

Common Random Fixed Point Theorems of Contractions in Partial Cone Metric Spaces over Non normal Cones

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1. ABSTRACT: The purpose of this paper is to prove existence of common random fixed point in the setting of partial cone metric space over the non-normal cones.

Key wards: common fixed point,cone metric space, random variable

2. INTRODUCTION AND PRELIMINARIES

Random nonlinear analysis is an important mathematical discipline which is mainly concerned with the study of random nonlinear operators and their properties and is needed for the study of various classes of random equations. The study of random fixed point theory was initiated by the Prague school of Probabilities in the 1950s [4, 13, and 14]. Common random fixed point theorems are stochastic generalization of classical common fixed point theorems. The machinery of random fixed point theory provides a convenient way of modeling many problems arising from economic theory and references mentioned therein. Random methods have revolutionized the financial markets. The survey article by Bharucha-Reid [1] attracted the attention of several mathematicians and gave wings to the theory. Itoh [18] extended Spacek's and Hans's theorem to multivalued contraction mappings. Now this theory has become the full edged research area and various ideas associated with random fixed point theory are used to obtain the solution of nonlinear random system (see [2,3,7,8,9]). Papageorgiou [11, 12], Beg [5,6] studied common random fixed points and random coincidence points of a pair of compatible random and proved fixed point theorems for contractive random operators in Polish spaces.

In 2007, Huang and Zhang [9] introduced the concept of cone metric space and establish some fixed point theorems for contractive mappings in normal cone metric spaces. Subsequently, several other authors [10, 17,] studied the existence of fixed points and common fixed points of pings satisfying contractive type condition on a normal cone metric space. In 2008, Rezapour and Hamlbarani [17] omitted the assumption of normality in cone metric space, which is a milestone in developing fixed point theory in cone metric space. In this paper we prove existence of common random fixed point in the setting of cone random metric spaces under weak contractive condition. Recently, Dhagat et al. [19] introduced the concept of cone random metric space and proved an existence of random fixed point under weak contraction condition in the setting of cone random metric spaces. The purpose of this paper to find common random fixed point theorems of contractions in partial cone metric spaces over non normal cones.

Definition 2.1. Let X be a nonempty set and let P be a cone of a topological vector space E. A partial cone metric on X is a mapping $\mathrm{p}:\Omega\times\mathrm{X}\times\mathrm{X}\to\mathrm{P}$ such that, for each $f(t), g(t), h(t)\in X$, $t\in\Omega$,

$(p1)\ \ p(f(t),g(t)) = p(f(t),f(t)) = p(g(t),g(t)) \Leftrightarrow f(t) = g(t),$

$(p2)\ p(f(t),g(t)) = p(g(t),f(t))$

$(p3)\ \ p(f(t),f(t)) \le p(f(t),g(t))$

$(p3)\ \ p(f(t),g(t)) \le p(f(t),h(t)) + p(h(t),g(t)) - p(h(t),h(t))$

$\forall t \in \Omega$

The pair (X,p) is called a partial cone metric space over *P*.

Definition 2.2. A function f: Ω → C is said to be measurable if $f^{-1}(B \cap C) \in \sum$ for every Borel subset B of H.

Definition 2.3. A function $F:\Omega\times C\to C$ is said to be a random operator if $F(.,x):\Omega\to C$ is measurable for every $x\in C$

Definition 2.4. A measurable $g:\Omega\to C$ is said to be a random fixed point of the random operator $F:\Omega\times C\to C$ if $F(t,g(t)) = g(t)$ for all $t\in\Omega$

Definition 2.5. A random operator F: Ω×C → C is said to be continuous if for fixed $t\in\Omega,\ F(t,.):C\to C$ is continuous.

Lemma:2.6 Let P be solid cone of a topological vector space E and $\{f_n(t)\},\{g_n(t)\},\{h_n(t)\}\subset E$.if $f_n(t)\le h_n(t)\le g_n(t)\ \forall n,$ and there exists some $\omega(t)\in E$ Such that $f_n(t)\xrightarrow{\omega}\omega(t)\ \ and\ \ g_n(t)\xrightarrow{\omega}\omega(t)\ \ then\ \ h_n(t)\xrightarrow{\omega}\omega(t)$

Lemma:2.7 Let P be solid cone of a normed vector space $(E,\|.\|)$then for each sequence $\{f_n(t)\}\subset E.$ $f_n(t)\xrightarrow{\|.\|}\omega(t)\ \ implies\ \ f_n(t)\xrightarrow{\omega}\omega(t)$ moreover if P is normal, then $f_n(t)\xrightarrow{\omega}\omega(t)$ $implies\ \ f_n(t)\xrightarrow{\|.\|}\omega(t)$

Lemma:2.8 Let P be solid cone of a normed vector space $(E,\|.\|),\{K_n\}\subset\zeta\ \ and\ \ \{f_n(t)\}\subset P.$ $f_n(t)\xrightarrow{\omega}\theta\ and\ \sup_n\|K_n\|<+\infty,\ then\ K_n f_n(t)\xrightarrow{\omega}\theta.$

Theorem 2.9 Let $(X\times\Omega,p)$ be partial cone metric space. The mapping $T,S:X\times\Omega\to X$ are called contractions restricted with variable positive linear bounded mappings if there exist $L_i:X\times X\to\zeta\ (i=1,2,3,4)$ such that

$$\begin{aligned}p(T(f(t),t)) \le\ & L_1(f(t),g(t))p(f(t),g(t)) + L_2(f(t),g(t))p(f(t),T(f(t),t))\\ & + L_3(f(t),g(t))p(g(t),S(g(t),t)\\ & + L_4(f(t),g(t))p(f(t),S(g(t),t) + p(g(t),T(f(t),t))\end{aligned}$$

$$f(t),g(t)\in X\quad ---(*)$$

In particular if (*) is holds with $L_i(f(t),g(t))\equiv A_i\quad and\ \ A_i\in\zeta\ (i=1,2,3,4)$

then T and S are called contractions restricted with positive linear bounded mappings

3. Main Result:

Theorem 3.1. Let (X, p) be a θ-complete partial cone metric space over a solid cone P of a normed vector space $(E, \|.\|)$ and let $T, S : X \times \Omega \to X$ be contractions restricted with variable positive linear bounded random mappings. If

$p(L_3(f(t), g(t)) + L_4(f(t), g(t))) < 1$ and $p(L_2(f(t), g(t)) + L_4(f(t), g(t))) < 1$

$\forall\ f(t), g(t) \in X$ ---------------------------------------(1)

$l_1 l_2 < 1$ and $l_3 < \infty$, where $p(.)$ denotes the spectral radius of linear bounded mappings,

$$l_1 = \sup_{f(t), g(t) \in X} \|K_1(f(t), g(t))\|$$
$$l_2 = \sup_{f(t), g(t) \in X} \|K_2(f(t), g(t))\|$$
$$l_3 = \sup_{f(t), g(t) \in X} \|K_3(f(t), g(t))\|, \text{ ---------------------------------(2)}$$

$$K_1(f(t), g(t)) = \overline{L_1}(f(t), g(t))[L_1(f(t), g(t)) + L_2(f(t), g(t)) + L_4(f(t), g(t))]$$
$$K_2(f(t), g(t)) = \overline{L_2}(f(t), g(t))[L_1(f(t), g(t)) + L_3(f(t), g(t)) + L_4(f(t), g(t))]$$
$$K_3(f(t), g(t)) = \overline{L_2}(f(t), g(t))[I + L_3(f(t), g(t)) + L_4(f(t), g(t))]$$
$\forall\ f(t), g(t) \in X$ ---(3)

where $\overline{L_1}(f(t), g(t))$ and $\overline{L_2}(f(t), g(t))$ denote the inverse of $I - L_3(f(t), g(t)) - L_4(f(t), g(t))$ and $I - L_2(f(t), g(t)) - L_4(f(t), g(t))$ respectively, then T and S have common random fixed point in X Moreover if

$p(L_1(f(t), g(t)) + L_2(f(t), g(t)) + L_3(f(t), g(t)) + 2L_4(f(t), g(t))) < 1 \quad \forall\ f(t), g(t) \in X$ -----------(4)

then T and S have unique common random fixed point $h(t) \in X$ such that, for each $h_n(t) \in X$, $h_n(t) \xrightarrow{\tau_p} h(t)$ where $h_n(t)$ is defined by

$$h_{n+1}(t) = \begin{cases} T(h_n(t), t) & ;n \text{ is even number} \\ S(h_n(t), t) & ;n \text{ is odd number} \end{cases} \text{ -------------------------(5)}$$

Proof. For each $x, y \in X$ by (1), the inverse of $1 - L_3\ (f(t), g(t)) - L_4(f(t), g(t))$ and $1 - L_2\ (f(t), g(t)) - L_4(f(t), g(t))$ exist. then, it is clear that $\overline{L_1}$ and $\overline{L_2}$ are meaningful and K_1, K_2, K_3 are well defined.

$$\overline{L_1}(f(t), g(t)) = \sum_{i=0}^{\infty} [L_3\ (f(t), g(t)) + L_4(f(t), g(t))]^i$$

$$\overline{L_2}(f(t), g(t)) = \sum_{i=0}^{\infty} [L_2\ (f(t), g(t)) + L_4(f(t), g(t))]^i \text{ -----------------(6)}$$

$x, y \in X$, which is together with $L_i : X \times X \to \zeta\ (i = 2,3,4)$ implies that $L_i : X \times X \to \zeta\ (i = 1,2)$ and hence $K_i : X \times X \to \zeta\ (i = 1,2,3)$. by(*),(5), (p4) $L_4 : X \times X \to \zeta$

$$\begin{aligned}
p(f_{2k+1}(t), f_{2k+2}(t)) &= p(T(f_{2k}(t),t), S(f_{2k+1}(t),t) \\
&\leq L_1(f_{2k}(t), f_{2k+1}(t))\, p(f_{2k}(t), f_{2k+1}(t)) \\
&+ L_2(f_{2k}(t), f_{2k+1}(t))\, p(f_{2k}(t), f_{2k+1}(t)) \\
&+ L_3(f_{2k}(t), f_{2k+1}(t))\, p(f_{2k+1}(t), f_{2k+2}(t)) \\
&+ L_4(f_{2k}(t), f_{2k+1}(t))[p(f_{2k}(t), f_{2k+2}(t)) + p(f_{2k+1}(t), f_{2k+1}(t))] \\
&\leq L_1(f_{2k}(t), f_{2k+1}(t))\, p(f_{2k}(t), f_{2k+1}(t)) \\
&+ L_2(f_{2k}(t), f_{2k+1}(t))\, p(f_{2k}(t), f_{2k+1}(t)) \\
&+ L_3(f_{2k}(t), f_{2k+1}(t))\, p(f_{2k+1}(t), f_{2k+2}(t)) \\
&+ L_4(f_{2k}(t), f_{2k+1}(t))[p(f_{2k}(t), f_{2k+1}(t)) + p(f_{2k+1}(t), f_{2k+2}(t))]\ \forall k \in N
\end{aligned}$$

$$--------(7)$$

And so

$$\begin{aligned}
&[I - L_2(f_{2k}(t), f_{2k+1}(t)) - L_4(f_{2k}(t), f_{2k+1}(t))]\, p(f_{2k+1}(t), f_{2k+2}(t)) \\
&\leq \begin{bmatrix} L_1(f_{2k}(t), f_{2k+1}(t)) + L_2(f_{2k}(t), f_{2k+1}(t)) \\ + L_4(f_{2k}(t), f_{2k+1}(t)) \end{bmatrix} p(f_{2k}(t), f_{2k+1}(t))]\ \forall k \in N
\end{aligned}$$

$$---------(8)$$

Act the above inequality with $\overline{L_1}(f_{2k}(t), f_{2k+1}(t))$; then, $\overline{L_1} : X \times X \to \zeta$,

$$p(f_{2k+1}(t), f_{2k+2}(t)) \leq K_1(f_{2k}(t), f_{2k+1}(t))\, p(f_{2k}(t), f_{2k+1}(t)),\ \forall k \in N\ ----------(9)$$

Similarly ,by (*),(p3), (p4) and $L_4 : X \times X \to \zeta$

$$\begin{aligned}
p(f_{2k+2}(t), f_{2k+3}(t)) &= p(f_{2k+3}(t), f_{2k+2}(t)) \\
&= p(T(f_{2k+2}(t),t), S(f_{2k+1}(t),t) \\
&\leq L_1(f_{2k+2}(t), f_{2k+1}(t))\, p(f_{2k+2}(t), f_{2k+1}(t)) \\
&+ L_2(f_{2k+2}(t), f_{2k+1}(t))\, p(f_{2k+2}(t), f_{2k+3}(t)) \\
&+ L_3(f_{2k+2}(t), f_{2k+1}(t))\, p(f_{2k+1}(t), f_{2k+2}(t)) \\
&+ L_4(f_{2k+2}(t), f_{2k+1}(t))[p(f_{2k+2}(t), f_{2k+2}(t)) + p(f_{2k+1}(t), f_{2k+3}(t))] \\
&\leq L_1(f_{2k+2}(t), f_{2k+1}(t))\, p(f_{2k+2}(t), f_{2k+1}(t)) \\
&+ L_2(f_{2k+2}(t), f_{2k+1}(t))\, p(f_{2k+2}(t), f_{2k+3}(t)) \\
&+ L_3(f_{2k+2}(t), f_{2k+1}(t))\, p(f_{2k+1}(t), f_{2k+2}(t)) \\
&+ L_4(f_{2k+2}(t), f_{2k+1}(t))[p(f_{2k+2}(t), f_{2k+1}(t)) + p(f_{2k+2}(t), f_{2k+3}(t))]\quad \forall k \in N
\end{aligned}$$

$$----------(10)$$

And so

$$[I - L_2(f_{2k+2}(t), f_{2k+1}(t)) - L_4(f_{2k+2}(t), f_{2k+1}(t))]p(f_{2k+2}(t), f_{2k+3}(t))$$
$$\leq \begin{bmatrix} L_1(f_{2k+2}(t), f_{2k+1}(t)) + L_3(f_{2k+2}(t), f_{2k+1}(t)) \\ + L_4(f_{2k+2}(t), f_{2k+1}(t)) \end{bmatrix} p(f_{2k+2}(t), f_{2k+1}(t))] \; \forall k \in N$$
$$---------(11)$$

Act the above inequality with $\overline{L_2}(f_{2k+2}(t), f_{2k+1}(t))$; then, by: $\overline{L_2} : X \times X \to \zeta$,

$$p(f_{2k+2}(t), f_{2k+1}(t) \leq K_2(f_{2k+2}(t), f_{2k+1}(t))p\ (f_{2k+1}(t), f_{2k+2}(t)), \forall k \in N. ---------(12)$$

Moreover $K_1, K_2 : X \times X \to \zeta$,

$$p(f_{2k+1}(t), f_{2k+2}(t)) \leq K_1(f_{2k}(t), f_{2k+1}(t)) \times K_2(f_{2k}(t), f_{2k-1}(t))........$$
$$........K_1(f_0(t), f_1(t))p(f_0(t), f_1(t)), \forall k \in N ------------(13)$$

In the following, we will prove that

$$p(f_n(t), f_m(t)) \xrightarrow{\omega} \theta ----------------------------------(14)$$

For m>n, we have four cases

$(1)\ m = 2p+1, n = 2q+1; (2)\ m = 2p+1, n = 2q\ (3) m = 2p, n = 2q+1 (4)\ m = 2p, n = 2q$

Where p and q are two non negative integers such that $p > q$. We only show that (14) holds for case (1) the proof of three cases is similar. It follows for (p4) and (13) that

$$\theta \leq p(f_n(t), f_m(t))$$
$$= p(f_{2q+1}(t), f_{2p+1}(t))$$
$$\leq p(f_{2q+1}(t), f_{2q+2}(t)) + p(f_{2q+2}(t), f_{2q+3}(t)) + ... + p(f_{2p-1}(t), f_{2p}(t)) + p(f_{2p}(t), f_{2p+1}(t))$$
$$\leq pK_1K_2(f_0(t), f_1(t))$$
$$= K_1(f_{2q}(t), f_{2q+1}(t)) \times K_2(f_{2q}(t), f_{2q-1}(t))........K_1(f_0(t), f_1(t))p(f_0(t), f_1(t))$$ By
$$+ K_2(f_{2q+2}(t), f_{2q+1}(t))K_1(f_{2q}(t), f_{2q+1}(t)) \times K_2(f_{2q}(t), f_{2q-1}(t)).....K_1(f_0(t), f_1(t))p(f_0(t), f_1(t)) + ...$$
$$+ K_1(f_{2q-2}(t), f_{2q-1}(t)) \times K_2(f_{2p-2}(t), f_{2p-3}(t)).....K_1(f_0(t), f_1(t))p(f_0(t), f_1(t))$$
$$+ K_2(f_{2p}(t), f_{2q-1}(t)) \times K_1(f_{2p-2}(t), f_{2p-1}(t)).....K_1(f_0(t), f_1(t))p(f_0(t), f_1(t)) \forall p > q$$
$$.-----(15)$$

$$l_1 l_2 < 1,$$
$$\|pK_1K_2(f_0(t), f_1(t))\|$$
$$\leq l_1^{q+1}l_2^{q} + l_1^{q+1}l_2^{q+1} + ... + l_1^{p+1}l_2^{p} + l_1^{p}l_2^{p} \|p(f_0(t), f_1(t))\|$$
$$= \left(l_1 \sum_{i=q}^{p} (l_1 l_2)^i + l_1 \sum_{i=q+1}^{p} (l_1 l_2)^i + \right) \|p(f_0(t), f_1(t))\|$$
$$\leq \frac{(l_1 + l_1 l_2)(l_1 l_2)^q \|p(f_0(t), f_1(t))\|}{1 - l_1 l_2} \quad \forall\ p > q, \ ------------------------(16)$$

Which implies that $p(f_n(t), f_m(t)) \xrightarrow{\|.\|} \theta$,and hence $p(f_n(t), f_m(t)) \xrightarrow{\omega} \theta$ by lemma(2.7) Thus by (15) and lemma (2.6) $p(f_n(t), f_m(t)) \xrightarrow{\omega} \theta$;that is (14) holds. It is prove that $\{f_n(t)\}$is a θ Cauchy sequence in (X, p),and so by the θ completeness of (X, p), there exits $h(t) \in X$ such that $f_n(t) \xrightarrow{\tau_p} h(t)$ and $p(h(t), h(t)) = \theta$; that is,

$$p(f_n(t),h(t)) \xrightarrow{\omega} \theta \text{ ------------------------------------(17)}.$$

For all $k \in N$, by (*) and (p4),

$$\begin{aligned}
p\big(T(h(t),t),h(t)\big) &\le p\big(T(h(t),t),h(t), f_{2k}(t)\big) + p(f_{2k}(t),h(t)) \\
&= p\big(T(h(t),t), S(f_{2k-1}(t),t)\big) + p(f_{2k}(t),h(t)) \\
&\le L_1(h(t), f_{2k-1}(t))\, p(h(t), f_{2k-1}(t)) \\
&\quad + L_2(h(t), f_{2k-1}(t))\, p\big(h(t),T(h(t),t)\big) \\
&\quad + L_3(h(t), f_{2k-1}(t))\, p(f_{2k-1}(t), f_{2k}(t)) \\
&\quad + L_4(h(t), f_{2k-1}(t))\big[p(h(t), f_{2k}(t)) + p\big(f_{2k-1}(t),T(h(t),t)\big)\big] \\
&\quad + p(f_{2k}(t),h(t))
\end{aligned}$$

$$\begin{aligned}
&\le L_1(h(t), f_{2k-1}(t))\, p(h(t), f_{2k-1}(t)) \\
&\quad + L_2(h(t), f_{2k-1}(t))\, p\big(h(t),T(h(t),t)\big) \\
&\quad + L_3(h(t), f_{2k-1}(t))\big[p(f_{2k-1}(t),h(t))) + p(h(t), f_{2k}(t))\big] \\
&\quad + L_4(h(t), f_{2k-1}(t))\begin{bmatrix} p(h(t), f_{2k}(t)) + p\big(f_{2k-1}(t),(h(t),t)\big) \\ + p\big(h(t),T(h(t),t)\big) + p(f_{2k}(t),h(t)) \end{bmatrix} \\
&\quad + p\big(h(t),T(h(t),t)\big) + p(f_{2k}(t),h(t)) \text{ ------------------------(18)}
\end{aligned}$$

And so

$$\begin{aligned}
&\big[I - L_2(h(t), f_{2k-1}(t)) - L_4(h(t), f_{2k-1}(t))\big] p(T(h(t),t),h(t)) \\
&\le \big[L_1(h(t), f_{2k-1}(t)) + L_3(h(t), f_{2k-1}(t)) + L_4(h(t), f_{2k-1}(t))\big] p(h(t), f_{2k-1}(t)) \\
&\quad + \big[I + L_3(h(t), f_{2k-1}(t)) + L_4(h(t), f_{2k-1}(t))\big] p(f_{2k}(t),h(t)) \forall k \in N
\end{aligned}$$

$$\text{---------(19)}$$

Act the above inequality with $\overline{L_2}(h(t), f_{2k-1}(t))$;then, $\overline{L_1} : X \times X \to \zeta$,

$$\theta \le p(T(h(t),t,h(t)) \le K_{2,2k-1}\, p(h(t), f_{2k-1}(t) + K_{3,2k-1}\, p(f_{2k}(t),h(t)) \forall k \in N \text{ -----(20)}$$

Where $K_{2,2k-1} = K_2\, p(h(t), f_{2k-1}(t))$ and $K_{3,2k-1} = K_2\, p(h(t), f_{2k-1}(t))$.

It is clear that { $K_{2,2k-1}$ },{ $K_{3,2k-1}$ } subsets of ζ and $\sup_k \|K_{2,2k-1}\| < +\infty, \sup_k \|K_{3,2k-1}\| < +\infty$ by $l_1 l_2$ <1and $l_3 < +\infty$.then its follows from the lemma (3) and (17)that

$$K_{2,2k-1}\, p(h(t), f_{2k-1}(t) + K_{3,2k-1}\, p(h(t), f_{2k-1}(t) \xrightarrow{\omega} \theta, \text{ ----------------(21)}$$

Which together with lemma (2.6)and (20) implies that $p\big(h(t),T(h(t),t)\big) = \theta$

Therefore $T(h(t),t) = h(t)$ by (p1) and(p3).similarly we can show that $S(h(t),t) = h(t)$.

Hence $h(t)$ is common random fixed point of T and

Now we show the uniqueness of fixed point. Let $f(t)$ and $h(t)$ be two common random fixed point of T and S then by (*) and (p3) $L_i : X \times X \to \zeta \ (i = 2,3)$

$$
\begin{aligned}
p(h(t), f(t)) &= p(T(h(t),t)), S(f(t),t)) \\
&\le L_1(h(t), f(t))p(h(t), f(t),) + L_2(h(t), f(t))p(h(t), T(h(t),t) + L_3(h(t), f(t))p(f(t), S(f(t),t)) \\
&\quad + L_4(h(t), f(t))\left[p(h(t), S(f(t),t)) + p(f(t), T(h(t),t))\right] \\
&= \left[L_1(h(t), f(t)) + 2L_4(h(t), f(t))\right]p(h(t), f(t)) \\
&\quad + L_2(h(t), f(t))p(h(t), h(t)) + L_3(h(t), f(t))p(f(t), f(t)) \\
&\le \left[L_1(h(t), f(t)) + L_2(h(t), f(t)) + L_3(h(t), f(t)) + 2L_4(h(t), f(t))\right]p(h(t), f(t)), \text{-------}(22)
\end{aligned}
$$

And so

$$\left[I - L_1(h(t), f(t)) - L_2(h(t), f(t)) - L_3(h(t), f(t)) - 2L_4(h(t), f(t))\right]p(h(t), f(t)), \text{-------}(23)$$

It follow from (3) that the inverse of $\left[I - L_1(h(t), f(t)) - L_2(h(t), f(t)) - L_3(h(t), f(t)) - 2L_4(h(t), f(t))\right]$ exists(denoted by) $\left[I - L_1(h(t), f(t)) - L_2(h(t), f(t)) - L_3(h(t), f(t)) - 2L_4(h(t), f(t))\right]^{-1}$ and $\left[I - L_1(h(t), f(t)) - L_2(h(t), f(t)) - L_3(h(t), f(t)) - 2L_4(h(t), f(t))\right]^{-1} \in \zeta$ by Neumann's formula with $\left[I - L_1(h(t), f(t)) - L_2(h(t), f(t)) - L_3(h(t), f(t)) - 2L_4(h(t), f(t))\right]^{-1}$;then $p(h(t), f(t)) = \theta$ and hence $h(t) = f(t)$ by (p1) and (p3)the proof is completed.

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