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Some properties of g*i- Closed Sets in Topological Space

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Abstract

In this paper we introduce a new class of sets called generalized *i-closed sets in topological spaces (briefly g*i-closed set). Also we study some of its basic properties and investigate the relations between the associated topology.

Keywords: g-closed, i-open, gi-closed.

1-Introduction

In 1970 Levine [6], first considered the concept of generalized closed (briefly, g-closed) sets were defined and investigated. Arya and Nour [1], defined generalized semi open sets [briefly, gs-open] using semi open sets. Maki Devi and Balachandram [2, 3]. On generalized α-closed maps and semi-generalized homeorphisems. Dontchev and Maki, in 1999 [4, 5], introduced the concept of (δ-generalized, θ-generalized) respectively. Mohammed and Askander [7], in 2011, introduced the concept of i-open sets. Mohammed and Jardo [8], in 2012, introduced the concept of generalized i-closed sets.

We introduced a new class of sets called g*i-closed sets and study some properties.

2- Preliminaries

Throughout this paper (X, τ) or simply X represent nonempty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of (X, τ), $cl(A)$ and $int(A)$, represent the closure of A and the interior of A respectively. A subset A of a space (X, τ) is called semi-open [2] (resp; α-open[11], b- open[9]), if $A \subseteq cl(int(A))$; (resp, $A \subseteq int(cl(int(A)))$, $A \subseteq cl(int(A)) \cup int(cl(A))$). The family of all semi-open (resp; α- open, b-open) sets of (X, τ) denoted by $SO(X)$ (resp; $\alpha O(X)$, $BO(X)$). The complement of a semi-open (resp; α-open, b-open) set is said to be semi-closed (resp; α-closed, b-closed). The semi closure, α-closure, b-closure of A are similarly defined and are denoted by $Cls(A)$, $Cl\alpha(A)$, $Clb(A)$. And a subset A of (X, τ) is called(δ-open, θ-open) set [10], if $A=cl\delta(A)$ where $cl\delta(A) = \{x \in X : int(cl(U)) \cap A \neq \emptyset, U \in \tau$ and $x \in U\}$, $A=cl\theta(A)$ where $cl\theta(A) = \{x \in X :$

$int(cl(U)) \cap A \neq \emptyset,\ U \in \tau$ *and* $x \in U\}$ respectively, and the family of all (δ-open, θ-open) sets of *(X, τ)* denoted by *δO(X), θO(X)* respectively, and the complement of *(δ*-open, *θ*-open) sets is called (*δ*-closed, *θ*-closed) sets the family of all (δ-closed, θ-closed) sets of *(X, τ)* is denoted by *δC(X), θC(X)* respectively. And a subset *A* of *(X, τ)* is called an i-open set [7], if A $\subset cl(A \cap G)$, if there exists an open set *G* whenever $(G \neq X, \emptyset)$, and the complement of i-open sets is called i-closed sets. The family of all i-closed sets of *(X, τ)* is denoted by *IC(X)*, and the family of all i-open sets of *(X, τ)* is denoted by *IO(X).* If *A* is a subset of a space *(X, τ)*, then the i-closure of *A*, denoted by *cli*(*A*) is the smallest an i-closed set containing *A*. The i-interior of *A* denoted by *inti*(*A*) is the larges an i- open set contained in *A.*

Some definitions used throughout this paper.

Definition 2.1:

For any subset *A* of topological spaces *(X, τ)* we have

1- Generalized closed (briefly g-closed) [6], if $cl(A) \subset U$ whenever $A \subset U$ and *U* is open in *X*, the complement of g-closed is called g-open.
2- Generalized α –closed (briefly gα-closed) [2], if $cl_\alpha(A) \subset U$ whenever $A \subset U$ and *U* is α-open in(*X, τ*), the complement of gα -closed is called gα-open.
3- α -Generalized closed (briefly αg-closed) [2], if $cl_\alpha(A) \subset U$ whenever $A \subset U$ and *U* is open in*(X, τ)*, the complement of αg -closed is called αg-open.
4- Generalized b-closed (briefly gb-closed) [9], if $cl_b(A) \subset U$ whenever $A \subset U$ and *U* is open in *(X, τ)*, the complement of gb -closed is called gb-open.
5- Generalized i-closed (briefly gi-closed) [8], if $cli(A) \subset U$ whenever $A \subset U$ and *U* is open in *(X, τ)*, the complement of gi -closed is called gi-open.
6- Generalized semi -closed (briefly gs-closed) [1], if $cls(A) \subset U$ whenever $A \subset U$ and *U* is open in *(X, τ)*, the complement of gs -closed is called gs-open.
7- Generalized θ-closed (briefly gθ-closed) [5], if $cl_\theta(A) \subset U$ whenever $A \subset U$ and *U* is open in*(X, τ)*, the complement of gθ -closed is called gθ-open.
8- Generalized δ-closed (briefly gδ-closed) [4], if $cl_\delta(A) \subset U$ whenever $A \subset U$ and *U* is open in*(X, τ)*, the complement of gδ -closed is called gδ-open.
9- Generalized g*closed (briefly g*-closed) [11], if $cl(A) \subset U$ whenever $A \subset U$ and *U* is g-open in *(X, τ)*, the complement of g* -closed is called g*-open.

Theorem 2.3[6]: Every open set is g-open set.
Theorem 2.4[6]: Every closed set is g-closed set.
Theorem 2.5[3]: Every semi-closed set is gs-closed set.

3- properties of g*i-closed sets in topological spaces

In this section, we introduce a new class of closed set called g*i-closed set and study some of their properties.

Definition 3.1: A subset A of topological spaces (X, τ) is called a g*i-closed set if $cli(A) \subset U$ whenever $A \subset U$, U is g-open in (X, τ), and $(G \neq X, \emptyset)$, the set of all family g*i-closed denoted by g*i C(X).

Theorem 3.2: Every closed set in a space X is g*i-closed, but converse need not be true in general.

Proof: Let A be a closed set in (X, τ) such that $A \subseteq U$, where U is g-open .Since A is closed ,that is $cl(A) = A$, since $cli(A) \subset cl(A) = A$, and $A \subset U$ therefore $cli(A) \subset U$. Hence A is g*i-closed set in (X, τ).

The converse of the above theorem is not true in general as shown from the following example.

Example 3.3: Consider the topological spaces $X = \{a, b, c\}$ with the topology

$\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$

$C(X) = \{\emptyset, \{b, c\}, \{c\}, X\}$

$GC(X) = \{\emptyset, \{c\}, \{b, c\}, \{a, c\}, X\}$

$GO(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$

$IO(X) = \{\emptyset, \{a\}, \{b\}, \{a, c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$IC(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$

g*iC(X) = $\{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$

Let $A = \{a, c\}$, here A is a g*i-closed set but not a closed set.

Theorem 3.4: Every i-closed in X is g*i-closed set

Proof: Let A be i-closed in X such that $A \subset U$, where U is g-open. Since A is an i-closed set, then $cli(A) = A$,and $A \subset U$, therefore $cli(A) \subset U$. Hence A is g*i-closed set in X.

The converse of the above theorem is not true in general as shown in the following example.

Example 3.5: Let $X = \{a, b, c\}$, with the topology $\tau = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$

$C(X) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$

$GC(X) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$

$GO(X) = \{\emptyset, \{a, b\}, \{b, c\}, \{b\}, X\}$

$IO(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$

$IC(X) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

g*iC(X) = *{Ø, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, X}*

Let A = {b}. So A is g*i-closed set but not i-closed in (X, τ)

Theorem 3.6: Every g*i-closed set in topological spaces (X, τ) is gi-closed set.

Proof: let A be a g*i-closed set in X such that $A \subset U$, where U is open. Since every open set is g-open by Theorem (2.3), and A is g*i-closed, $cli(A) \subset U$. Hence A is gi-closed.

The converse of the above theorem is not true in general as shown from the following example.

Example 3.7: let $X = \{a, b, c\}$, $\tau = \{Ø, \{a\}, X\}$

$C(X) = \{Ø, \{b, c\}, X\}$.
$IO(X) = \{Ø, \{a\}, \{a, b\}, \{a, c\}, X\}$

$IC(X) = \{Ø, \{b\}, \{c\}, \{b, c\}, X\}$

$GC(X) = \{Ø, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$GO(X) = \{Ø, \{a\}, \{c\}, \{b\}, \{a, b\}, \{a, c\}, X\}$

g*i$C(X) = \{Ø, \{b\}, \{c\}, \{b, c\}, X\}$

giC(X) = $\{Ø, \{b\}, \{c\}, \{a, c\}, \{b, c\}, \{a, b\}, X\}$

Let A = {a, b}, then A is gi-closed set but not g*i-closed set.

Theorem 3.8: Every semi-closed set in topological spaces (X, τ) is g*i-closed set.

Proof: let A be semi-closed set in (X, τ) , such that $A \subset U$, where U is g-open. Since A is semi-closed and by Theorem (2.2), then $cli(A) \subset cls(A) \subset U$

, therefore $cli(A) \subset U$ and U is g-open . Hence A is g*i-closed set.

The converse of the above theorem is not true in general as shown from the following example.

Example 3.9: let $X = \{a, b, c\}$ and $\tau = \{Ø, \{c\}, \{a, c\}, \{b, c\}, X\}$, then

$C(X) = \{Ø, \{a\}, \{b\}, \{a, b\}, X\}$, and

$IO(X) = \{Ø, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$

$IC(X) = \{Ø, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$SO(X) = \{Ø, \{c\}, \{a, c\}, \{b, c\}, X\}$

$SC(X) = \{Ø, \{a\}, \{b\}, \{a, b\}, X\}$

$GC(X) = \{Ø, \{a\}, \{b\}, \{a, b\}, X\}$

GO(X) = {Ø, {c}, {a, c}, {b, c}, X}

g*iC(X) = *{Ø, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, X}*

Let *A ={c}*. Then *A* is g*i-closed set but not semi-closed set.

Theorem 3.10: Every g*-closed set in topological spaces (X, τ) is g*i-closed set.

Proof: let *A* be a g*-closed set in (X, τ) such that $A \subset U$, where *U* is g-open .Since *A* is g*-closed and by Theorem (2.2), then $cli(A) \subset cl(A) \subset U$. Hence *A* is a g*i-closed set in (X, τ).

The converse of the above theorem is not true in general as shown from the following example.

Example 3.11: Let *X = {a, b, c}*, τ *= {Ø, {b}, {a, b}, {b, c}, X}*

C(X) = {Ø, {a}, {c}, {a, c}, X}

IO(X) = {Ø, {a}, {b}, {c}, {a, b}, {b, c}, X}

IC(X) = {Ø, {a}, {c}, {a, b}, {a, c}, {b, c}, X}

GC(X) = {Ø, {a}, {c}, {a, c}, X}

GO(X) = {Ø, {b}, {a, b}, {b, c}, X}

g*i*C(X) = {Ø, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, X}*

g**C(X) = {Ø, {a}, {c}, {a, c}, X}*

Let *A* = {b}, so *A* is a g*i-closed set but not g*-closed set of (X, τ)

Corollary 3.12: Every g-closed set in (X, τ) is g*i-closed.

Proof: By Theorem (2.4), and Theorem (3.2).

Corollary 3.13: Every gs-closed set in (X, τ) is g*i-closed.

Proof: By Theorem (2.5), and by Theorem (3.8).

Theorem 3.14: Every δg-closed set in topological spaces(X, τ) is g*i-closed set.

Proof: Let *A* be a δg-closed set in (X, τ) such that $A \subset U$ where *U* is g-open. Since *A* is δg-closed and by Theorem (2.2), then $cli(A) \subset cl\delta(A) \subset U$, so we get $cli(A) \subset U$. Hence *A* is g*i-closed set.

The converse of the above theorem is not true in general as shown from the following example.

Example 3.15: let *X = {a, b, c}*, τ *= {Ø, {a}, {b}, {a, b}, X}*

$C(X) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$

$IO(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$IC(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$

$GC(X) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$

$GO(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$

$\delta O(X) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$

$\delta C(X) = \{\emptyset, \{c\}, \{a, c\}, \{b, c\}, X\}$

$\delta GC(X) = \{\emptyset, \{c\}, \{b, c\}, \{a, c\}, X\}$

$g^*iC(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$

Let $A = \{a\}$, so A is g*i-closed set in (X, τ), but not δg-closed set.

Theorem 3.16: Every θg-closed set in topological spaces (X, τ) is g*i-closed set.

Proof: Let A be θg-closed set in (X, τ), such that $A \subset U$ where U is g-open. Since A is θg-closed and by Theorem (2.2), then $cli(A) \subset cl\theta(A) \subset U$, so we have $cli(A) \subset U$. Hence A is a g*i-closed set.

The converse of the above theorem is not true in general as shown from the following example.

Example 3.17: Let $X = \{a, b, c\}$, and $\tau = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$, *then* $C(X) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$, $\theta O(X) = \{\emptyset, X\}$, *and* $\theta C(X) = \{\emptyset, X\}$

$IO(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, X\}$

$IC(X) = \{\emptyset, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$GC(X) = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$

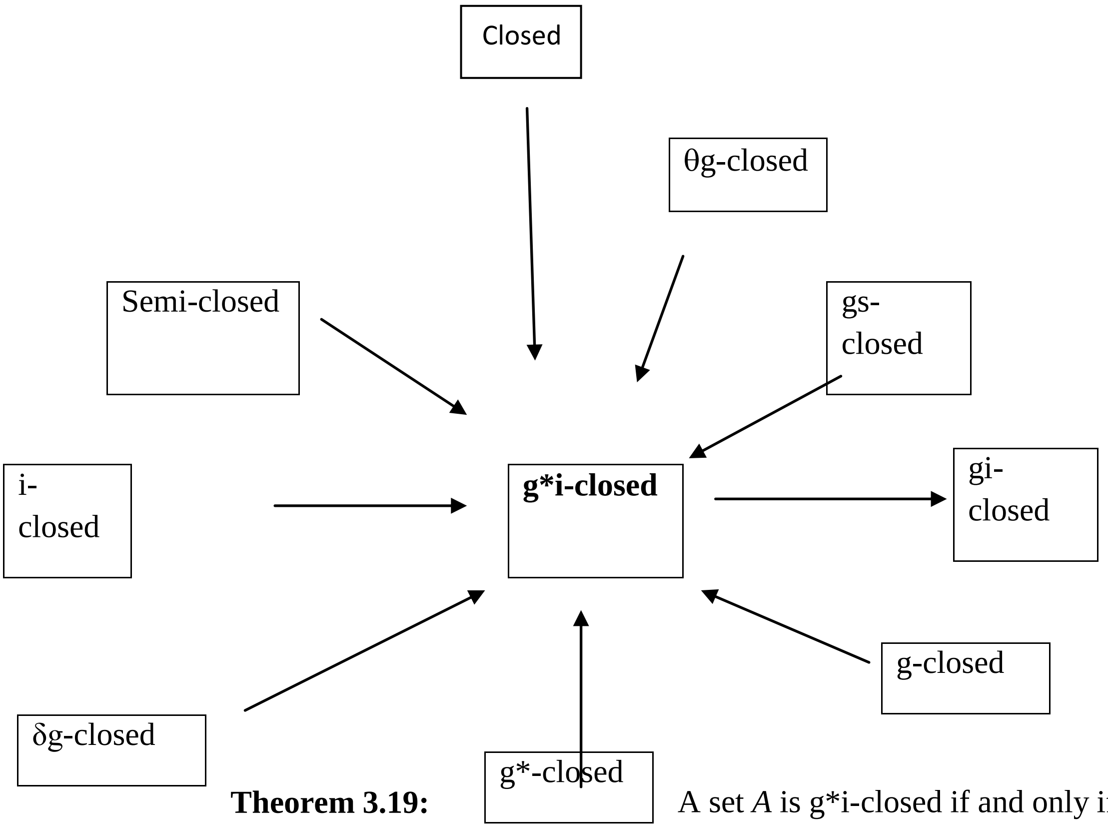
$GO(X) = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, X\}$

$g^*iC(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$

$\theta GC(X) = \{\emptyset, \{a, c\}, X\}$

Let $A = \{a, b\}$. So A is g*i-closed set in (X, τ) but not θg-closed set.

Remark 3.18: By the above results we have the following diagram.



Theorem 3.19: A set A is g*i-closed if and only if $cli(A)\backslash A$ contains no non-empty g-closed set .

Proof : Necessity let F be a g-closed set in (X, τ) such that $F \subset cli(A)\backslash A$ then $cli(A) \subset X \backslash F$. This implies $F \subset X \backslash cli(A)$, so $F \subset (X \backslash cli(A)) \cap (cli(A)\backslash A) \subset (X \backslash cli(A)) \cap cli(A)=\emptyset$. Therefore $F = \emptyset$.

Sufficiency: Assume that $cli(A)\backslash A$ contains no non –empty g-closed set. And let $A \subset U$, U is g-open. Suppose that $cli(A)$ is not contained in U, $cli(A) \cap U$ is a non-empty g-closed set of $cli(A)\backslash A$ which is a contradiction. Therefore $cli(A) \subset U$, Hence A is g*i-closed .

Theorem 3.20: A g*i-closed set A is an i-closed set if and only if $cli(A)\backslash A$ is an i-closed set.

Proof: If A is an i-closed set, then $cli(A)\backslash A=\emptyset$. Conversely, suppose $cli(A)\backslash A$ is an i-closed set in X. Since A is g*i-closed. *Then* $cli(A)\backslash A$ contain no non-empty g-closed set in X. Then $cli(A)\backslash A =\emptyset$. Hence A is an i-closed set.

Theorem 3.21: If A and B are two g*i-closed, then $A\cap B$ is g*i-closed.

Proof: Let A and B be two g*i-closed sets in X. And let $A \cap B \subset U$,

U is g-open set in X. Since A is g*i-closed, then $cli(A) \subset U$, whenever

$A \subset U$, and U is g-open in X. Since B is g*i-closed, then $cli(B) \subset U$ whenever $B \subset U$, and U is g-open in X. Now $cli(A)\cap cli(B) \subset U$, therefore $A\cap B$ is g*i-closed.

Corollary 3.22: The intersection of g*i-closed set and closed set is g*i-closed set .

Proof: By Theorem (3.2) and Theorem (3.21) we get the result.

Note. If *A* and *B* are g*i-closed then their union need not be g*i-closed as shown in the following example.

Example 3.23: let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$

$g^*iC(X) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$

Let $A = \{a\}$ and $B = \{b\}$ are g*i-closed but $A \cup B = \{a, b\}$ is not g*i-closed.

Theorem 3.24: If *A* is both g-open and g*i-closed set of *X*, then *A* is an i-closed set.

Proof: Since *A* is g-open and g*i-closed in *X*, then $cli(A) \subset U$, *U* is g-open.

Since $A \subset U$ and *A* is g-open then $cli(A) \subset A$. but always $A \subset cli(A)$, therefore $A = cli(A)$. Hence *A* is an i-closed set.

Theorem 3.25: For $x \in X$, the set $X \setminus \{x\}$ is g*i-closed or g-open.

Proof: Suppose $X \setminus \{x\}$ is not g-open. Then X is the only g-open set containing $X \setminus \{x\}$. This implies $cli(X \setminus \{x\})$ is g*i-closed. Then $X \setminus \{x\}$ is g*i-closed.

Theorem 3.26: If *A* is g*i-closed and $A \subset B \subset cli(A)$, then *B* is g*i-closed.

Proof: Let *U* be g-open set of *X* such that $B \subset U$. Then $A \subset U$. Since *A* is g*i-closed. Then $cli(A) \subset U$, now $cli(B) \subset cli(cli(A)) = cli(A) \subset U$. Therefore *B* is g*i-closed.

Theorem 3.27: Let $A \subset Y \subset X$, and suppose that *A* is g*i-closed in *X*, then *A* is g*i-closed relative to *Y*

Proof: Given that $A \subset Y \subset X$ and *A* is g*i-closed in *X*. To show that *A* is g*i-closed relatives *Y*. Let $A \subset Y \cap U$, where *U* is g-open in *X*. Since *A* is g*i-closed $A \subset U$, implies $cli(A) \subset U$. It follows that $Y \cap cli(A) \subset Y \cap U$. Thus *A* is g*i-closed relative to *Y*.

Proposition 3.28: If a set *X* is finite and a topology τ *on X* is T_1 – space, then gi-closed = g*i-closed

Proof: Let *X* be a finite set and T_1 – space, let $A \in$ gi-closed. If $A = \emptyset$, then $A \in$ g*i – closed. If $A \neq \emptyset$, then let $A \subset X$ and for each $x \in A$. $\{x\}$ is closed therefore $A = \cup_{x \in A} \{x\}$, then *A* is closed. By Theorem (3.2), $A \in$ g*i-closed. Hence gi-closed $\subset$ g*i-closed but by Theorem (3.6), g*i-closed $\subset$ gi-closed therefore gi-closed = g*i-closed.

Corollary 3.29: If a topological τ on *X* is discrete topology, then

gi-closed = g*i-closed.

Proof: Obvious

Proposition 3.30: For a subset A of a topological space (X, τ) the following statements are true.

1- If A is gi-closed then *cl (A)* $\subset$ g*i-closed.
2- If A is g*i-closed then gi-*int (A)* $\subset$ g*i-closed.
3- If A is g*i-closed then gi-*cl (A)* $\subset$ g*i-closed.

Proof: Obvious.

Definition 3.31: A subset A of a space X is called g*i-open if $X \setminus A$ is g*i-closed. The family of all g*i-open subset of a topological space (X,τ) is denoted by g*iO(X,τ) or g*iO(X).

All of the following results are true by using complement.

Proposition 3.32: The following statements are true:

1- Every open is g*i-open.
2- Every g*i-open is gi-open.
3- Every i-open is g*i-open.
4- Every δ-open is g*i-open.

Proof: By using the complement of the definition of g*i-closed.

Proposition 3.33: Let A be subset of a topological space (X,τ). If A is g*i-open, then for each $x \in A$ there exists g*i-open set be such that $x \in B \subset A$.

Proof: Let A be g*i-open set in a topological space (X,τ) then for each $x \in A$, put $A = B$ is g*i-open containing x such that $x \in B \subset A$.

4. Some properties of g*i-open and g*i-closed sets in a topological space

Definition 4.1: Let (X,τ) be a topological space and $x \in X$. A subset N of X is said to be g*i-neighborhood of x if there exists g*i-open set Y in X such that $x \in Y \subset N$.

Definition 4.2: Let A be subset of a topological space (X, τ), a point $x \in X$ is called g*i-interior point of A, if there exist g*i-open set U such that $x \in U \subset A$. The set of all g*i-interior points of A is called g*i-interior of A and is denoted by g*i-*int(A)*.

Proposition 4.3: For any subsets A and B of a space X, the following statements hold:

1. g*i-*int*$(\emptyset) = \emptyset$ and g*i-*int*$(X) = X$.
2. g*i-*int (A)* is the union of all g*i-open sets which are contained in A.
3. g*i-*int(A)* is g*i-open set in X.
4. g*i-*int (A)* $\subset A$.
5. If $A \subset B$, then g*i-*int(A)* $\subset$ g*i-*int (B)*.
6. If $A \cap B = \emptyset$, then g*i-*int(A)* $\cap$ g*i-*int(B)* $= \emptyset$.
7. g*i-*int* ((g*i-*int(A)*) = g*i-*int(A)*
8. A is g*i-open if and only if A = g*i-*int (A)*.

9. g*i-$int(A) \cup$ g*i-$int(B) \subset$ g*i-$int(A \cup B)$.
10. g*i-$int(A \cap B) \subset$ g*i-$int(A) \cap$ g*i-$int(B)$.

Proof : The prove of (1), (2), (3), (4), (5), (6), (7) and (8) is Obvious, only to prove (9) and (10)

Proof (9): Let A and B be subset of X, since $A \subset A \cup B$ and $B \subset A \cup B$. Then by Proposition (4.3) (5), we have g*i-$int(A) \subset$ g*i-$int(A \cup B)$ and g*i-$int(B) \subset$ g*i-$int(A \cup B)$. Hence g*i-$int(A) \cup$ g*i-$int(B) \subset$ g*i-$int(A \cup B)$

Proof (10): Since $A \cap B \subset A$ and $A \cap B \subset B$ then by Proposition (4.3) (5), we have g*i-$int(A \cap B) \subset$ g*i-$int(A)$ and g*i-$int(A \cap B) \subset$ g*i-$int(B)$. Hence g*i-$int(A \cap B) \subset$ g*i-$int(A) \cap$ g*i-$int(B)$.

In general the equalities of (9) and (10) and the converse of (5) does not hold, as shown in the following example.

Example 4.4: From Example (3.7)

g*i$O(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $A = \{a\}$ and $B = \{b\}$, then g*i-$int(\{a\}) = \{a\}$ and g*i-$int(\{b\}) = \emptyset$.

g*i-$int(\{a\}) \cup$ g*i-$int(\{b\}) = \{a\} \cup \emptyset = \{a\}$

g*i-$int(\{a\} \cup \{b\})$ = g*i-$int(\{a, b\}) = \{a, b\}$

g*i-$int(A \cup B) \not\subset$ g*i-$int(A) \cup$ g*i-$int(B)$

Example 4.5: From Example (3.3)

g*i$O(X) = \{\emptyset, X \{a\}, \{a, b\}, \{b\}, \{a, c\}, \{b, c\}\}$

Let $A = \{a, c\}$, $B = \{b, c\}$

g*i-$int(A \cap B)$ = g*i-$int(\{a, c\} \cap \{b, c\})$ = g*i-$int\{c\} = \emptyset$.

g*i-$int(A) \cap$ g*i-$int(B) = \{a, c\} \cap \{b, c\} = \{c\}$.

g*i-$int(A) \cap$ g*i-$int(B) \not\subset$ g*i-$int(A \cap B)$.

Example 4.6: From Example (3.7)

g*i$O(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $A = \{b, c\}$,and $B = \{a, c\}$

g*i-$int(A) = \emptyset$ and g*i-$int(B) = \{a, c\}$

g*i-$int(A) \subset$ g*i-$int(B)$, but $A \not\subset B$.

Proposition 4.7: For any subset A of X, g*i-$int(A) \subset$ gi-$int(A)$

Proof: Let A be a subset of a space X and let $x \in$ g*i-$int(A)$, then $x \in \cup \{G : G$ is g*i-open, $G \subset A\}$. Then there exists a g*i-open set G such that $x \in G \subset A$. Since every g*i-open set is gi-open then there exists a gi-open set G such that $x \in G \subset A$, this implies that $x \in$ gi-$int(A)$. Hence g*i-$int(A) \subset$ gi-$int(A)$.

The converse of the above proposition is not true in general as shown from the following example.

Example 4.8: From Example (3.7)

g*i$O(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$, let $A=\{b\}$

g*i-$int\{b\}=\emptyset$, gi-$int\{b\}=\{b\}$, this implies gi-$int(A) \not\subset$ g*i-$int(A)$.

Definition 4.9: Let A be a subset of a space X. A point $x \in X$ is called to be g*i-limit point of A if for each g*i-open set U containing x, $U \cap (A \setminus \{x\}) \neq \emptyset$

The set of all g*i-limit points of A is called the g*i-derived set of A and is denoted by g*i-$D(A)$.

Proposition 4.10: Let A and B be subsets of a space X, then we have the following properties:

1- g*i-$D(\emptyset) = \emptyset$.
2- If $x \in$ g*i-$D(A)$, then $x \in$ g*i-$D(A \setminus \{x\})$.
3- If $A \subset B$, Then g*i-$D(A) \subset$ g*i-$D(B)$.
4- g*i-$D(A) \cup$ g*i-$D(B) \subset$ g*i-$D(A \cup B)$.
5- g*i-$D(A \cap B)$ = g*i-$D(A) \cap$ g*i-$D(B)$.
6- g*i-D(g*i-$D(A)) \setminus A \subset$ g*i-$D(A)$.
7- g*i-$D(A \cup$ g*i-$D(A) \subset A \cup$ g*i-$D(A)$.

Proof: We only prove (6) and (7) since the other part can be proved obviously.

6- If $x \in$ g*i-D(g*i-$D(A)) \setminus A$, then $x \in$ g*i-D(g*i-$D(A)$ and $x \notin A$,and U is g*i-open set containing x. Then $U \cap$ (g*i-$D(A) \setminus \{x\} \neq \emptyset$. Let $y \in U \cap$ (g*i-$D(A) \setminus \{x\}$. Since $y \in U$ and $y \in$ g*i-$D(A)$. $U \cap (A \setminus \{y\}) \neq \emptyset$, Let $z \in U \cap (A \setminus \{y\})$, Then $z \neq x$ for $z \in A$, and $x \notin A$, $U \cap (A \setminus \{x\}) \neq \emptyset$, Therefore $x \in$ g*i-$D(A)$.

7- Let $x \in$ g*i-$D(A \cup$ g*i-$D(A))$. If $x \in A$ the result is obvious. Let $x \notin A$, and

$x \in$ g*i-$D(A \cup$ g*i-$D(A)) \setminus A$ then for any g*i-open set U containing x, $U \cap (A \cup$ g*i-$D(A)) \setminus \{x\} \neq \emptyset$. It following similarly from (6). Thus $U \cap$ (g*i-$D(A)) \setminus \{x\} \neq \emptyset$. Then $(U \cap A) \setminus \{x\} \neq \emptyset$. Hence $x \in$ g*i-$D(A)$. Therefore g*i-$D(A \cup$ g*i-$D(A)) \subset A \cup$ g*i-$D(A)$.

The converse of above proposition (3) and (4) is not true in general as shown the following example.

Example 4.11: From example (3.7)

g*i$O(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{a, c\}\}$

Let $A = \{b, c\}$ and $B = \{a, c\}$

g*i-$D(A) = \emptyset$, g*i-$D(B) = \{b, c\}$, then g*i-$D(A) \subset$ g*i-$D(B)$, but $A \not\subset B$

Example 4.12: From example (3.3)

g*i$O(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{b\}, \{a, c\}, \{b, c\}$

Let $A = \{a\}$ *and* $B = \{b\}$

g*i-$D(A) = \emptyset$ and g*i-$D(B) = \emptyset$, then g*i-$D(A \cup B) = \{c\}$

g*i-$D(A) \cup$ g*i-$D(B) \subset$ g*i-$D(A \cup B)$, but g*i-$D(A \cup B) \not\subset$ g*i-$D(A) \cup$ g*i-$D(B)$.

Proposition 4.13: If X a topological space and A is subset of X, then gi-$D(A) \subset$ g*i-$D(A)$.

Proof: Let $x \notin$ g*i-$D(A)$. This implies that there exists g*i-open set U containing x such that $U \cap (A \setminus \{x\}) = \emptyset$, U is g*i-open. Since every g*i-open is gi-open. Then U is gi-open set containing x and $U \cap (A \setminus \{x\}) = \emptyset$, then $x \notin$ g*i-$D(A)$. Hence gi-$D(A) \subset$ g*i-$D(A)$

The converse of above proposition is not true in general as shown the following example .

Example 4.14: Let $X = \{a, b, c\}$, $\tau = \{\emptyset, \{c\}, X\}$

Let $A = \{c\}$, gi-$D(A) = \emptyset$ and g*i-$D(A) = \{a, b\}$

gi-$D(A) \subset$ g*i-$D(A)$ but g*i-$D(A) \not\subset$ gi-$D(A)$.

Definition 4.15: For any subset A in space X, the g*i-closure of A, denoted by g*i-$cl(A)$, and defined by the intersection of all g*i-closed sets containing A.

Proposition 4.16: Let X be a topological space . If A and B are subsets of space X, then

1. The g*i-closure of A is the intersection of all g*i-closed sets containing A
2. g*i-$cl(X) = X$ *and* g*i-$cl(\emptyset) = \emptyset$.
3. $A \subset$ g*i-$cl(A)$.
4. g*i-$cl(A)$ is g*i-closed set in X.
5. if g*i-$cl(A) \cap$ g*i-$cl(B) = \emptyset$, then $A \cap B = \emptyset$.
6. If B is any g*i-closed set containing A. Then g*i-$cl(A) \subset B$.
7. If $A \subset B$ then g*i-$cl(A) \subset$ g*i-$cl(B)$.
8. g*i-cl(g*i-$cl(A)$ = g*i-$cl(A)$.
9. A is g*i-closed if and only if g*i-Cl(A)=A.

Proof: It is obvious

Proposition 4.17: If *A* and *B* are subset of a spare *X* then

1- g*i-$cl(A) \cup$ g*i-$cl(B) \subset$ g*i-$cl(A \cup B)$.
2- g*i-$cl\ (A \cap B) \subset$ g*i-$cl(A) \cap$ g*i-$cl(B)$.

Proof: Let *A* and *B* be subsets of space *X*

1- Since $A \subset A \cup B$ and $B \subset A \cup B$. Then by Proposition (4.16)(7), the g*i-$cl(A) \subset$ g*i-$cl(A \cup B)$ and g*i-$cl(B) \subset$ g*i-$cl(A \cup B)$. Hence g*i-$cl(A) \cup$ g*i-$cl(B) \subset$ g*i-$cl(A \cup B)$.
2- Since $A \cap B \subset A$ and $A \cap B \subset B$. Then by Proposition (4.16)(7). The g*i-$cl(A \cap B) \subset$ g*i-$cl(A)$ and g*i-$cl\ (A \cap B) \subset$ g*i-$cl(B)$. Hence g*i-$cl(A \cap B) \subset$ g*i-$cl(A) \cap$ g*i-$cl(B)$.

The converse of the above Proposition is not true in general as shown the following example.

Example 4.18: From example (3.3)

g*i$O(X) = \{\emptyset, X, \{a\}, \{a, b\}, \{b\}, \{a, c\}, \{b, c\}\}$

g*I $= \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, c\}, \{b, c\}, X\}$

g*i-$cl(\{a\}) = \{a\}$, g*i-$cl(\{b\}) = \{b\}$

g*i-$cl(A \cup B) =$ g*i-$cl(\{a, b\}) = X$

g*i-$cl(A) \cup$ g*i-$cl(B) = \{a\} \cup \{b\} = \{a, b\}$

g*i-$cl(A \cup B) = X \nsubseteq \{a, b\}$

1- Let $A = \{a, b\}$, *and* $B = \{c\}$, then $A \cap B = \{a, b\} \cap \{c\} = \emptyset$, therefore g*i-$cl(A \cap B) = \emptyset$, but g*i-$cl(A) = X$ and g*i-$cl\ (B) = \{c\}$.Therefore g*i-$cl(A) \cap$ g*i-$cl\ (B) = X \cap \{c\} = \{c\}$, but g*i-$cl(A \cap B) = \emptyset$, implies that g*i-$cl(A) \cap$ g*i-$cl\ (B) \nsubseteq$ g*i-$cl(A \cap B)$.

Proposition 4.19: If A is subset of a space X. Then gi-$cl(A) \subset$ g*i-$cl(A)$

Proof: Let *A* be a subset of a space *X*. By Definition of g*i-closed.

g*i-$cl(A) = \cap$ { $F : A \subset F$ is g*i-closed}, since $A \subset F \in$ g*i-closed . Then by Theorem (3.6), $A \subset F \in$ gi-closed and by Proposition (4.16),(7), gi-$cl(A) \subset F$, therefore gi-$cl(A) \subset \cap \{F : A \subset F \in$ g*i-closed } = g*i-$cl(A)$. Hence gi-$cl(A) \subset$ g*i-$cl(A)$.

Proposition 4.20: A subset *A* of a topological space is g*i-closed if and only if it contains the set of all g*i-limit points.

Proof : Assume that *A* is g*i-closed we will prove that *A* it contains the set of its g*i-limit points. And assume that if possible that *x* is g*i-limit points of *A* which be longs to $X \setminus A$.

Then $X \setminus A$ is g*i-open set containing the g*i-limit point of A. Therefore by definition of g*i-limit points $A \cap X \setminus A \neq \emptyset$ which is contradiction.

Conversely, assume that A contains that set of its g*i-limit points. For each $x \in X \setminus A$, there exists g*i-open set U containing x such that $A \cap U = \emptyset$. That is $x \in U \subset X \setminus A$, by Proposition (3.33), $X \setminus A$ is g*i-open set. Hence A is g*i-closed.

Proposition4.21: Let A be subset of space X, then g*i- $cl(A) = A \cup$ g*i- $D(A)$

Proof: Since $A \subset$ g*i-$Cl(A)$ and g*i-$D(A) \subset$ g*i-$Cl(A)$, then ($A \cup$ g*i-$D(A)$) $\subset$ g*i-$cl(A)$. To prove that g*i-$cl(A) \subset A \cup$ g*i-$D(A)$, but g*i-$Cl(A)$ is the smallest g*i-closed containing A, so we prove that $A \cup$ g*i-$D(A)$ is g*i-closed, let $x \notin A \cup$ g*i-$D(A)$. This implies that $x \notin A$ and $x \notin$ g*i-$D(A)$. Since $x \in$ g*i-$D(A)$, there exists g*i-open set G_x of x which contains no point of A other than x but $x \notin A$. So G_x Contains no point of A. Then $G_x \subset X \setminus A$, agin $G_x$is an g*i-open set of each of its points but as G_x does not contain any point of A no point of G_x can be g*i-limit points of A, this implies that $G_x \subset X \setminus$ g*i-$D(A)$, hence $x \in G_x \subset X \setminus A \cap Cl(A) \subset A \cup X \setminus$ g*i-$D(A) \subset X \setminus$ ($A \cup$g*i-$D(A)$). Therefore $A \cup$ g*i-$D(A)$ is g*i-closed. Hence g*i-$cl(A) \subset A \cup$ g*i-$D(A)$. Thus g*i-$cl(A)=A \cup$ g*i-$D(A)$.

Proposition 4.22: Let A be subset of a topological space (X, τ) .And for any $x \in$ X , then $x \in$ g*i-Cl(A) if and only if $A \cap U \neq \emptyset$ for every g*i-open set U containing x.

Proof : Let $x \in X$ and $x \in$ g*i-$cl(A)$.We will prove $A \cap U \neq \emptyset$ for every g*i-open set U containing x, we will proved by contradiction, suppose that there exists g*i-open set U containing x such that $A \cap U = \emptyset$. Then $A \subset X \setminus U$ and X\A is g*i-closed. g*i-$cl(A) \subset X \setminus U$, then $x \notin$ g*i-$cl(A)$. which is contradiction. Hence $A \cap U \neq \emptyset$.

Conversely, let $A \cap U \neq \emptyset$. For every g*i-open set U containing x, we will proved by contradiction. Suppose $x \notin$ g*i-$cl(A)$. Then there exists a g*i-closed set F containing A such that $x \notin F$, hence $x \in X \setminus F$ and $X \setminus F$ is g*i-open set, then $A \cap X \setminus F = \emptyset$ which is contradiction.

Proposition 4.23: For any subset A of a topological space X, the following statements are true:

1- $X \setminus$ g*i-$cl(A)$ = g*i-int $(X \setminus A)$.
2- $X \setminus$ g*i-$int(A)$ = g*i-$cl(X \setminus A)$.
3- g*i-$cl(A)$ = $X \setminus$ g*i-$int(X \setminus A)$.
4- g*i-$int(A)$ = $X \setminus$ g*i-$cl(X \setminus A)$.

Proof:

1- For any $x \in X$, then $x \in X \setminus$ g*i-$cl(A)$ implies that $x \notin$ g*i-$cl(A)$, then there exists g*i-open set G containing x such that $A \cap G = \emptyset$, then $x \in G \subset X \setminus A$. Thus $x \in$ g*i-$int(X \setminus A)$. conversely, by reversing the above steps, we can prove this part.
2- Let $x \in X \setminus$g*i-$int(A)$, then $x \notin$ g*i-$int(A)$, so for any g*i-open set B containing x

$B \not\subset A$. This implies that every g*i-open set B containing x ,$B \cap X \backslash A \neq \emptyset$. This means $x \in$ g*i-cl($X \backslash A$). Hence ($X \backslash$g*i-*int(A))* ⊂ g*i-*cl*($X \backslash A$). Conversely, by reversing the above steps, we can prove this part

Proposition 4.24: Let A be subset of a space X. If A is both g*i-open and g*i-closed, then A= g*i-*int*(g*i-*cl(A))*

Proof: If A is both g*i-open and g*i-closed, then g*i-*int*(g*i-*cl(A*))= g*i-*int(A)=A*.

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