial values used were: $S(0)=3.5\times10^7$, $E(0)=2\times10^5$, $I(0)=1\times10^5$, $R(0)=2\times10^5$, $S_1(0)=1.29\times10^9$, $E_1(0)=250$, $I_1(0)=89$, $R_1(0)=2\times10^5$ (Zhang *et al*. 2011). The reduction in the number of infectives in Figure 2(b) and Figure 3 shows positive impact or effect of pre-exposure vaccination of humans in the spread of rabies.

Now, assuming that pre-exposure vaccination rate of humans differs from the post-exposure vaccination rate, and let α denote the pre-exposure vaccination rate. Then the system of equations (ii) becomes equation (iii),

$$\left.\begin{aligned}
\frac{dS}{dt} &= A+\lambda R+\sigma(1-r)E-\beta SI-(m+k)S\\
\frac{dE}{dt} &= \beta SI-(m+\sigma+k)E\\
\frac{dI}{dt} &= \sigma E-(m+\mu)I\\
\frac{dR}{dt} &= k(S+E)-(m+\lambda)R\\
\frac{dS_1}{dt} &= B+\lambda_1 R_1+\sigma_1(1-r_1)E_1-\beta_1 S_1 I-(m_1+\alpha)S_1\\
\frac{dE_1}{dt} &= \beta_1 S_1 I-(m_1+\sigma_1+k_1)E_1\\
\frac{dI_1}{dt} &= \sigma_1 r_1 E_1-(m_1+\mu_1)I_1\\
\frac{dR_1}{dt} &= \alpha S_1+k_1 E_1-(m_1+\lambda_1)R_1
\end{aligned}\right\} \quad \ldots \quad \text{(iii)}$$

Evaluating the system of equations (iii) for different values of α along side the other parameters defined in Table II we obtain the following result in Figure 4.

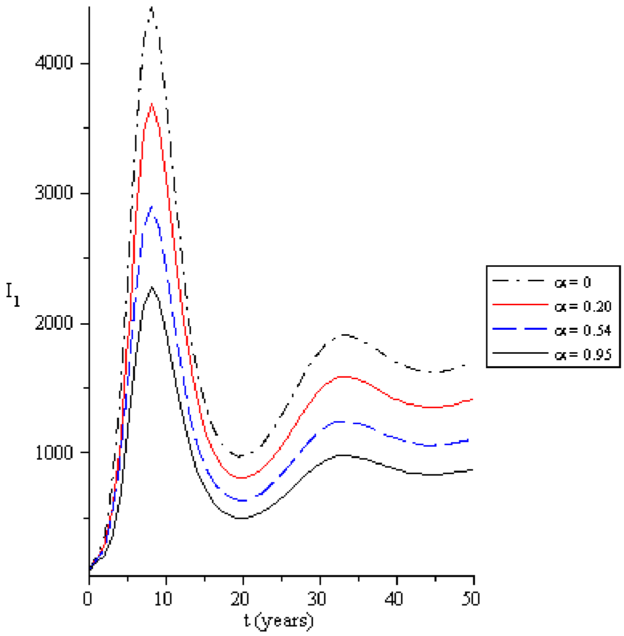


Figure 4: The effect of pre-exposure vaccination of humans in the spread of rabies

4. Summary and Conclusion

The results from the numerical simulations revealed the tendency of rabies epidemics with time which tends to oscillate for some period of time before it stablizes. Comparing both models, Figure 2 and 3 showed that the modified model had less number of human rabies cases in the dynamics of the spread of rabies. By implication, the administration of pre-exposure vaccination of humans prevented many of the populace from being infected with the virus.

To further explore the impact of pre-exposure vaccination of humans in the transmission dynamics of rabies, we assumed that the pre-exposure vaccination rate differs from the post-exposure vaccination rate of humans. Evaluating for different parameter values of pre-exposure vaccination rate of humans, Figure 4 shows that pre-exposure vaccination of humans tends to reduce the number of rabies cases in humans. The higher the rate of pre-exposure vaccination of humans, α , the lower the number of people infected with the rabies virus.

Hence, based on the results of these analyses, we can state that the pre-exposure vaccination of humans is of importance in the transmission dynamics and control of the spread of rabies, particularly in China being the case study. Consequently, as a control measure, in addition to culling and vaccination of dogs, the study suggests that pre-exposure rabies vaccination be administered to humans, especially people at higher risk of rabies infection such as the Veterinarians, Veterinary technicians, Veterinary students, Laboratory workers, Animals control officers, etc. in order to minimize the spread of rabies.

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