Modeling a Small Scale Business Outfit With M/M/1 Markovian Queuing Process and Revenue Analysis

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ABSTRACT

This article presents a queuing model on the operation of a small business which are often visited with a lot of set- backs and uncertainties. Probability models were derived for two system characteristics namely the customers' inter-arrival rate (I_t), the service time per customer (Y_t), and the Service charge (C_t) per customer was also recorded. This data collected were subjected to single factor analysis of variance with fixed effect to confirm whether the data came from the same population distribution. However it was observed that the inter-arrival time for the customers was exponential, service rate also exponential with a traffic intensity of $\rho = 0.30$ and the server was idle for about 70% in every one hour. With an average service charge of 32.52 per customer the business makes N1,062.33.28 per year.

Key words: stochastic model, queue, inter-arrival rate, service rate, Traffic intensity.

1.0 INTRODUCTION

Most of us spend part of our walking hours waiting for something - to pay at the checkout counter at a market, mail a letter in the post office, get food in a cafeteria, borrow a book from library, land at an airport etc.

- Frustrated, we may ask can't enough checkout clerks be provided, enough toll booths, landing strips etc? Why is there always insufficient capacity?
- To make optimum decisions, it becomes necessary to predict the operating characteristics of waiting line systems such as the average number of people waiting, people being served, and attendants idling etc.
- This paper aims at reporting such analysis carried out from a photocopying centre based in Aba. When a waiting line system is designed, management is usually interested in overall performance and economic consequences as time goes by. Thus decisions are based on performance characteristics such as:

(i) The average (expected) number of customers arriving per hour.
(ii) The average time to service a customer.
(iii) The number of customers in the queue.
(iv) The number of customers being served, etc are all random variables.

Thus we are dealing here with probability processes and so we need probability models.

2.0 LITERATURE REVIEW

The literature on queuing or waiting line model and its application are extensive. There are many situations where queues are experienced, such as Aircrafts waiting to land, files in the "in-tray", cars awaiting for routing service etc. Thierauf etal(1995) observed that the theory of waiting line emanated from the work of A.K. Erband.

He explained that Erland started his work in 1909 when he experimented on a problem dealing with the congestion of telephone traffic. Smith(1977) opined through his schematic approval to the system that queuing system comprises of several components such as, source or calling population, calling unit, service discipline and service facility.
Frank(1963) Opined that when a customer arrives at a service facility, the customer often finds the server busy and he thus has to wait, while he is waiting other customers may arrive, a fairly long waiting line or queue may develop. Attahiru (2003) in his work titled "Comparative queue analysis" argued that most work on queue analysis assumed individual inter-arrival times.

Ryan Berry (2002) in his paper “queuing theory”, emphasized that the first paper on queuing theory was published by A.K. Erlang where the problem of determining how many telephone circuits were necessary to phone service.

Honda etal (2008) in their paper “Solving of waiting times models in the airport using queuing theory model and linear programming” observed that waiting lines and service systems are important parts of the business world. Alexander (2006) considered a model where multiple queues are served by servers whose capacity varies randomly.

3.0 METHODOLOGY

The period for data collection spanned four weeks. The business opens by 8.30am and closes 5.30pm each day, Monday to Saturday. For accuracy, a stop-watch and a calculator were used. The variables inter-arrival (I_t), the service time (Y_t), the waiting time (W_t) and the service charge (C_t) per customer were computed and recorded. The summary statistics of the computed operational variables I_t, Y_t, W_t and C_t are presented in appendix

table 1.

- The data were further put to test, to test for equality of mean on the data for inter-arrival time (I_t), service time (Y_t), and waiting time (W_t).

- We picked a random sample of size 10 (using systematic sampling) from each of the three variables data (I_t, Y_t, W_t) for each of the four weeks. Factor considered were the four weeks. We used the method of single factor analysis of variance with fixed effect.

- The model is $Y_{ij} = \mu + \alpha_i + e_{ij} \quad i = 1,2, \dots, 4; \quad j = 1,2, \dots, 10$

  Where

$$Y_{ij} = \text{the jth observatio n of the ith week}$$
$$\mu = \text{the universal constant}$$
$$\alpha_i = \text{the effect due to the ith week}$$
$$e_{ij} = \text{the error due to the jth observation of the ith week}$$

$$SST = \sum_{i=1}^{4}\sum_{j=1}^{10} Y_{ij}^2 - \frac{T_{..}^2}{(kn)}; \quad \mathrm{k} = 4, \ \mathrm{n} = 10$$

$$SS\alpha = \sum_{i=1}^{4} \frac{Y_{i.}^2}{n} - \frac{T_{..}^2}{(kn)}$$

$$\mathrm{H}_0 : \mu_{ij} = 0$$

Hypotheses $$\mathrm{H}_1 : \mu_{ij} \neq 0$$

Level of significance, $\alpha = 0.05$

Critical Region $= F_{(\alpha;k-1,N-k)}$ i.e. $F_{0.05,(3,36)} = 2.92$

Test Statistic: F-ratio = 0.78, 0.597 and 1.81 for the three variables. The computations and analysis are presented in appendix II table 2

Decision Rule: We do not accept H_0. if $Fcal > F \propto$

Conclusion: From appendix table 2 it shows (*ns) not significance at 5% for all the variables. We concluded that the sample means of the variables I_t, W_t and Y_t were equal. We note that equality in mean only suggests, but do not guarantee equality in the population distribution of the variables.

4.0 ANALYSIS

Based on the above conclusions we now had the basis to pool the data on the variables for fitting probability models on them.

(i) Fitting probability model on inter-arrival time data (It)

From appendix table 1, the average customers inter-arrival time (pooled) = $\overline{X} = 18.34$

$$\left[arrivalRate\ (A) = \frac{1hr}{\overline{X}} = \frac{60}{18.34} = 3\ \text{customers per hour)} \right]$$

$\chi^2 - test$ for exponential fit (goodness-of-fit) on the inter-arrival data. See appendix table 3

Hypotheses

$H_0: I_t = \ Exponential$

$H_0: I_t \neq Exponential$

Level of significance, $\alpha = 0.05$

$$\text{Critical Region}: \chi^2_{(\alpha,k-1)} \chi^2_{(0.05,39)} = 11.07$$

$$\text{Test statistic:} \quad \chi^2_c = \sum \frac{(o_i - e_i)^2}{e_i} = 9.5310$$

Decision: Since $\chi^2_\propto > \chi^2_c$

We do not reject H_0: but concluded that the I_t data were Exponentially distributed. (See appendix table 3)

(ii) <u>Fitting Probability model on service time data (Yt)</u>

From appendix table I, the Average service time per customer (pooled)

$$\overline{X} = 5.29$$

$$Service\ Rate(\mu) = \frac{1hr}{\overline{X}} = \frac{60\min}{5.29} = 11.34 \text{ customers per hour}$$

χ^2 - test for exponential fit on the service time data.

Hypotheses:

$H_0: Y_t \sim Exponential$

$H_i: Y_t \neq Exponential$

$\chi^2_{(0.05,\ 4)} = 9.488$ *and* $\chi^2_{calcated} = 4.6982$ (*see appendix table* 4)

Since $\chi^2_\propto > \chi^2_c$

We do not reject H_0: that the Y_t – Data are exponentially distributed wit.

Hence, we concluded in both cases that the inter-arrival rate for customers was exponentially distributed with 3 customers arriving per hour, also the service time was exponentially distributed with 11.34 customers receiving service in every 1 hour.

(iii) Determining the Traffic Intensity

From the appendix table 1, the average inter-arrival time (I_t) is 18.34 minutes, therefore arrival rate $\lambda = \frac{60}{18.34} = 3.26$ customers per hour.

Also from the appendix table 1, the average service time (Y_t) is 5.29, therefore average service rate $\mu = \frac{60}{5.29} = 11.34$ customers per hour.

Hence, for the system we adopted a queuing model of type m/m/1.

Meaning: Exponential inter-arrival, Exponential service rate with one server (i.e. queue of type m/m/1).

Therefore the traffic intensity $\rho = \frac{\lambda}{\mu}$

Where λ = arrival rate

μ = service rate

$is\ \rho = \frac{3.26}{11.34} = 0.29$

That is the system is busy approximately 30% of every 1 hour

and idle 70% or 42 minutes in every 60 minutes.

(iv) Revenue Analysis

Notations:

Let S = rate of service per hour

h = total hours open for services in a month

E(N) = expect number of customers served in a months

C_t = Service charge

E(R) = the Expected Revenue.

The center renders service for 24 days in a month, the center renders service for 10 hours in a day.

Therefore E(N) = Service rate(s) X (24 days x 10 hours)

E(N) = 11.34 x 24 x 10 = 2722 customers per month

Given from appendix table I that average service charge C_t=N32.52 per customer

Then revenue model E(R) = E(N) * C_t

∴ Expected revenue in a month E(R) = 2722 x 32.52 = 88,519.44

While,

Expected revenue in a year E(R) = E(N) * C_t.(12 months)

E(R) = N88,519.44x 12 = N1,062,233.28 per year.

5.0 CONCLUSION/RECOMMENDATIONS

- The traffic intensity (utilization factor) of 0.30 reveals that the business centre were only busy 30% of every one hour or 18 minutes in every 60 minutes in a day and the servers are idle for 70% of every one hour.
- This could be due to the location of the business centre, and no other business activities to keep the servers busy.
- We discovered that each customer who calls to the centre pays an average of N32.52 and
- The estimated gross income revenue for the year ending was N1,062,233.28.
- We recommend that
- Any interested investor on this type of business should do feasibility study on location before setting-up the business.
- The proprietors of such business should co-opt or introduce computer services, spiral binding, ID-Card lamination services to mop-up the idleness of the servers and increase the revenue generation.
- Finally, we strongly recommend popular spots like schools' premises, government secretariats etc for setting-up such business.

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Appendix 1.

Table 1: SUMMARY STATISTICS OF DATA ON I_t, Y_t, W_t AND C_t FOR FOUR WEEKS

	VARIABLES	WEEK1	WEEK2	WEEK3	WEEK4	AVERAGE
Inter-arrival time (I_t)	S N	20.65 18.45 83	16.97 16.33 88	16.20 16.91 84	19.54 16.27 82	18.34 16.99 337
Service time (Y_t)	S N	13.87 13.64 83	13.20 11.48 88	13.56 14.34 84	12.96 11.88 82	13.40 12.84 337
Waiting time (W_t)	S N	8.96 8.69 83	7.65 8.48 88	8.13 7.79 84	6.43 7.74 82	7.79 8.18 337
Service charge (C_t)	N	N36.28 83	N31.63 88	N35.29 84	N26.87 82	N32.52 337

APPENDIX TABLE 2: COMBINED ANOVA TABLE FOR I_t, Y_t and W_t

Sources of variation	Sum of squares	Df	Ms	f-ratio	$F_{(0.05)}$	Decision
I_t	834.3	3	278.1	0.78	2.92	(ns)
Y_t	52.98	3	17.66	0.597	2.92	(ns)
W_t	76.1	3	25.37	1.81	2.92	(ns)
I_t	1283.2	36				
Y_t	849.79	36				
W_t	534.2	36				
I_t	13665.50	39				
Y_t	902.77	39				
W_t	610.4	39				

*(ns) = not significant at $\propto = 0.05$ for the three variables.

Appendix Table 3: χ^2 - Test for Exponential fit on I_t - Data

Y	F(y)	P_i	F	e_i	$(f_i - e_i)^2 / e_i$
9.5	0.4029	0.4029	136	135.78	0.0004
19.5	0.6531	0.2502	90	84.32	0.3826
29.5	0.7985	0.1454	60	49.00	2.4694
39.5	0.8829	0.0844	31	28.44	0.2304
49.5	0.9320	0.0491	10	16.55	2.5923
78.5	0.9867	0.0547	10	18.43	3.8560
				TOTAL	**9.5310**

$Y =$ class intervals for I_t - Data

$f_\circ(y) =$ cumulative distribution value $\left(1 - e^{-y}\right)$

$p_i =$ cumulative boundary difference of $f_\circ(y)$

$f_i =$ the frequency of I_t - Data

$e_i =$ Expected frequency $(np_i), (n = 337)$

Table 4: Test for Exponential fit on Yt – Data

Y	F_i	F(y)	P_i	e_i	$\frac{(f_i - e_i)^2}{e_i}$
4.5	180	0.5728	0.5728	193.03	0.8796
9.5	92	0.8340	0.2612	88.02	0.1800
14.5	40	0.9355	0.1015	34.21	0.9810
19.5	19	0.9749	0.0394	13.28	0.24637
29.5	6	0.9962	0.0213	7.18	0.1939
Total					4.6982