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Mathematical Formulation Model for a School Bus Routing Problem with Small Instance Data

Denis M. Manumbu [1] , Egbert Mujuni [2] , Dmitry Kuznetsov [3]

[1] School of Computational and Communication Science and Engineering, The Nelson Mandela African Institution of Science and Technology

(NM-AIST) P.O Box 447, TengeruArusha Tanzania

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Abstract

This paper aims to describe the mathematical formulation model and an exact optimal solution analyses for a school bus routing problem with small instance data. The formulated model has been used to compute the optimal solution of time spent by students at all bus stops, apart from that the bus stops are not necessary be linearly ordered. We also listed down five procedures of mathematical formulation model to reach an exact optimal solution for a school bus routing problem with small instance data. We assume that each bus has fixed pick up points, these generates the many possible routes for a bus, the number of routes that generated is equal to permutation of pick up points, for each route of a bus we computing the objective function and the route with smallest objective function value can be optimal route of a bus. The sample data from two schools located at Dar es Salaam are collected and validated in the model to shows the good performing of that model. The optimal solution results obtained shows that the students spent minimal minutes in new planned routes compared to current routes.

Keywords: bus stop, students, buses, optimal value, optimal solution, set, pick up.

1. Introduction

The School Bus Routing Problem (SBRP) seeks to plan an efficient schedule for a fleet of school buses that pick up students from various bus stops and deliver them to the school by satisfying various constraints such as the maximum capacity of a bus, the maximum transport cost, the maximum travelling time of students in buses, and the time window to reach at school. Also the school bus routing problem (SBRP) can be defined as problem of finding the optimal routes of drops off or pickup students from one or several pick up points with allocated number of students and transporting them from various pick up points to school at very minimal spent time by students within the bus in all pick up points.

The main goal of any school transportation system is providing safe, efficient and reliable transportation for its students. School transportation plays a long studying role in the lives of children from nursery, primary and secondary. Every year numerous schools which provided transportation to their students all over the country must evaluate the transportation needs of their students. Many times it happen that, a large number of students reside too far away from the school and cannot be expected to walk to school each day. Also it is illogical to expect the families to carry their children to and from school (Britt et al., 2005).

In some countries, students living within a certain distance to school are entitled by law free transportation to and from school. A bus stop should be located at a maximum distance from home of each student (eg 750m). Hence a set of potential bus stops is predefined in advance. From a hierarchical point of view, one has to first select the bus stops (and assign the students to the bus stops) and then defined the routes for the buses (Schittekat et al., 2012).

One solution to SBRP is busing; there are many schools which provide transportation routes. Sometimes these routes are not optimal; these types of routes waste time or money for those involved, necessitating an accurate school transportation model to increase efficiency. But solutions like this must also considered some factors like economic concerns, time issues, route efficacy, etc (Britt et al., 2005).

While designing the school bus routes, safety of the students should come first, students who are eligible for transportation shall board buses at selected pick up points, safety of students, road conditions, economy of operation, student convenience etc, should be considered before designing and planning the bus routes (EEAB, 1998).

In school bus transportation the two most visible problems are routing and scheduling. In the routing problem every students who can picked by bus is assigned to a bus stop and those particular stops are sum up to form routes. In the morning a bus follows these routes, from one stop to another, picking up the students and carrying them to school. In the scheduling problem, particular buses are assigned to particular route. For example, in morning bus A might begin at route 1, deliver the students from that route to their school, the bus B travel to the beginning of route 2, pick up the students along that route and take them to their school, this continue up to last bus to assigned a last route.

In this study we take a case study of Dar es salaam, Tanzania. This it is largest city in Tanzania with high populated than all cities in that country. One problem found in Dar es salaam is school busing and scheduling, that problem facing the schools which providing transport to their students from bus stops to reach school or from school and drops near to home. The school management lacks scientific method on how to route and schedule these school buses which lead students much more travelling time than expected to reach at school and home.

Ngonyani, (2013) formulated a mathematical model for SBRP in Dar es Salaam. The model used an assumption that bus-stops are linearly ordered. Thus, this work aimed at reformulating a mathematical model for SBRP without making this assumption. The objective model is to minimize the total time spent by students in travel at all points. We demonstrate by computing the 5 procedures flows to reach an exact optimal solution for the simple problems with small instance data of school bus routing problem. We use data from two schools in Dar es Salaam. The first data set consists of 5 buses which serves 64 students in 5 bus stops; the second consists of 3 bus, 11 stops and 60 students. Each school bus already has a route with specific pickup points with students allocated, we schedule and permutated this route to generate routes, for each route we compute its objective function value, smallest value is optimal of a bus. Lastly we compare the optimal value and current route value for the aims to get saving time in optimal route.

2. Literature review

SBRP is a special case of Vehicle Routing Problem, and it has received considerable attention among researchers since it was introduced by Newton and Thomas (1969). Below we present a brief survey on previous researches.

Bowerman, et al., (1995) proposed a new heuristic for urban school Bus Routing. The problem was formulated as a multi-objective model and a heuristic based on this formulation is developed. The study involves two interrelated problems. One has to do with the assignment of students to their respective bus stops and the second has to do with routing of buses to the bus stops. A problem of these characteristics is a location-routing problem. The nature of the formulation made it possible to organize their study into three layers, where layer one is the school, layer two is the bus stops and layer three the students. School buses routes cause interaction between layers one and two, while movements of student cause interaction between layers two and three. The heuristic approach to this problem involves two algorithms which catered for the multi-objective nature of the model. The first is a districting algorithm which groups students into clusters to be serviced by a unique school bus route. The second is a routing algorithm, which generates a specific school bus route that visits a sub set of potential bus stops sites.

Corberán et al., (2002) addressed the problem of routing school buses in rural areas. They approached this problem with a node routing model with multiple objectives that arise from conflicting viewpoints. From the point of view of cost, the number of buses used to transport students from their homes to school and back is minimized. From the service viewpoint, they minimized the time that a given student spend on route. The multi-objective employs a weighted function to combine individual objective functions into a single one. They developed a solution procedure that considered each objective separately and searched for a set of efficient solution instead of a single optimum. Their solution procedure is based on construction, improving and then combining solutions within the frame work of the evolutionary approach known as scatter search.

Schittekat, et al., (2006) formulated the school bus routing problem using a single objective integer programming model VRP by introducing several other interesting additional features. They considered a set of potential stops as well as a set of students who can walk to one or more of these potential stops. The goal of their routing problem is to select a subset of stops that will actually be visited by the buses; determine which stops each student should walk to; and develop a set of tours that minimize the total distance travelled by all buses. The problem was solved using a commercial integer programming solver and results on small instances were discussed.

Swersey and Ballard (1984) presented a work on scheduling of school buses. With the scheduling situation considered here, a set of routes each associated with a particular school is given. A single bus is assigned to each route to pickup students and arriving at their school within a specific time window. The problem includes finding the fewest buses needed to cover all routes whiles meeting the time window specifications. They presented two integer programming formulations of the scheduling problem and applied them to actual data from New Heaven, Connecticut for two different years as well as to 30 randomly generated problems. Linear programming relaxation of the integer programs was found to produce integer solutions more than 75% of the time. In the remaining cases, they observed the few functional values can be adjusted to integer values without increasing the number of buses needed. Their method reduces the number of buses needed by about 25% compared to the manual solutions developed by the New Heaven school bus scheduler.

Li and Fu (2002) presented a case study of the bus routing problem. It is formulated as a multi- objective combinatorial optimization problem. The objectives include minimizing the total number of buses required, the total travel time spent by all pupils at all pick-up points and the total bus travel time. They also aimed at balancing the loads and travel times between the buses. They proposed a heuristic algorithm, which was programmed and run efficiently on a PC. Numerical results were reported using test data from a kindergarten in Hong Kong. This proved to be effective as it save 29% of total travelling times when compared the system under practice

Spasovic et al, (2001) their work is a methodology for evaluating of school bus routing a case study of Riverdale, New Jersey. The techniques are evaluated using the case study of Riverdale, New Jersey, the case study involves a municipality with one elementary school and requires all of the buses to depart from and return to the school, the routes and operating costs vary for each of the methodologies used. They formulated mathematical model as integer programming, objective of the model is a cost minimization.

Anderson et al, (2005) their work is route optimization applied to school transports- a method combining column generation with greedy heuristics, the objective is to minimization of costs is viewed upon in an area with many schools and the vehicle routing problem with time windows. It is formulated mathematical model as integer programming with constraints as regarding the vehicle/ buses (the capacity of the fleet and individual load capacities of vehicles) and the bus stops (which have to be visited in certain time interval, the time windows). Proposed the greedy heuristic algorithm methods to solve the problem are implemented and evaluated, it concludes on the pros and cons of using the developed techniques for a practical planning case.

More researchers in the world have written papers of school bus routing problem, they formulate mathematical formulation model subject to their constraints, mostly of them take that bus stops are linearly ordered and the objectives are to minimize distance travel by buses, minimize cost consumed by school bus, minimize time window but few deals with simple problems and to compute it by using manual method to reach routes with exact optimal solution, and mostly of problems are NP hard with large instance data and it be impossible to obtain optimal solution, these include formal heuristic (or approximate) method to quickly find good transit routes (Chien and Yang,2000; Fan and Machemehl,2004; Mauttone and Urquhart,2009; Van Nes et al.,1988; Zhao and Gan,2003). The last of these is most widely used and perhaps the most important technique for the majority of transit agencies and sometimes referred to as manual route planning process in order to generate good route alternatives (Dubois et al.,1979; Moorthy,1997; Newell,1979).

Ngonyani, 2013 formulated mathematical model for a single school and the objective wasto minimize the total travel time spent by pupils at all point a case study of Dar es salaam such that the bus stops are linearly ordered subject to bus capacity and all buses visit the school constraints and they proposed tabu search algorithm to reach a solution. Patrick et al., 2006formulatedthemathematical model for a single school and their objective is to develop an integer programming formulation for the problem that minimize the total distance travelled by all buses subject to constraints of bus capacity, buses visit the school and bus stop is visited once by bus. Park and Kim, 2010mentioned five different sub-problems which are often treated separately in the SBRP literature: data preparation, bus stop selection, bus route generation, school bell time adjustment and bus scheduling. Taehyeong and Bum, 2013 they formulating mode as a mixed integer programming problem and use harmony heurist algorithm and exact method to reached optimal solution. Liong et al., 2008 studied the model objective and find a set of delivery routes satisfying some requirements or constraints and giving minimal total cost.Suman and Kuma, 2006 proposed heurist simulated annealing algorithm for solutions of single and multi objective optimization problems. Spasovic et al., 2001investigated the mathematical model based on the school routing and compared the results for three techniques used in school bus routing; a time savings heuristic, the ROUTER computer program and the sweep method. Schittekat et al., 2013 shown that the met heuristic exhibits excellent performance and finds optimal or close to optimal solutions of large instances of the SBRP in very limited computing times.

3. Model formulation for school bus routing problem

3.1 Model Assumptions

In order to formulate the model for optimizing the school scheduling of the school bus routing problem (SBRP) for Dar es Salaam city, the following assumptions are applied;

1. The number of buses start at school is equal to returning at school.
2. Each bus has only one route for transporting students to school in the morning and back to their home after classes.
3. The pick- up points visited and picked students by bus are scattered and not necessary to be linearly ordered.
4. Each school bus picks up students at least on one pick- up point.
5.
6. If the bus visiting a point it must picks up all students at that point.
7. The time spend by students within the bus from one pick up point to another includes jams, road condition, accident action and waiting time in traffic light.
8. Each pick up point is allocated to only one bus.

3.2 The objective of the model

The objective of the model is therefore to plan routes that will minimize the total travel time spent by students in all bus stops by using the non- linear mixed integer programming model

Sets

The following are the sets that are used in the model formulation.

1. $S = \{1, 2, \dots, N\}$ a set of all bus stops where one or more students are picked up whereby N is the total number of stops arranged scattered around the school and $N + 1$ denotes the school.
2. $K = \{1, 2, 3, \dots, B\}$ a set of the available buses to be used where B is the total number of available buses.

Parameters

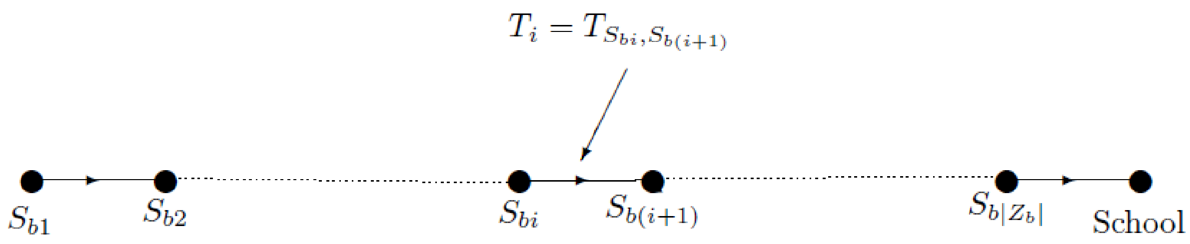
Proposed model uses the following parameters;

1. B represents the number of available buses for the school bus service.
2. T_{ij} represents the travel time from $i \in S$ to $j \in S$.
3. N is the total number of bus stops available
4. C_b denotes the capacity of bus $b \in K$
5. α is the average pick – up time of one student by bus at a pick-up point.
6. Y_i is the number of students at stop $i \in S$
7. Z_b is the set of pick-up point to be visited by bus $b \in K$
8. S_{bi} is represents the index number of pick-up point be visited by bus $b \in K$

Objective function

The objective function f of the model is to calculate total travel time spent by students at all points. The aim of the model is to minimize f. The objective function f is equal to Total travelling time (TTT) spent by students within the bus in all points plus Total pick up time (TPT) of students by bus at all points.

The Total Travelling time (TTT) is derived as follows: Suppose that a bus b picks students at stops S_{b1}, S_{b2}, ... $S_{b|Z_b|}$ in this order as show in the figure below. Let $T_i$be the travelling time from $S_{bi}$to $S_{b(i+1)}$.
The Total Travelling time (TTT) is derived as follows: Suppose that a bus b picks students at stops S_{b1}, S_{b2}, ... $S_{b|Z_b|}$ in this order as show in the figure below. Let $T_i$be the travelling time from $S_{bi}$to $S_{b(i+1)}$.



All students in S_{b1}, S_{b2}, ... S_{bi} must use the link $(S_{bi}, S_{b(i+1)})$. Since there are $Y_{S_{bl}}$ students at the stop S_{bl}. The total travelling by all students in this link is

$$T_i \sum_l^i Y_{S_{bl}} = T_{S_{bi},S_{b(i+1)}} \sum_l^i Y_{S_{bl}}, \text{ (since } T_i = T_{S_{bi},S_{b(i+1)}})$$

Therefore the sum of Total travelling time (TTT) of students at all points by all buses is

$$\sum_{b=1}^{B} \sum_{i=1}^{|Z_b|} T_{S_{bi},S_{b(i+1)}} (\sum_{l=1}^{i} Y_{S_{bl}})$$

Total pick up time (TPT) is derived as follows;

Given that; α is the average pick up time of one student by bus at a point

$Y_{S_{bi}}$ Is the number of students picked up by bus b at point $S_{bi} \epsilon S$

The sum of spent pick up time by students at all points on the route of bus b is

$$\alpha Y_{S_{b1}} + \alpha Y_{S_{b2}}(1 + Y_{S_{b1}}) + \alpha Y_{S_{b3}}(1 + Y_{S_{b1}} + Y_{S_{b2}}) + \alpha Y_{S_{b4}}(1 + Y_{S_{b1}} + Y_{S_{b2}} + Y_{S_{b3}}) + \alpha Y_{S_{b5}}(1 + Y_{S_{b1}} + Y_{S_{b2}} + Y_{S_{b3}} + Y_{S_{b4}}) + \cdots + \alpha Y_{S_{bi}}(1 + Y_{S_{b1}} + Y_{S_{b2}} + Y_{S_{b3}} + Y_{S_{b4}} + Y_{S_{b5}} + \cdots + Y_{S_{b(i-1)}} =$$

$$\sum_{i=1}^{|Z_b|} \alpha Y_{S_{bi}} (1 + \sum_{l=1}^{i-1} Y_{S_{bl}})$$

Therefore the sum of spent Total pick up time (TPT) by students at all points on the routes of buses is

$$\sum_{b=1}^{B} \sum_{i=1}^{|Z_b|} \alpha Y_{S_{bi}} (1 + \sum_{l=1}^{i-1} Y_{S_{bl}})$$

The objective function $f = TTT + TPT$

$$f = \sum_{b=1}^{B} \{ \sum_{i=1}^{|Z_b|} (T_{S_{bi},S_{b(i+1)}} (\sum_{l=1}^{i} Y_{S_{bl}}) + \alpha Y_{S_{bi}} (1 + \sum_{l=1}^{i-1} Y_{S_{bl}}))\} .$$

Therefore, the mathematical model becomes:

$$minimize\ f = \sum_{b=1}^{B}\{\sum_{i=1}^{|Z_b|}(T_{S_{bi},S_{b(i+1)}}(\sum_{l=1}^{i} Y_{S_{bl}}) + \alpha Y_{S_{bi}}(1+\sum_{l=1}^{i-1} Y_{S_{bl}}))\}$$

Subject to;

1. $\sum_{i=1}^{|Z_b|} Y_{S_{bi}} \leq C_b ,\quad b = 1,2,\ldots,B$
2. $S_{b(|Z_b|+1)} = N+1,\ \ b = 1,2,\ldots,B$
3. $\sum_{i=1}^{N} Y_i = \sum_{b=1}^{B}\sum_{i=1}^{|Z_b|} Y_{S_{bi}}$
4. $Y_i \geq 0\ and\ an\ integer$

Constraints, (1) ensures that the sum of students picked up in all points by bus must not exceed the bus capacity; (2) ensures that all buses finished their routes at a school; (3) ensures that all students are picked up; (4) the number of students at each bus stop is nonnegative.

4. The procedures of mathematical model to reach exact optimal solution of sbrp with small instance data.

The small instance data's are collected from two primary schools located in Dar Es Salaam city, each school bus picks up students at most from four bus stops and ends to school. Here we have shown the procedures of derived mathematical formulation model to reach an exact optimal solution of objective function subject to their constraints of school bus routing problem with small instance data.

In one school they have five buses and other school they have three buses served students at the stops from home to their school, each bus have one route and a stop is allocated to one bus. The capacity of each bus from school one is to serve 15 students and capacity of each bus from school two is to serve 23 students in their route, also the time to pick up one student at a stop is 0.5 minute.

4.1 Stops allocated to the buses

In school one they are 15 stops allocated none linearly ordered to the five buses as follows;

Bus 1 served the students at stops 1, 11, 14, 9 and ends to school, bus 2 served the students at stops 7, 2, 15 and ends to school, bus 3 served the students at stops 3, 13, 5 and ends to school, bus 4 served the students at stops 8, 4, 12 and ends to school, and bus 5 served the students at stops 10, 6 and ends to school.

In school two known as Green acres primary school has 11 stops allocated also none linearly ordered to the three bus as follows; Bus 1 served the students at stops 1, 3, 6, 11 and ends to school, bus 2 served the students at stops 4, 5, 8, 10 and ends to school and bus 3 served the students at stops 2, 7, 9, and ends to school.

4.2 Number of students served by bus at a stop.

The first procedure is to producing a table that can shows the number of students served by bus at a stop.

Table 1. Bus stops allocated to the buses for serves students.

Bus stop	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Number of student	4	4	2	6	2	4	5	2	2	5	4	7	6	5	6
Allocated Bus	1	2	3	4	3	5	2	4	1	5	1	4	3	1	2

Table 2. Data of Green acres primary school-Mabibo

Stop	1	2	3	4	5	6	7	8	9	10	11
Students	5	7	5	4	6	7	3	7	5	5	6
Allocated Bus	1	3	1	2	2	1	3	2	3	2	1

4.3 Travelling time matrix developed

The second procedure is to developing travelling time matrix. This matrix has shows the time in minutes consumed by students from one stop to another stop. The developed travelling time matrix as follows;

Table 3: Travelling time matrix between the stops for School 1

0	4	8	9	7	4	10	5	6	7	8	10	12	9	8	16
4	0	7	9	6	12	7	8	9	10	12	11	10	8	7	12
8	7	0	7	8	9	8	10	12	14	12	10	8	7	9	10
9	6	7	0	4	8	9	10	12	13	12	8	9	10	12	13
7	9	8	4	0	7	9	10	8	9	8	10	12	9	10	12
4	12	9	8	7	0	7	8	9	10	14	15	16	12	12	18
10	7	8	9	6	7	0	7	8	9	10	12	14	13	8	12
5	8	10	10	10	8	7	0	7	9	10	12	13	14	15	12
6	9	12	12	8	9	8	7	0	8	9	10	11	12	13	12
7	10	14	13	9	10	9	9	8	0	7	8	9	10	14	14
8	12	12	12	8	14	10	10	9	7	0	8	7	9	12	16
10	11	10	8	10	15	12	12	10	8	8	0	8	10	10	12
12	10	8	9	12	16	14	13	11	9	7	8	0	10	12	14
9	8	7	10	9	12	13	14	12	10	9	10	10	0	14	12
8	7	9	12	10	12	8	15	13	14	12	10	12	14	0	10
16	12	10	13	12	8	12	12	12	14	16	12	14	12	10	0

Table 4: Travelling time matrix between the stops for School 2

0	6	12	8	7	6	10	9	8	12	5	11
6	0	8	8	6	5	15	8	9	12	4	10
12	8	0	12	13	14	7	12	16	8	9	14
8	8	12	0	16	14	12	10	11	14	12	15
7	6	13	16	0	16	15	10	8	9	14	12
6	5	14	14	16	0	14	9	10	11	9	7
10	15	7	12	15	14	0	12	12	9	10	12
9	8	12	10	10	9	12	0	8	9	12	16
8	9	16	11	8	10	12	8	0	14	13	28
12	12	8	14	9	11	9	9	14	0	12	14
5	4	9	12	14	9	10	12	13	12	0	22
11	10	14	15	12	7	12	16	28	14	22	0

4.4 Neighborhood structure sets of routes

The third procedure is to producing neighborhood structure sets. The number of neighborhood structure sets is equal to the permutation number of the bus stops, this contain the state of existing route and possible routes scheduled in the same stops of a bus route. The states are produced by taking stop, flows another stops and ends to a school, but during that scheduling the stop must not repeated in the route. The process is produce the first set of a route, flows the second set of a route, continue to produce another sets and ends to the last set of a route. For a school 1; the neighborhood structure sets of a bus 1 are follows;

{(1,11,14,9,16),(1,11,9,14,16),(1,14,11,9,16),(1,14,9,11,16),(1,9,11,14,16),(1,9,14,11,16),(11,1,14,9,16),(11,1,9,14,16),(11,14,1,9,16),(11,14,9,1,16),(11,9,1,14,16),(11,9,14,1,16),(14,1,11,9,16),(14,1,9,11,16),(14,11,1,9,16),(14,11,9,1,16),(14,9,1,11,16),(14,9,11,1,16),(9,1,11,14,16),(9,1,14,11,16),(9,11,1,14,16),(9,11,14,1,16),(9,11,14,1,16),(9,14,1,11,16),(9,14,11,1,16)}=4! =24.

For bus 2 are; {(7,2,15,16),(7,15,2,16),(2,7,15,16),(2,15,7,16),(15,7,2,16),(15,2,7,16)}=3! =6.

For bus 3 are; {(3,13,5,16),(3,5,13,16),(13,3,5,16),(13,5,3,16),(5,3,13,16),(5,13,3,16)}=3! =6.

For bus 4 are; {(8,4,12,16),(8,12,4,16),(4,8,12,16),(4,12,8,16),(12,8,4,16),(12,4,8,16)}=3! =6.

For bus 5 are; {(10, 6, and 16), (6, 10, and 16)} =2! = 2.For a school 2 known as Green acres primary school. Neighborhood structure sets of a bus 1 are follows:

{(1,3,6,11,12),(1,3,11,6,12),(1,6,3,11,12),(1,6,11,3,12),(1,11,3,6,12),(1,11,6,3,12),(1,11,6,3,12),(3,1,11,6,12),(3,6,1,11,12),(3,6,11,1,12),(3,11,1,6,12),(3,11,6,1,12),(6,1,3,11,12),(6,1,11,3,12),(6,3,1,11,12),(6,3,11,1,12),(6,11,1,3,12),(6,11,3,1,12),(11,1,3,6,12),(11,1,6,3,12),(11,3,1,6,12),(11,3,6,1,12),(11,6,1,3,12),(11,6,3,1,12)} = 4! =24.For bus 2 neighborhood structure sets are;

{(4,5,8,10,12),(4,5,10,8,12),(4,8,5,10,12),(4,8,10,5,12),(4,10,5,8,12),(4,10,8,5,12),(5,4,8,10,12),(5,4,10,8,12),(5,8,4,10,12),(5,8,10,4,12),(5,10,4,8,12),(5,10,8,4,12),(8,4,5,10,12),(8,4,10,5,12),(8,5,4,10,12),(8,5,10,4,12),(8,10,4,5,12),(8,10,5,4,12),(10,4,5,8,12),(10,4,8,5,12),(10,5,4,8,12),(10,5,8,4,12),(10,8,4,5,12),(10,8,5,4,12)} = 4! =24

For bus 3 are; {(2,7,9,12),(2,9,7,12),(7,2,9,12),(7,9,2,12),(9,2,7,12),(9,7,2,12)} = 3! =6

4.5. Value of the objective function subject to their constraints for each set of a bus.

The fourth procedure is to computing values of objective function subject to their constraints for each set of a bus. Therefore here we produced the sets of value of objective function for each bus, from that set; the smallest value is optimal value of a bus.

Example1

For school 1; the objective function value of current route for a bus 1 is computing as follows,

Route; 1, 11, 14, 9 and ends to school (16)

$$f = \sum_{b=1}^{B}\{\sum_{i=1}^{|Z_b|}(T_{S_{bi,}S_{b(i+1)}}(\sum_{l=1}^{i} Y_{S_{bl}}) + \alpha Y_{S_{bi}}(1+\sum_{l=1}^{i-1} Y_{S_{bl}}))\}$$

$$= \sum_{i=1}^{4} T_{S_{1i,}S_{b(i+1)}}(\sum_{l=1}^{i} Y_{S_{bl}}) + \alpha Y_{S_{bi}}(1+\sum_{l=1}^{i-1} Y_{S_{bl}})$$

$$i = 1,\quad T_{S_{11,}S_{12}}(Y_{S_{11}}) + \alpha Y_{S_{11}} = 8(4) + 0.5(4) = 34$$

$$i = 2,\quad T_{S_{12,}S_{13}}(Y_{S_{11}} + Y_{S_{12}}) + \alpha Y_{S_{12}}(1 + Y_{S_{11}}) = 9(4+4) + 0.5(4)(1+4) = 82$$

$$i = 3,\quad T_{S_{13,}S_{14}}(Y_{S_{11}} + Y_{S_{12}} + Y_{S_{13}}) + \alpha Y_{S_{13}}(1 + Y_{S_{11}} + Y_{S_{12}}) = 12(4+4+5) + 0.5(5)(1+4+4) \quad = 178.5$$

$$i = 4,\quad T_{S_{14,}S_{15}}(Y_{S_{11}} + Y_{S_{12}} + Y_{S_{13}} + Y_{S_{14}}) + \alpha Y_{S_{14}}(1 + Y_{S_{11}} + Y_{S_{12}} + Y_{S_{13}})$$
$$= 12(4+4+5+2) + 0.5(2)(1+4+4+5) = 194$$

$$= 34 + 82 + 178.5 + 194 = 488.5\ minutes$$

Example 2

Route; 9, 1, 11, 14 and ends to school (16)

$$f = \sum_{i=1}^{4} T_{S_{1i,}S_{1(i+1)}}(\sum_{l=1}^{i} Y_{S_{1l}}) + \alpha Y_{S_{1i}}(1+\sum_{l=1}^{i-1} Y_{S_{1l}})$$

$$= T_{S_{11,}S_{12}}(Y_{S_{11}}) + \alpha Y_{S_{11}} + T_{S_{12,}S_{13}}(Y_{S_{11}} + Y_{S_{12}}) + \alpha Y_{S_{12}}(1 + Y_{S_{11}}) + T_{S_{13,}S_{14}}(Y_{S_{11}} + Y_{S_{12}} + Y_{S_{13}})$$
$$+ \alpha Y_{S_{13}}(1 + Y_{S_{11}} + Y_{S_{12}}) + T_{S_{14,}S_{15}}(Y_{S_{11}} + Y_{S_{12}} + Y_{S_{13}} + Y_{S_{14}}) + \alpha Y_{S_{14}}(1 + Y_{S_{11}} + Y_{S_{12}} + Y_{S_{13}})$$

$$= 6(2) + 0.5(2) + 8(2+4) + 0.5(4)(1+2) + 9(2+4+4) + 0.5(4)(1+2+4) + 12(2+4+4+5)$$
$$+ 0.5(5)(1+2+4+4) = 378.5\ minutes$$

For school 1; Therefore the set of values for a bus 1 is {488.5, 452.5, 462.5, 531.5, 396.5, 483.5, 488.5, 428.5, 423.5, 498.5, 390.5, 495.5, 462.5, 477.5, 423.5, 480.5, 478.5, 499.5, 378.5, 453.5, 384.5, 459.5, 463.5, 463.5}. Optimal value is 378.5 minutes and the planned route that minimize the time spent by students within a bus 1 at all points is starts stop 9, 1, 11, 14, and ends to school. So after school bus 1 scheduling routes the new route saving 110 minutes (22.5%) compare with current route. The set of values for a bus 2 is {292.5, 341.5, 294.5, 332.5, 349.5, 336.5}.The optimal value is 292.5 and that is the value of existing route, so no time to saving because the optimal solution is current route which is starts at a stop 7, 2, 15 and ends to school. The set of values for bus 3 is {251, 223, 251, 255, 207, 207}. Optimal value is 207 minutes and the planned route that minimize the time spent by students within a bus 3 at all points is starts stop 5, 3, 13 and ends to school or starts at a stop 5, 13, 3 and ends to school. So after school bus 3 scheduling routes the new route saving 44 minutes (17.5%) to compare with current route. The set of values for a bus 4 is {302.5, 332.5, 377.5, 425.5, 410.5, and

407.5}.Optimal value is 302.5 and that is the value of existing route, so no time to saving because the optimal solution is current route which is starts at a stop 8, 4, 12 and ends to school. The set of values for a bus 5 is {226.5, 180.5}.Optimal value is 180.5 minutes and the planned routes that minimize the time spent by students within a bus 5 at all points is starts stop 10, 6, and ends to school. So after school bus 5 scheduling routes the new route saving 46 minutes (20.3%) to compare with current route.

For school 2; the set of values(in minutes) of bus 1 is

{969,547,967,732,619,808,889,525,843,631,467,615,955,696,943,659,776,7569,657,780,553,679,780,815}.

Optimal value is 467 minutes and the planned new route that minimize the time spent by students within a bus 1 at all points is starts stop 3, 11,1, 6 and ends to school. So after school bus 1 scheduling routes the new route saving 502 minutes (51.8%).

For bus 2 set is

{725.5,741.5,711.5,647.5,739.5,661.5,757.5,823.5,836.5,859.5,810.5,763.5,807.5,732.5,922.5,869.5,851.5,889.5,816.5,684.5,823.5,765.5,785.5,883.5}.Optimal value is 647.5 minutes and the planned new route that minimize the time spent by students within a bus 2 at all points is starts stop 4, 8,10, 5 and ends to school. So after school bus 2 scheduling routes the new route saving 78 minutes (10.75%) to compare with current route. The set of values for a bus 3 is {688, 430, 598, 301, 448, 373,}.Optimal value is 301 minutes and the planned new route that minimizes the time spent by students within a bus 3 at all points is starts stop 7, 9, 2 and ends to school. So after school bus 3 scheduling routes the new route saving 387 (56.25%).

4.6. Optimal solution of objective function for all buses

The fifth procedure is to adding all optimal value of each bus for obtains optimal solution of objective function for a single school.

4.7. Result analyses.

The result analyses shows how schools benefit from this work. In school one time saved by planned new route compare with current route is analyzed as follows:

Table 5. Data analyses of current route and new route for a school 1

Bus	Stop	Current route (Cr)	Time spent of Cr	New route (Nr)	Time of Nr	Saved	Saved (%)
1	4	1, 11, 14, 9, 16	488.5	9, 1, 11, 14, 16	378.5	110	22.5%
2	3	7, 2, 15, 16	292.5	7, 2, 15, 16	292.5	0	0%
3	3	3, 13, 5, 16	251	5, 3, 13, 16	207	44	17.5%
4	3	8, 4, 12, 16	302.5	8, 4, 12, 16	302.5	0	0%
5	2	10, 6, 16	226.5	6, 10, 16	180.5	46	20.3%
Total	15		f = 1561		f = 1361	200	12.8%

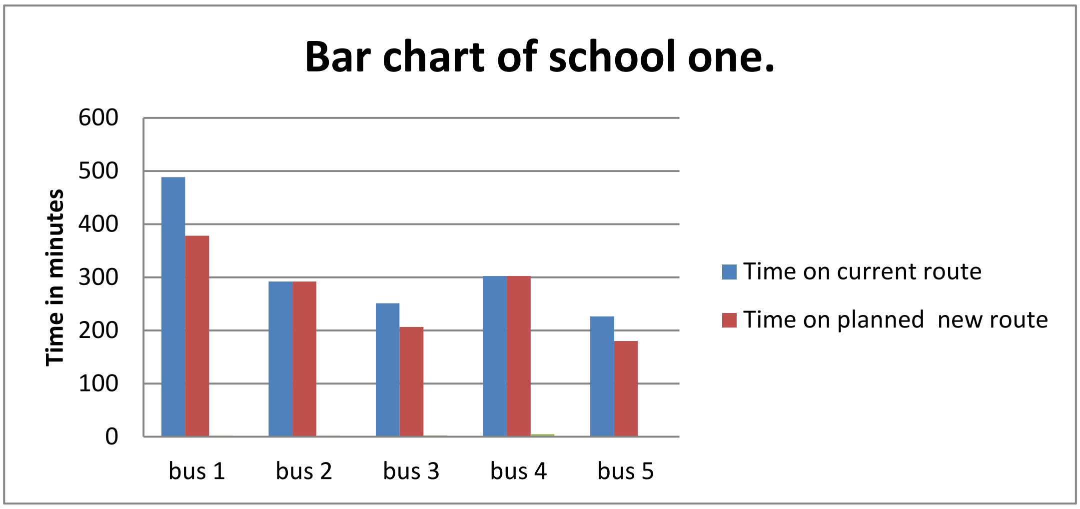


Figure 1. Comparisons between current and planned new routes for a school 1

In school two time saved by planned new route compare with current route is analyzed as follows;

Table 6. Data analyses of current route and new route for a school

Bus	Stop	Current route(Cr)	Time spent on Cr(in minutes)	New route(Nr)	Time on Nr (in minutes)	Saved	Saved (%)
1	4	1,3,6,11,12	969	3,11,1,6,12	467	502	51.8%
2	4	4,5,8,10,12	725.5	4,8,10,5,12	647.5	78	10.7%
3	3	2,7,9,12	688	7,9,2,12	301	347	56.25%
Total	11		$f = 2382.5$		$f = 1415.5$	967	40.59%

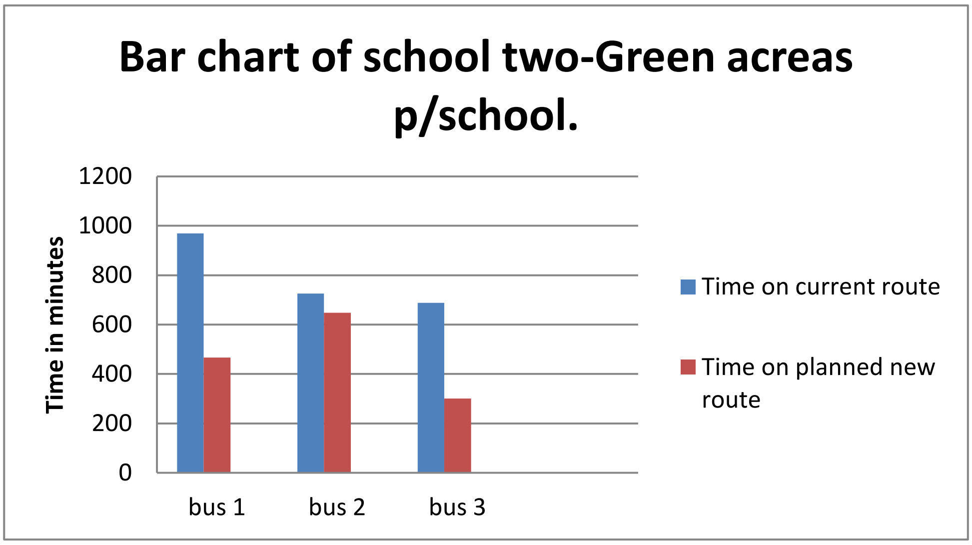


Figure 2. Comparisons between current and planned new routes for a school 2

5. Conclusions and future work

In this paper we have proposed mathematical formulation model for solving the student bus routing problem, the objective of the model is to minimize the time spent by students at all points to school every school day we assumed that the bus stops are not necessary be linearly ordered, already the students allocated to the pick-up points, also their existing bus route that bus is allocated to picks students in pick-up points and the bus has fixed pick- up points. We are required to assign a bus into pickup points and to schedule the bus routing for each school bus route assumed that a bus has the fixed pickup points, for each generated route we computing objective function value and the route with smaller value is proposed to school management to use it. The time travel by students within the bus from one stop to another stop collected by school bus conductor on existing route and proposed new route, they approximate time so the time not exactly collect handled percent. We follow five procedures to computing objective function value for solving a school bus routing problem. In this study, we used five procedures to computing optimal value of each bus for a single school. The real data from two schools located at Dar es salaam is validated in models. The results show that schools benefits with this study because it saves 200 minutes and 967 minutes from proposed new routes of the school buses compare to current routes, in future the researchers should extend this work by adding more constraints, proposed heuristic to solve the proposed model, use this study as a literature review and improves the way to collect time between pick-up points by use very efficient collected data method.

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