Effect of couple stress fluid on MHD peristaltic motion and heat transfer with partial slip in an inclined asymmetric channel

Ghusoon G.mhammad ,Ahmed M. Abdulhadi
Department of Mathematic, College of Science, University of Baghdad, Baghdad, Iraq.
email: ghusoon.ghazi@yahoo.com , ahm6161@yahoo.com

The research is financed by Asian Development Bank. No. 2006-A171(Sponsoring information)

Abstract

In this investigation we have analyzed the effect of variable couple stress fluid on the peristaltic flow of non-Newtonian fluid in an inclined asymmetric channel . The relevant equations have been modeled. Analysis has been carried out in the presence of velocity and thermal slip conditions. Expressions for stream function, temperature, pressure gradient and heat transfer coefficients are derived. Numerical integration has been performed for pressure rise per wavelength. Plots are presented and analyzed for various embedded parameters into the problem. This study is done through drawing many graphs by using the MATHEMATICA package.

Keywords: Heat transfer ,Non - Newtonian fluid, Couple stress, magnetic field, Partial slip, inclined..

1.Introduction

several researchers have analyzed the phenomenon of peristaltic transport under various assumptions. It is noticed from the available literature that much has been reported on the peristaltic transport of hydrodynamic viscous and non-Newtonian fluids. Some recent investigations regarding such fluids are mentioned in the studies [1-11].The couple-stress fluid may be considered as a special case of a non-Newtonian fluid which is intended to take into account the particle size effects. Moreover, the couple stress fluid model is one of the numerous models that proposed to describe response characteristics of non-Newtonian fluids. The constitutive equations in these fluid models can be very complex and involving a number of parameters, also the out coming flow equations lead to boundary value problems in which the order of differential equations is higher than the Navier–Stokes equations. Some recent investigations regarding such fluids are mentioned in the studies [12-21]. Mekheimer[22] has discussed the effects of the induced magnetic field on peristaltic flow of a couple stress fluid in a slit channel. Magneto hydrodynamics (MHD) is the science which deals with the motion of a highly conducting fluids in the presence of a magnetic field. The motion of the conducting fluid across the magnetic field generates electric currents which change the magnetic field, and the action of the magnetic field on these currents gives rise to mechanical forces which modify the flow of the fluid[23]. Recently Akbar et al. [24] analyzed the simultaneous effects of partial slip and heat transfer on the peristaltic flow of third fluid in an inclined asymmetric channel.

With the above discussion in mind, the goal of this investigation is to study the effect of couple stress fluid on MHD peristaltic motion and heat transfer with partial slip in an inclined asymmetric channel. The governing equations are simplified using long wavelength approximation. An exact solutions of velocity, stream function, energy equation and pressure gradient has found. The expressions for pressure rise has been calculated using numerical integration by software Mathematica. The effects of pertinent parameters on the velocity, energy equation, pressure gradient, and stream functions are presented graphically.

2. Mathematical formulation and analysis

We Consider an incompressible magnetohydrodynamic (MHD) fluid in an inclined asymmetric channel of width $d_1 + d_2$. The angle of inclination is α. A sinusoidal wave propagating with constant speed c on the channel walls induces the flow. The heat transfer process is maintained by considering temperatures T0 and T1 to the lower and upper walls of a channel, respectively.
Fig.1 shows the representation of physical model. The wall surfaces are chosen as

$$Y = H_1 = d_1 + a_1 \cos[\frac{2\pi}{\lambda}(X - ct)],\ Y = H_2 = -d_2 - b_1 \cos[\frac{2\pi}{\lambda}(X - ct) + \phi] \qquad (1)$$

where a1 and b1 are the waves amplitudes, λ is the wave length, $d_1 + d_2$ is the channel width, c is the wave speed, t is the time, X is the direction of wave propagation and Y is perpendicular to X. The phase difference ϕ varies in the range $0 \le \phi \le \pi$. For $\phi = 0$ the symmetric channel with waves out of phase can be described and when $\phi = \pi$,the waves are in phase.

Moreover, a_1, b_1, d_1, d_2 and ϕ meet the following relation:

$$a_1^2 + b_1^2 + 2a_1 b_1 \cos\phi \le (d_1 + d_2)^2 .$$

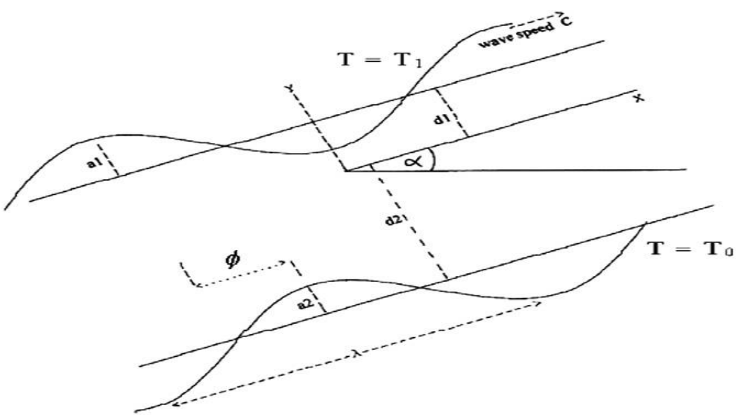


Fig. 1. Geometry of the problem.

The flow is governed by the following expressions

$$\left.\begin{aligned}
&\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \\
&\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \upsilon(\frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2}) - \frac{\upsilon}{K}U - \frac{\sigma B_0^2 U}{\rho} - \frac{\eta}{\rho}\nabla^4 U + g\sin\beta \\
&\frac{\partial V}{\partial t} + U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial Y} + \upsilon(\frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2}) - \frac{\upsilon}{K}V - g\cos\beta \\
&c'[\frac{\partial T}{\partial t} + U\frac{\partial T}{\partial X} + V\frac{\partial T}{\partial Y}] = \frac{K'}{\rho}\nabla^2 T + \upsilon\phi + Q_0 .
\end{aligned}\right\} \qquad (2)$$

Where $\nabla^2 = \frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2}, \nabla^4 = \frac{\partial^4}{\partial X^4} + \frac{\partial^4}{\partial Y^4} + 2\frac{\partial^4}{\partial X^2 \partial Y^2},$

$$\phi = [2(\frac{\partial U}{\partial X})^2 + 2(\frac{\partial V}{\partial X})^2 + (\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial X})^2],$$

U, V are the velocities in X and Y directions in fixed frame, ρ is constant density, P is the pressure, υ is the kinematics viscosity, σ is the electrical conductivity, K' is the thermal conductivity, c' is the specific heat and T is the temperature, ϕ is the dissipation factor and Q_0 the constant heat absorption parameter .

Introducing a wave frame (x, y) moving with velocity c away from the fixed frame (x,y) by the transformations:

$$x = X - ct, y = Y, u = U - c, v = V, p(x) = P(X,t) \tag{3}$$

Defining

$$\left.\begin{aligned} &\bar{x} = \frac{x}{\lambda}, \bar{y} = \frac{y}{d_1}, \bar{u} = \frac{u}{c}, \bar{v} = \frac{v}{c}, \delta = \frac{d_1}{\lambda}, d = \frac{d_2}{d_1}, \bar{p} = \frac{d_1^2 p}{\mu c \lambda}, \bar{t} = \frac{ct}{\lambda}, \\ &h_1 = \frac{H_1}{d_1}, h_2 = \frac{H_2}{d_2}, a = \frac{a_1}{d_1}, b = \frac{b_1}{d_1}, \mathrm{Re} = \frac{cd_1}{\upsilon}, \bar{\psi} = \frac{\psi}{cd_1}, \overline{K} = \frac{K}{d_1^2}, \\ &\theta = \frac{T - T_0}{T_1 - T_0}, \ Br = Ec\ \mathrm{Pr}, F_r = \frac{c^2}{g d_1}, Ec = \frac{c^2}{c'(T_1 - T_0)}, \mathrm{Pr} = \frac{\rho \nu c'}{K'}, \\ &M = \sqrt{\frac{\sigma}{\mu}} B_0 d_1, \ \alpha = d_1 \sqrt{\frac{\mu}{\eta}}, \beta_1 = \frac{Q_0 d_1^2}{(T_0 - T_1)\nu c'}, \ N^2 = \frac{1}{K} + M^2 \end{aligned}\right\} \tag{4}$$

Using the above non-dimensional quantities and neglecting the terms of order δ and higher, the resulting equations in terms of stream function ψ ($u = \frac{\partial \psi}{\partial y}, v = -\delta \frac{\partial \psi}{\partial x}$) can be written as:

$$\psi_{yyyy} - N^2 \psi_{yy} - \frac{1}{\alpha^2} \psi_{yyyyyy} = 0, \tag{5}$$

$$\theta_{yy} = -Br\ \psi_{yy}^2 - \beta_1 \mathrm{Pr} \tag{6}$$

Where Br is the Brinkman number and Pr Prandtl number.

Since we are considering the partial slip on the wall, therefore, the corresponding boundary conditions for the present problem can be written as

$$\psi = \frac{q}{2} \quad \text{at} \quad y = h_1 = 1 + a\cos 2\pi x,$$

$$\psi = -\frac{q}{2} \quad \text{at} \quad y = h_2 = -d - b\cos(2\pi x + \phi),$$

$\frac{\partial \psi}{\partial y} + L\frac{\partial^2 \psi}{\partial y^2} = -1$ at $y = h_1$, $\frac{\partial \psi}{\partial y} - L\frac{\partial^2 \psi}{\partial y^2} = -1$ at $y = h_2$,

The finishing couple stress boundary condition are:

$\frac{\partial^3 \psi}{\partial y^3} = 0$ at $y = h_1$, $\frac{\partial^4 \psi}{\partial y^4} = 0$ at $y = h_2$.

And the boundary condition for heat transfer are :

$$\theta = 0 \text{ on } y = h_1, \theta = 1 \text{ on } y = h_2. \quad (7)$$

Where q is the flux in the wave frame, a, b, ϕ and d satisfy the relation

$$a^2 + b^2 + 2ab\cos\phi \le (1+d)^2$$

The solution of the momentum equation straight forward can be written as

$$\psi = f_0 + f_1 y + f_2 \cosh m_1 y + f_3 \sinh m_1 y + f_4 \cosh m_2 y + f_5 \sinh m_2 y \quad (8)$$

Where

$$m_1 = \sqrt{\frac{\alpha^2 - \sqrt{\alpha^2(-4N^2 + \alpha^2)}}{2}}, m_2 = \sqrt{\frac{\alpha^2 + \sqrt{\alpha^2(-4N^2 + \alpha^2)}}{2}} \quad (9)$$

The functions $f_0, ..., f_5$ are large expressions will not mentioned here for sake of simplify.

The flux and average volume flow rate is defined

$$Q = \frac{1}{T}\int_0^T \overline{Q} dt = \frac{1}{T}\int_0^T (q + h_1 - h_2) dt = q + 1 + d \quad (10) \qquad \frac{dp}{dx} = \psi_{yyy} - N^2 \psi_y - \frac{1}{\alpha^2}\psi_{yyyyy} + \frac{\text{Re}}{Fr}\sin(\beta)$$

(11)

$$\Delta P = \int_0^1 \frac{dp}{dx} dx \quad (12)$$

The axial velocity component in the fixed frame is given as

$$U(X,Y,t) = 1 + \psi_y = 1 + f_1 + f_2 m_1 \sinh m_1 y + f_3 m_1 \cosh m_1 y + f_4 m_2 \sinh m_2 y + f_5 m_2 \cosh m_2 y \quad (13)$$

Where $h_1 = 1 + a\cos[2\pi(X - t)]$ And $h_2 = -d - b\cos[(2\pi(X - t) + \phi]$ By using Eq. (8) the solution of Eq. (6)satisfying the boundary conditions(7)can be written as:

$$\theta = \theta(f_0, f_1, f_2, f_3, f_4, f_5, m_1, m_2, Ec, \text{Pr}, \beta_1, y). \quad (14)$$

where c_1 and c_2 are constant can be determined from the boundary conditions (7).

3. Result and discussion

In this section,the results are discussed through the graphical illustrations for different physical quantities. Figs.2-6 show the pressure gradient for different value of Permeability parameter K, couple stress α , magnetic field M , amplitude ratio ϕ, partial slip L , Froude number F_r, Reynolds number Re and the inclination of the channel β .It is noticed that pressure gradient is maximum at X=0.5 for α **=1**, K=.1, M=4 , L=.1, F_r =1, $\sin(\beta)$ =1,and Re=4. and the pressure gradient increas when the parameters M, $\sin(\beta)$ and Re increases as shown in Figs.5,8,9 and the pressure gradient decrease when the other pramerters incrseas. Pressure rise is important physical measures in peristaltic mechanism,so Figs.7-10 show the effect of L, α , M ,K, ϕ ,Re, F_r and $\sin(\beta)$ on pressure rise.We noticd that increases in α ,L, K,Re and $\sin(\beta)$ the Δp increases and increases in M,Fr and Q the Δp will be decreases. Figs.18-22 illusrate the velocity field for different values of α ,K,M and L.It is observed that the velocity field increase when K increase and velocity field decrease when the other parameters increase and finally the shapes looks like a parabola and it can be noticed that the velocity take the maximum value in the middle. The temperature field for different value of L, M ,K, Pr ,Ec, α and β_1 .are shown in Figs.23 – 28.It is noticed from the figures that the increase in L,M and α the temperature filed decreases while the increase in Pr, Ec and β_1 , the temperature filed increases.

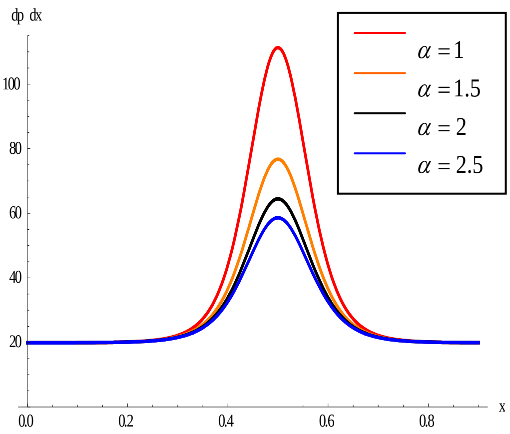


Fig.2.variation of dp/dx with x for different values of α at k=1000

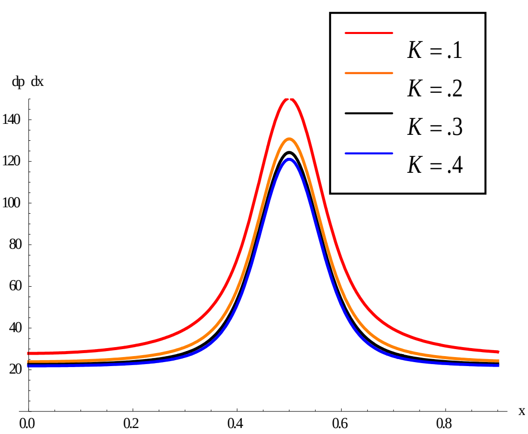


Fig.3.variation of dp/dx with x for different values of K, at, α =1

and the other parameter are M=0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, ϕ =0.001, F_r =.4, $\sin(\beta) = .8$,Re=1

Finally ,the Figs.29-33 describe the stream line and trapping phenomena and the effect of $\alpha, L, M, Q,$ and ϕ we noticed that all diagram were not symmetric and the trapping is about the center line and the trapped bolus decrease in size as α, M, L increase and slowly disappear for the large value while increase of the parameter Q the trapping will be increase.

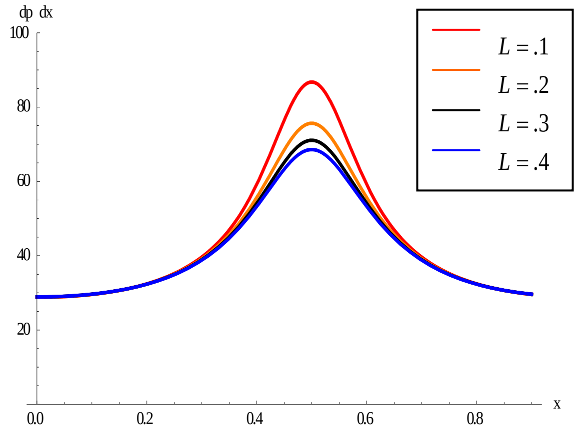


Fig.4. variation of dp/dx with x for different values of L at, M=1

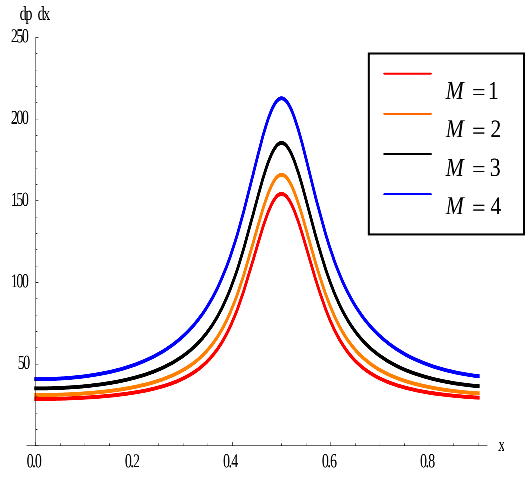


Fig.5. variation of dp/dx with x for different values of M at L=0

and the other parameter are α =1, K =0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, ϕ =0.001, F_r =.4, $\sin(\beta) = .8$,Re=10

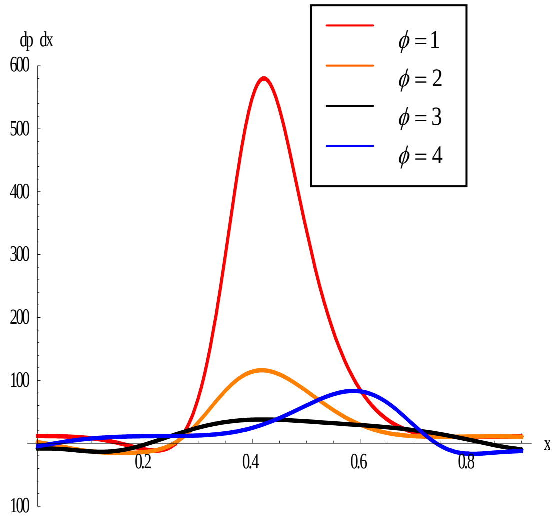


Fig.6. variation of dp/dx with x for different values of ϕ at F_r =.4

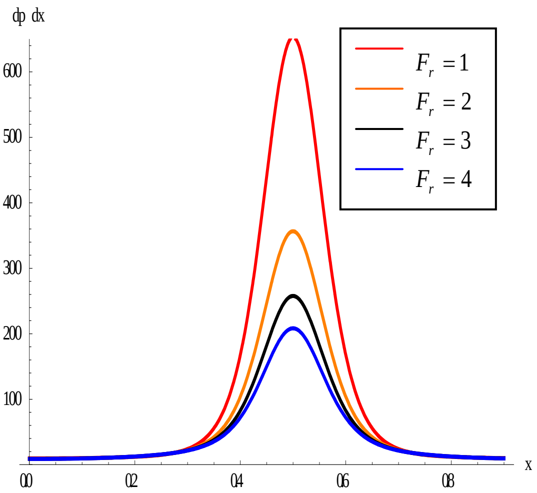


Fig.7. variation of dp/dx with x for different values of F_r at ϕ =0.001

and the other parameter are α =1, K =0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, M=1, K =0.1, $\sin(\beta) = .8$,Re=10

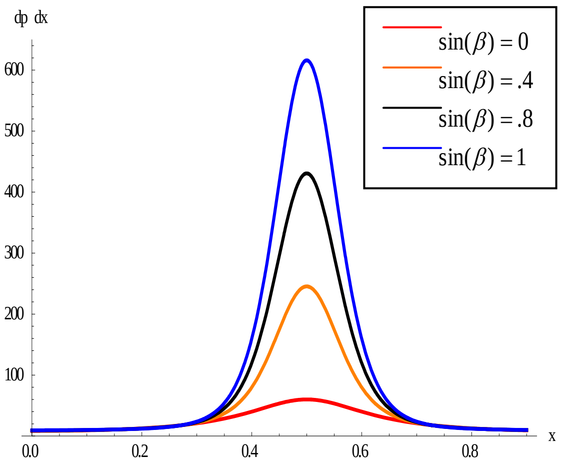


Fig.8. variation of dp/dx with x for different values of $\sin(\beta)$ at Re=10

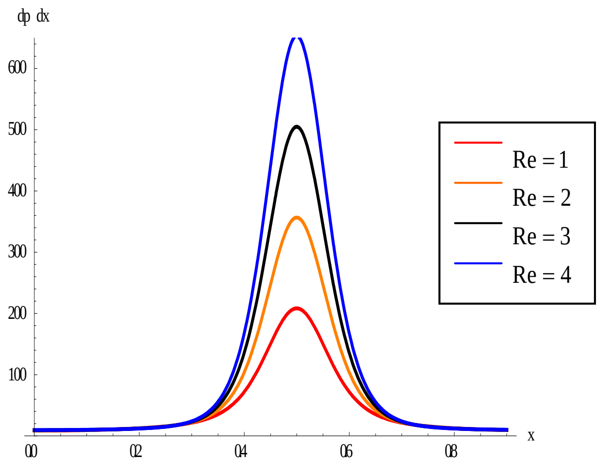


Fig.9. variation of dp/dx with x for different values of Re at $\sin(\beta)=.8$

and the other parameter are α =1, K =0.1, d=2, Q=-1, a=0.7, b=1.2, L=0, M=1, K =0.1, L =0.001, F_r =.4

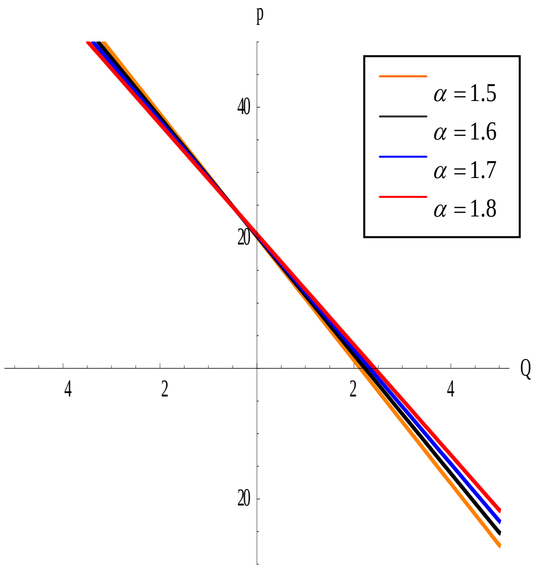


Fig.10. variation of Q with Δp for different values of α at L=0.02 ,k=.2

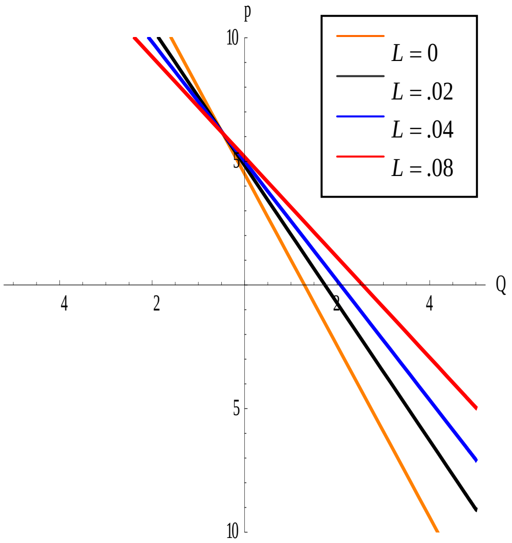
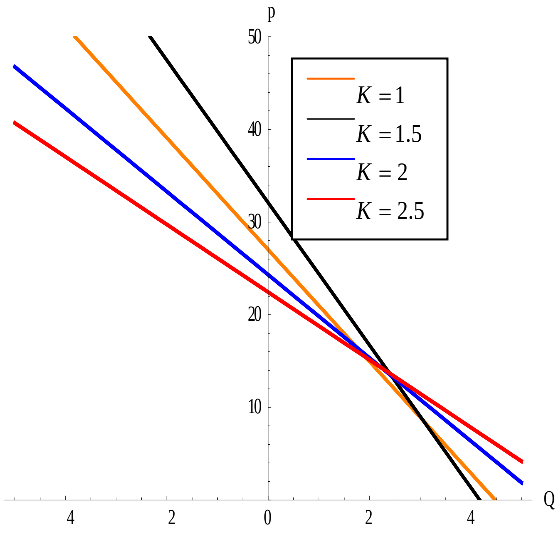


Fig.11. variation of Q with Δp for different values of L at $\alpha=2$,k=.1000

and the other parameter are ,M=2,d=2,a=0.7,b=1.2, , $\phi=\pi/6$,y=1.4, F_r =.4, $\sin(\beta)=.8$,Re=10



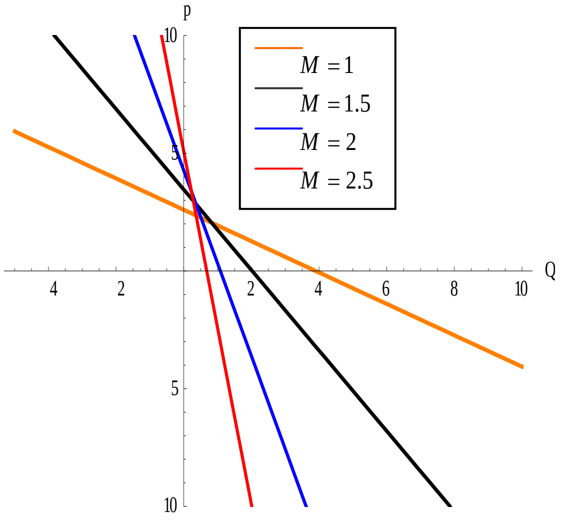
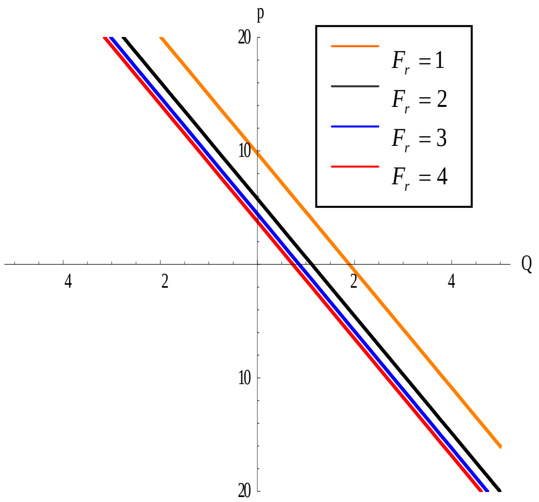


Fig.12. variation of Q with Δp for different values of K at $\alpha=2$,M=2,

Fig.13. variation of Q with Δp for different values of M at $\alpha=1$,K=10,

and the other parameter are , d=2,a=0.7,b=1.2,,L=0.04, $\phi=\frac{\pi}{6}$,y=1.4, F_r=.4, $\sin(\beta)=.8$,Re=10



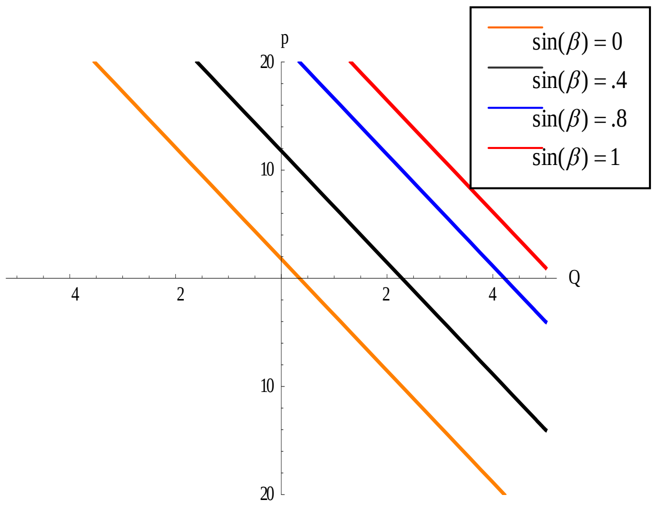
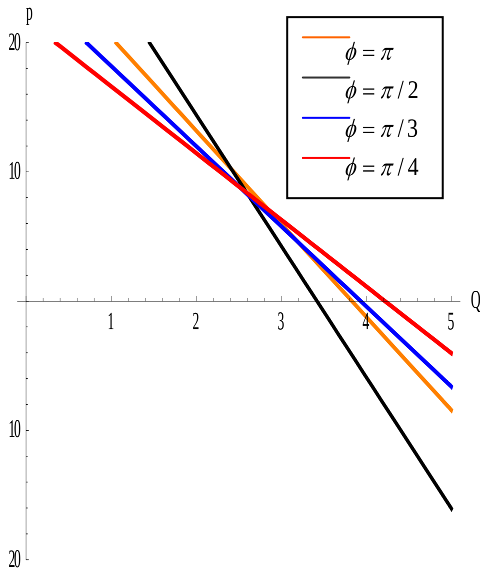


Fig.14. variation of Q with Δp for different values of F_r at $\sin(\beta)=.8$

Fig.15. variation of Q with Δp for different values of $\sin(\beta)$ at F_r=.4,

and the other parameter are $\alpha=1$, K=10, d=2, M=2 ,a=0.7 ,b=1.2 ,L=0.04, $\phi=\pi/6$, y=1.4 ,Re=10



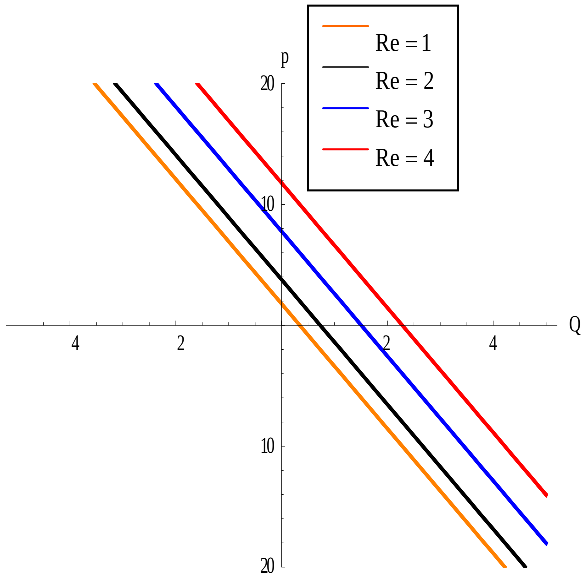
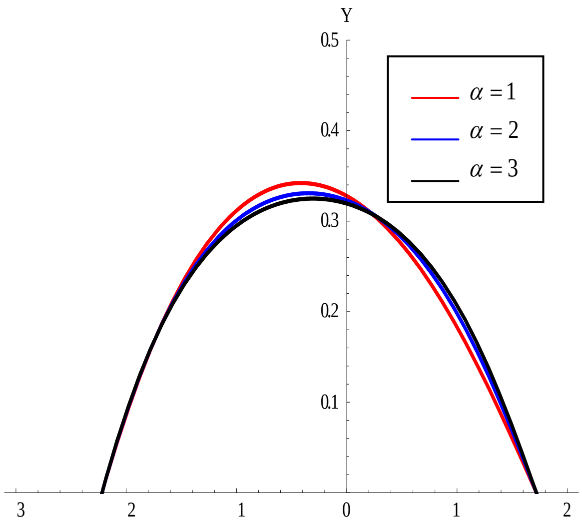


Fig.16. variation of Q with Δp for different values of ϕ at Re=10

Fig.17. variation of Q with Δp for different values of Re at $\phi = \pi / 6$

and the other parameter are $\alpha = 1$,K=10,d=2,a=0.7,b=1.2,,L=0.04,M=2 ,y=1.4, F_r =.4, $\sin(\beta) = .8$



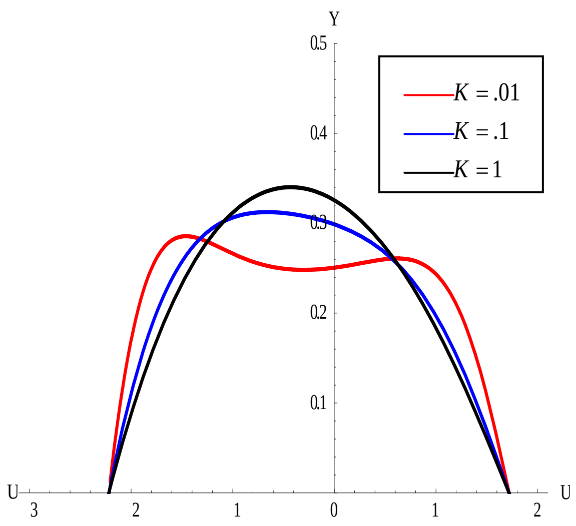


Fig.18. the velocity at Q=-1, a=0.7, b=1.2 , ϕ =0, x=1 k=2, M=1, d=1, ,L=0.02

Fig.19. the velocity at α =1, M=1, d=1, Q=-1, a=0.7, b=1.2 ,L=0.02, ϕ =0, x=1

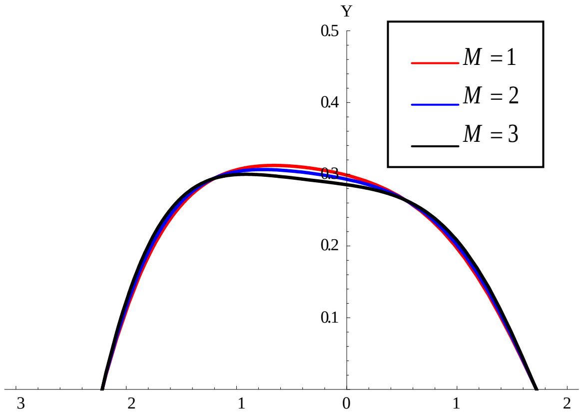


Fig.20.the velocity at α =1, k=2, d=1, Q=-1, a=0.7, b=1.2,L=0.02,

a=0.7,b=1.2, ϕ =0, x=1

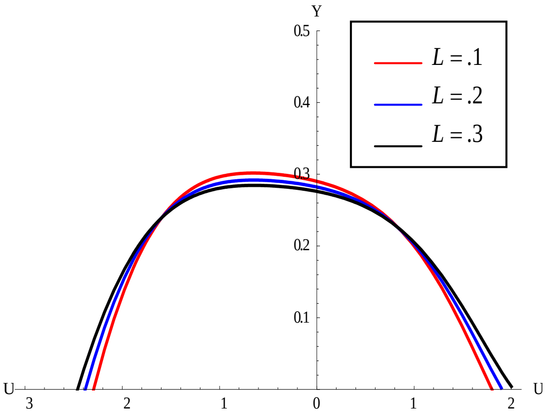


Fig.21. the velocity at α =1, k=2, M=1, d=1, Q=-1,

a=0.7,b=1.2, ϕ =0, x=1

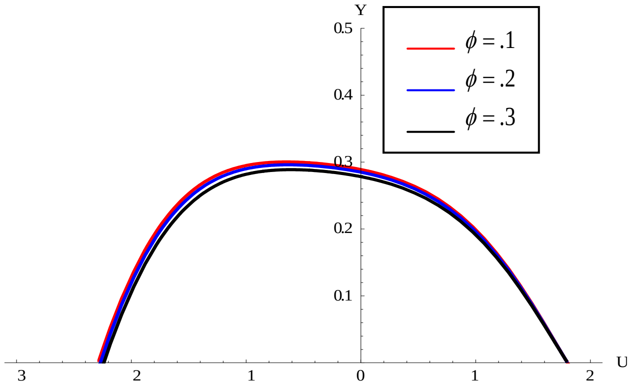


Fig.22. the velocity at α =1,k=2,M=1,d=1,Q=-1,a=0.7,b=1.2,L=0.02,x=1

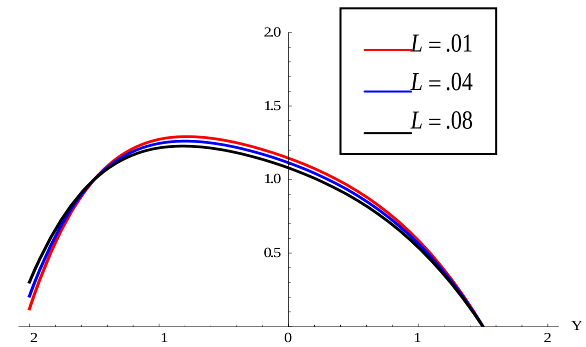


Fig.23.variation of temperature θ with Y for different

value of L at α =1,k=1000,M=1,d=1.5, ϕ =Pi/2, Q=1.4,

a=0.5,b=1.2,x=1,Ec=1,Pr=1,L=.01, β_1 =.3.

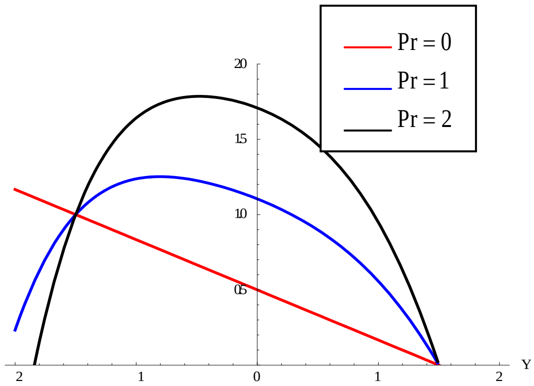


Fig.24.variation of temperature θ with Y for different

value of Pr at α =1,k=1000,M=1,d=1.5,Q=1.4, ϕ =Pi/2,

a=0.5, b=1.2, L=.01, x=1,Ec=1, β_1 =.3

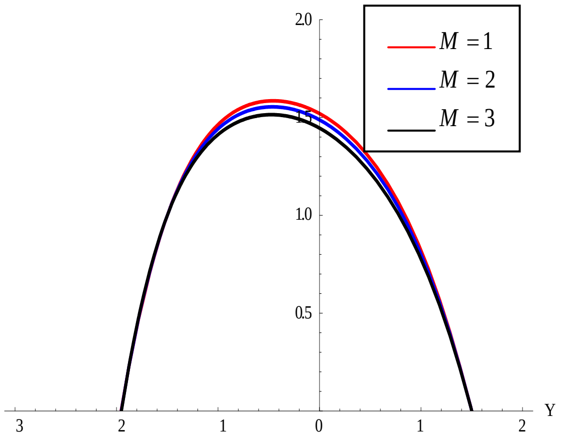


Fig.25.variation of temperature θ with Y for different value of M at α =1,k=1000,d=1.5,Q=1.4,a=0.5, b=1.2,L=.01, ϕ =Pi/2,x=1, Ec=1,Pr=1, β_1 =.3

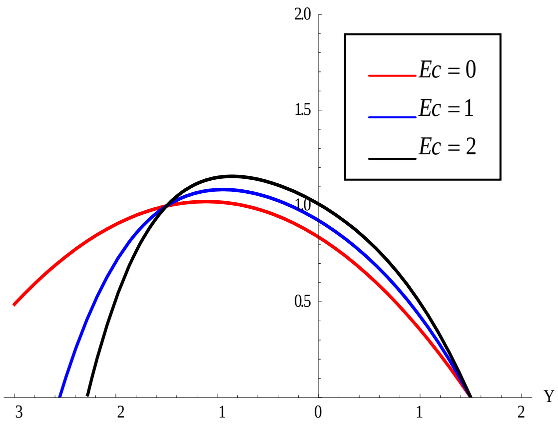


Fig.26.variation of temperature θ with Y for different value of Ec at, α =1, k=1000, M=1, d=1.5, Q=1.4, a=0.5, b=1.2, L=.01, ϕ =Pi/2, x=1, Pr=1, β_1 =.3

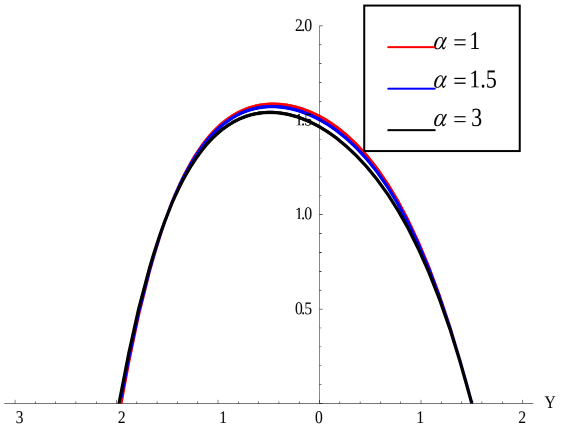


Fig.27.variation of temperature θ with Y for different α at k=1000,M=1,d=1.5,Q=1.4,a=0.5,b=1.2,L=.01, ϕ =Pi/2 ,x=1,Ec=1,Pr=1, β_1 =.3

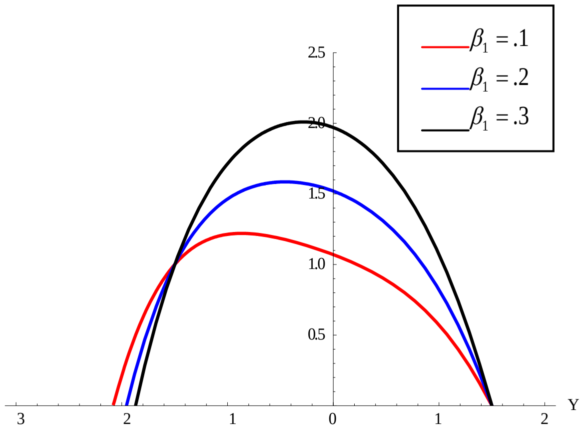


Fig.28.variation of temperature θ with Y for different value of value of β_1 at α =1, k=1000,M=1,d=1.5,Q=1.4,a=0.5,b=1.2, L=.01, ϕ =Pi/2 ,x=1, Ec=1,Pr=1

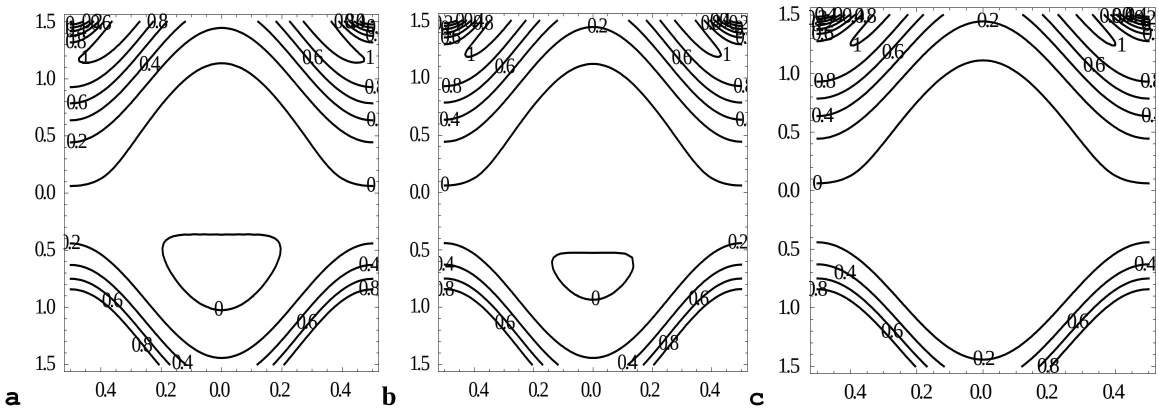


Fig.29.stream line for different values of M.(a) M=.1,(b)M=1.5,(c)M=2 and the other parameters are α =3,k=0.2,d=1,Q=1.5,a=0.5,b=0.5,L=0.02, ϕ =0

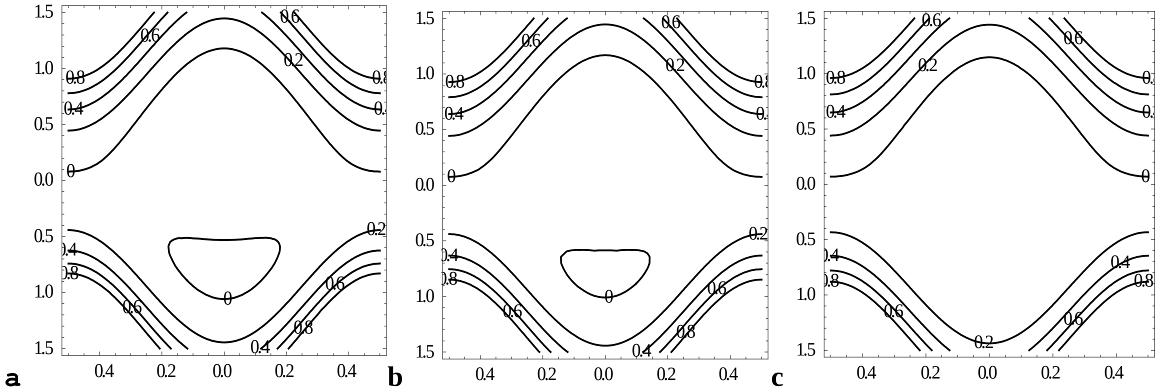


Fig.30.stream line for different values of L.(a)L=0.01,(b)L=0.04,(c)L=0.07 and the other parameters are α =1,k=0.2,d=1,Q=1.5,a=0.5,b=0.5, ϕ =0, M=2.

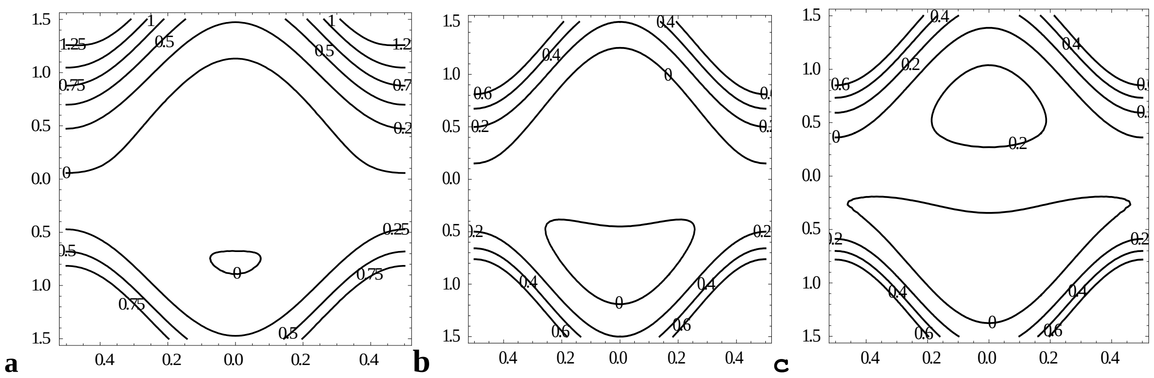


Fig.31.stream line for different values of Q.(a) Q=1.45,(b)Q=1.6,(c)Q=1.8,and the other parameters are α =1,k=0.2,d=1,a=0.5,b=0.5, ϕ =0, M=2,L=0.02.

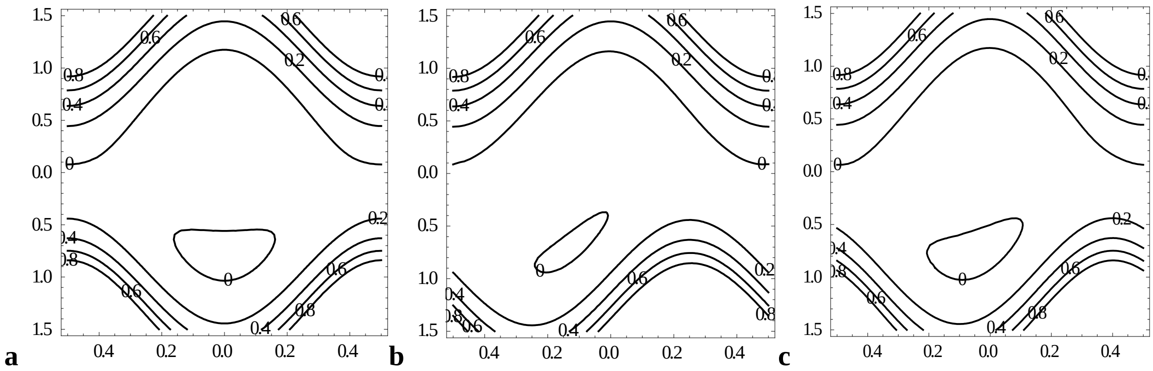


Fig.32.stream line for different values of ϕ .(a) ϕ =0, (b) ϕ =Pi/2,(c)= ϕ Pi/5 and the other parameters are α =1,k=0.2,d=1,Q=1.5,a=0.5,b=0.5,L=0.02, M=2.

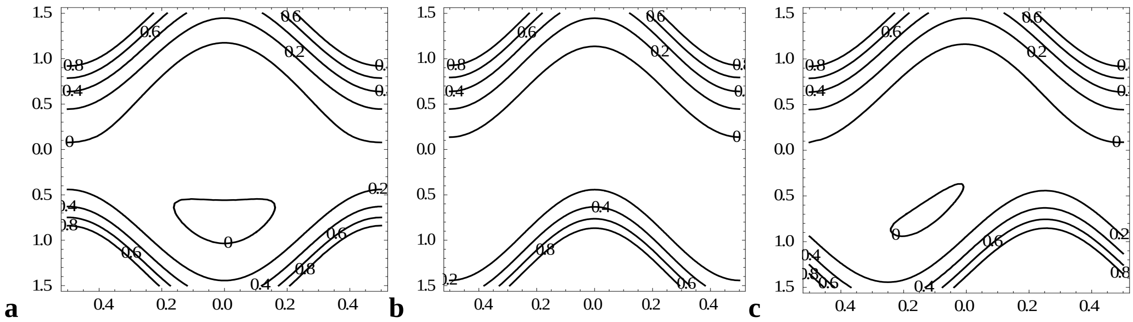


Fig.33.stream line for different values of α .(a) α =.5,(b) α =2,(c)=2.4, and the other parameters are k=0.2,d=1,Q=1.5,a=0.5,b=0.5,L=0..02, M=2.

4. Conclusion

we have discussed the influence of couple stress with heat transfer and magnetic field on the peristaltic flow of a non-Newtonian fluid with partial slip in inclined channel. the governing equations of motion and energy equation have been calculated under the assumptions of long wave length approximation The results are discussed through graphs. We have concluded the following observations:

1. The pressure gradient decreases with the increases in M.
2. The pressure rise decreases with the increases in Land increases when α ,M and K increases
3. The velocity field increases with the increase in k and decreases with the increase in M, α, L, ϕ .
4. an inclination of the channel does not effect on the velocity field and streamlines.
5. The temperature field decreases with the increase in L,M and α ,while with the increase in Pr,Ec and β_1 the temperature field increases.
6. The size of the trapping bolus decreases by increasing in L, α, M .
7. The size of the trapping bolus increases by increasing Q.

References

1. Ebaid, A., ‘‘Effects of Magnetic Field and Wall Slip Conditions on the PeristalticTransport of a Newtonian Fluid in an Asymmetric Channel,’’ Phys. Lett. A, 372,4586 (2008).

2. Elshahed, M. and M. H. Haroun, ‘‘Peristaltic Transport of Johnson–Segalman Fluidunder Effect of a Magnetic Field,’’ Math. Prob. Eng., 6, 663 (2003).
3. Elshehawey, E. F., E. M. E. Elbarbary, and N. S. Elgazery, ‘‘Effect of Inclined Magnetic ield on Magneto Fluid Flow through a Porous Medium between Two InclinedWavy Porous Plates,’’ Appl. Math. Comput., 135, 85 (2003).
4. Haroun, M. H., ‘‘Effect of Deborah Number and Phase Difference on PeristalticTransport of a Third-order Fluid in an Asymmetric Channel,’’ Commun. NonlinearSci. Numer. Simulat., 12, 1464 (2007a).
5. Haroun, M. H., ‘‘Non-linear Peristaltic Flow of a Fourth grade Fluid in an InclinedAsymmetric Channel,’’ Comput. Mater. Sci., 39, 324 (2007b).
6. Kothandapani, M. and S. Srinivas, ‘‘Non-linear Peristaltic Transport of a NewtonianFluid in an Inclined Asymmetric Channel through a Porous Medium,’’ Phys. Lett. A,372, 1265 (2008).
7. Mekheimer, and Y. Abd Elmaboud, ‘‘The Influence of Heat Transfer and Magnetic Fieldon Peristaltic Transport of a Newtonian Fluid in a Vertical Annulus: an Applicationof an Endoscope,’’ Phys. Lett. A, 372, 1657 (2008a).
8. Mekheimer, Kh .S, ‘‘Effect of the Induced Magnetic Field on Peristaltic Flow of a CoupleStress Fluid,’’ Phys. Lett. A, 372, 4271 (2008b).
9. Nadeem, S. and N. S. Akbar, ‘‘Effects of Heat Transfer on the Peristaltic Transportof MHD Newtonian Fluid with Variable Viscosity: Application of AdomianDecomposition Method,’’ Commun. Nonlinear Sci. Numer. Simulat., 14, 3844(2009a).
10. Nadeem, S. and N. S. Akbar, ‘‘Influence of Heat Transfer on Peristaltic Transport ofHerschel–Bulkley Fluid in a Non-uniform Inclined Tube,’’ Commun. Nonlinear Sci.Numer. Simulat., 14, 4100 (2009b).
11. Srinivas, S. and M. Kothandapani, ‘‘The Influence of Heat and Mass Transfer on MHDPeristaltic Flow through a Porous Space with Compliant Walls,’’ Appl. Math.Comput., 213, 197 (2009).
12. Srinivas, S. and V. Pushparaj, ‘‘Non-linear Peristaltic Transport in an Inclined AsymmetricChannel,’’ Commun. Nonlinear Sci. Numer. Simulat., 13, 1782 (2008).
13. Abd El Hakeem, Abd El Naby, A.E.M. El Misiery, Appl. Math. Comput. 128 (2002) 19.
14. T. Hayat, Y. Wang, A.M. Siddiqui, K. Hutter, S. Asghar, Math. Models Methods Appl. Sci. 12 (2002) 1691.
15. T. Hayat, A.H. Kara, E. Momoniat, Int. J. Non-Linear Mech. 38 (2003) 1533.
16. C. Fetecau, C. Fetecau, Z. Angew. Math. Phys. 56 (2005) 1098.
17. C. Fetecau, C. Fetecau, Int. J. Eng. Sci. 43 (2005) 781.
18. T. Hayat, A.H. Kara, Comput. Model. 43 (2006) 132.
19. T. Hayat, N. Ali, Physica A 371 (2006) 188.
20. T. Hayat, N. Ali, S. Asghar, Appl. Math. Comput. 186 (1) (2007) 309.
21. T. Hayat, N. Ali, Appl. Math. Comput. 193 (2) (2007) 535.
22. Mekheimer, Kh.S.: Effect of induced magnetic field on peristaltic flow of a couple stress fluid. Phys. Lett. A. 372, 4271–4278 (2008)
23. V.C.A. Ferraro, An Introduction to Magneto-Fluid Mechanics, Clarendon Press,Oxford, 1966.
24. N.S. Akbar, T. Hayat, S. Nadeem, Awatif A. Hendi, Effects of slip and heat transfer on the peristaltic flow of a third order fluid in an inclined asymmetric channel, Int. J. Heat Mass Transfer. 54 (2011) 1654–1664.