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Analysis of the Behaviour of Stocks of Dar es Salaam Stock Exchange (DSE)

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Abstract

A stock market is a place where investors trade certificates that indicate partial ownership in businesses for a set price. Different countries in the world have stock markets where other countries started their stock markets long time ago like the USA and they have investigated the trend of their market if it is normally distributed or not. Also they have strong models that assist them in making predictions and also help the investors on the choice of the stocks to invest so as to gain the profit in the future. On the other hand other countries just started few years ago. Tanzania is among the countries where stock markets has just started recently and hence there is a need to study the nature of the stocks distribution and see whether the Dar-Es-Salaam Stock of Exchange (DSE) market do follow the theoretical conclusions or not. Thus in this study we adapt the Markowitz modern portfolio theory (MPT) and using the mean variance analysis theory together with the DSE data to investigate if the DSE stock market follows a normal distribution or not. The analysis shows that the DSE stocks log returns are reasonably normally distributed and its prices do change according to the change in other factors like the inflation rate, consumers (investors) interest, the policy of the country, and other exogenous factors.

Keywords: portfolio, stock market, volatility (risk), expected return, covariance matrix.

1. Introduction

Financial status of any company or institution is not static. It changes over time due to a number of factors that can affect its financial flow. Among the factors that affects the financial flow includes inflation, decrease in human consumption rate caused by inflation, increase in investments, price fluctuation and others of the like. Due to this factor we can easily see that financial and insurance markets always operate under various types of uncertainties that can affect financial positions of companies and individuals. In financial and insurance theories these uncertainties are usually referred to as risks. And in stock markets where certificates of partial ownership of business are traded so as to raise the initial capital of the company for operation aspects, the stock market remains the major means of investment and can be used as an indicator of overall economic health. Due to this reason different countries have their own stock markets where prices are determined by the forces of demand and supply. Given certain states of the market, and the economy in general, one can talk about risk exposure. Any economic activities of individuals, companies and public establishments aiming for wealth accumulation assume studying risk exposure is of great important, Melnikov and Alexander, (2004).

Governments do own companies or have shares on different companies that acts as the financial security in case of financial crisis. The shareholder becomes part of the company ownership. During the financial crisis the shareholder can sell the shares he owns. The shares are sold at the stock exchange markets. Barro, (1990).

In a business, Investors have to find the best way to price their business that will have the minimum risk in time. And because different investments have different turnover over time due to changing of the sales caused by customer consumption and investment rate, one will need to have the financial flow throughout the year by investing in both of the two portfolios. Singleton, (2003).

Trade decisions are more concerned with the speed, costs, and risks associated with executing the transaction, while investing emphasizes selection of the security. To be able to make a good decision one needs to have a good knowledge on the pricing theory and pricing model. There are many models formulated that aims to reduce risk and maximize profit by providing a frame work for portfolio selection that have a minimum risk in time and among them is the Modern portfolio theory (MPT). It is used in pricing assets so as to have the minimum risk and gain the maximum return. Although it is more useful it does not operate in the same way in all the stock markets due to markets difference, and market governing rules. Edwin et al, (1997).

Since the DSE started its operations there is no research done to be used as a tool for risk minimization and finding the optimal portfolio and hence help the investors to invest in different portfolios that eventually leads to profit maximization and this is what this study does.

2 Literature review

Portfolio allocation is independent for each individual investor basing upon several factors, including age, investable years before retirement, risk, required necessary return, and current or future goals. The overall positioning of a portfolio is important in evaluating the portfolios. A goal in any portfolio is to achieve the greatest amount of return while taking the least amount of risk. Also each security has its own deviation from an expected return statistically known as standard deviation from the mean in finances it is called risk. The risk of an overall portfolio is expected to decrease as the number of securities increase. According to Markowitz it is not only about pricing securities, but it is about choosing the most appropriate allocation of securities because different securities brings in different concepts of risk. An investor seeking higher return he also intrinsically take an increased risk because return to risk ratio grows quickly at first with each unit of additional risk eventually brings less and less opportunity for the return. This implies that for an investor to benefit more he has to get the optimal portfolio that has minimum risk and brings the expected return. Markowitz, (1952) in his paper “Portfolio Selection," published in 1952 in the Journal of Finance, he introduced the MPT (Modern Portfolio Theory).in this paper Markowitz explain mathematics of diversification

Markowitz suggested that if we treat single-period returns for various securities as random variables, we can assign them expected values, standard deviations and correlations. And we can calculate the expected return and volatility of any portfolio constructed with those securities. Out of the entire universe of possible portfolios, we are sure that there is ones that will optimally balance risk and return. These comprise what Markowitz called an efficient frontier of portfolios. An investor should select a portfolio that lies on the efficient frontier.

The foundations of MPT resulted to the establishment of a formal risk-return framework for investment decision-making. By defining investment risk in quantitative terms, Markowitz gave investors a mathematical approach to asset selection and portfolio management by considering the mean and variances of portfolios. However, differing to its theoretical reputation, the mean- variance model has not been used extensively in its original form to construct a large-scale portfolio. Due to among the reason is that it is computationally difficulty and is associated with solving a large-scale quadratic programming problem with a dense covariance matrix. Several authors tried to alleviate this difficulty by using various approximation schemes Sharpe, (1967, 1971, Stone 1973) in the early years of the history.

To force the portfolio to the efficient frontier James Tobin, (1958) added a risk-free asset to the analysis and hence bringing up the concept of super-efficient portfolio and the capital market line. The capital asset pricing model (CAPM) makes strong assumptions that lead to interesting conclusions Harpe, (1964). Not only does the market portfolio sit on the efficient frontier, but it is actually Tobin's super-efficient portfolio. Konno, (1988, 1989), proposed a new portfolio optimization model using piecewise linear risk functions showing that their model can achieve the intention of Markowitz by solving a linear program instead of a difficult quadratic program and they emphasize on the use of LI risk model that leads to a linear program instead of a quadratic program, so that a large-scale optimization problem of more than 1,000 stocks may be solved on a real time basis. Various aspects of this phenomenon have been extensively studied in the literature on portfolio selection.

All these readings show that portfolio optimization is rooted from the Markowitz mean-variance model (MPT-model) and all the other models are directly or indirectly based on this model. However it is important to note that these models have been tested and used in few developed financial markets like the New York Stock Exchange (NYSE), most research papers analysis has been done using yahoo finance data, with some stocks like the S & P ‘100, Nikkei, FTSE 100’ appearing in most of the papers, Puelz, (2002), while different Financial Markets have different characteristics and their securities behave differently Konno and Yamazaki, (1991). hence its effectiveness in other markets is not guaranteed, therefore there is need to examine the applicability of these portfolio optimization models before implementing them in the budding stock markets like DSE.

Due to this, this study aims to develop a model that will be adapted to the Dar es Salaam Stock Exchange market, basing on Modern Portfolio Theory. And the contribution of this study will be to explore the relevance and applicability of the modern portfolio theory model to the Dar es Salaam Stock Market as far as portfolio optimization is concerned.

3 **Material and Methods**

The data was obtained at the Dar-Es-Salaam Stock exchange market (DSE) offices and there are about 17 registered stocks namely Tanzania Breweries Limited (TBL) Swissport Tanzania Limited (Swissport), TOL Gases Limited (TOL), Tanzania Portland Cement Company Limited (TWIGA),TATEPA LIMITED, Kenya Airways Limited (KA), Tanzania Cigaratte Company (TCC), East African Breweries Limited (EABL), Tanga Cement Company Limited (SIMBA), Jubilee Holdings Limited (JHL), Dar Es Salaam Community Bank (DCB),CRDB Bank PLC (CRDB), National Microfinance Bank (NMB), Kenya Commercial Bank (KCB). Eleven stocks were selected to represent the DSE market. And from these eleven stocks the general trend of the DSE market was studied.

The data was collected on 17th of April 2014, includes the closing prices, opening prices, trading dates, market capitalization, high and low prices, turnover and company names. The data provided was for nine years from 2006 to 2014. From the data it was observed that seven companies started selling their stocks in 2006, three companies started on 2007, three companies on 2008, one company on 2009, another one on 2011 and the other two companies on 2013. Also it was observed that some of the companies have sold their stocks ones, twice or thrice which it is difficult to get the real trend for these data. Even though we were provided all these data only the company names, closing dates and trading dates was employed in the calculations and its outputs were used in plotting the relevant graphs. The Excel 2007 spread sheet was used in calculating the returns by using the formulae

$$R_i = \frac{\text{clossing price - previous closing price}}{\text{previous closing price}},$$

Where the $R_i\text{'}s$ represent the daily stock returns, and to calculate the monthly stock returns the formulae

$$monthly\ returns = \frac{\sum_{i=1}^{n} R_i}{n}$$ where n is the total number of traded days in the month was used.

Using these monthly returns the log returns was calculated by using the inbuilt function 'LN', the mean, standard deviation, maximum and minimum values of both the returns and log returns were calculated by using the excel inbuilt functions of; STDEV, MAX, MIN and AVERAGE

3.1Model formulation and analysis

The methodology employed is to adopt the Markowitz model and simulate it with the data obtained from the Dar es Salaam stock of exchange market so as to come up with the optimal portfolio through solving the expectations and volatility of the portfolios. And upon doing the calculations the following equations was used.

Expected return:

According to Mukhopadyyay, (2000), the expected value of the random variable R_p is denoted by $E(R_p)$ or ER_p or $[R_p]$ and is defined as

$$E(R_p) = \sum_i w_i E(R_i) \tag{3.1}$$

$$\begin{aligned} where\ \ R_p &\rightarrow \text{ the return on the portfolio} \\ R_i &\rightarrow \text{ the return on asset } i \\ w_i &\rightarrow \text{weighting of the component asset } i \end{aligned}$$

And the Portfolio variance is found by using the equation

$$\sigma_p^2 = \sum_i w_i^2 \sigma_i^2 + \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{3.2}$$

$$\sigma_p^2 = \sum_i \sum_{j \neq i} w_i w_j \sigma_i \sigma_j \rho_{ij} \tag{3.3}$$

where $\rho_{ij} = 1$ for $i = j$

Now from equation (3.1) if we are finding the expected return for two stocks only it can be found by using the formulae

$$\begin{aligned} E(R_p) &= w_A E(R_A) + w_B E(R_B) \\ &= w_A E(R_A) + (1 - w_A) E(R_B) \end{aligned} \tag{3.1.1}$$

Where A means the first stock and B stands for the second stock. also the portfolio variance can be found by using the formulae

$$\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \sigma_{AB} \tag{3.2.1}$$

This trend will continue up to N where N is the number of securities or bonds in the given portfolio.

According to Hsu, (1997) the covariance of assets R_i and R_j is defined by the equation

$$\sigma_{ij} = E[R_i - E(R_i)][R_j - E(R_j)] \tag{3.2.2}$$

In the case of diversification, the investor has to hold combinations of instruments which are not perfectly correlated where correlation coefficient is between $-1 \leq \rho_{ij} < 1.$

To get the required expected return for a correct pricing mechanism the asset pricing model (MPT) has to be employed. and the CAPM which uses the equation $E(R_i) = R_f + \beta_i (E(R_m - R_f))$ (3.4) can be employed to derive the theoretical required expected return (i.e. discount rate) for an asset in a market, given the risk-free rate available to investors and the risk of the market as a whole and β is used as the measure of assets sensitivity to a movement in the overall market premium $(E(R_m - R_f))$, it is the expected excess return of the market portfolio's expected return over the risk – free asset.

The incremental impact on risk and expected return when an additional risky asset say 'A' is added to the market portfolio 'B' follows from the equation (3.1.1) for the two-asset portfolio and these results are used to derive the asset-appropriate discount rate.

The aim of investors is to maximize anticipated or expected profit and hence it follows the normal probability theories of predictions and for the investor to have higher probability of getting profit has to invest in the portfolio with high return and low risk.

In the Markowitz mean – variance portfolio theory the rate of return on assets are assumed to be random variables. And the goal is to choose the portfolio with optimal weighting factor and which in the Markowitz context is the one with acceptable expected baseline.

Due to the fact that Markowitz follows the principle of random variables in its operations, some statistical concepts were used as follows;-

3.2 Some concepts from elementary statistics

The following theorems from Feller (1968).

Theorem 1: If you have a number of random variables $R_1, R_2, ... R_n$ if R is a weighted sum (linear combination) of all the R_i's i.e. $R = a_1R_1 + a_2R_2 + ... + a_nR_n$ then R is also a random variable

Theorem 2: If $R_1, R_2, ... R_n$ are random variables with expectations, then the expectations of their weighted sum (linear combination) exists and is the sum of their expectation $E(R) = a_1ER_1 + a_2ER_2 + ... + a_nER_n$

That is $E(a_1R_1 + a_2R_2 + ... + a_nR_n) = E(a_1R_1) + E(a_2R_2) + ... + E(a_nR_n)$ . . . (3.5)

Theorem 3: let x be a random variable with finite expected values and C be any constant, then

$$E(cX) = cE(X) \qquad (3.5.1)$$

To define the variance of the weighted sum we have firstly to adopt the definition of the covariance as defined by Hsu (1997) that the covariance of R_i and R_j is denoted as $Cov(R_iR_j)$ or σ_{ij} it is defined by $\sigma_{ij} = E[[R_i - ER_i][R_j - ER_j]]$ this means the expected value of the deviation of R_i from its mean times the deviation of R_j from its mean results to σ_{ij} and this may be expressed in terms of correlation coefficient ρ_{ij}. The covariance between R_i and R_j is equal to their correlation times the standard deviation of $R_i\sigma_i$ times the standard deviation of $R_j\sigma_j$ in short it can be expressed as $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$

Theorem 4: If $R_1, R_2, ... R_n$ are random variables with finite variances $\sigma_1^2, \sigma_2^2, \sigma_3^2, ... \sigma_n^2$ and $R = a_1R_1 + a_2R_2 + ... + a_nR_n$ then $Var(R) = \sum_{i=1}^{n} a_i^2\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{i>1}^{n} a_1a_j\sigma_{ij}$ if we denote σ_i^2 as σ_{ii} then this equation can be rewritten as $Var(R) = \sum_{i=1}^{n}\sum_{i=1}^{n} a_ia_j\sigma_{ij}$ for more details about these theorems proofs the reader is referred to Mayanja, (2011) for example.

According to Feller, (1968) If $R_1, R_2, R_3, ...R_n$ are random variables with expectations, then the expectation of their weighted sum (linear combination) exists and is the sum of their expectation.

$E(R) = a_1E(R_1) + a_2E(R_2) + ... + a_nE(R_n)$. . . eqn (3.5.2) or it can be written as

$$E(a_1R_1 + a_2R_2 + a_3R_3 + ... + a_nR_n) = a_1ER_1 + a_2ER_2 + a_3ER_3 + ... + a_nER_n$$

Where by

R_i is the return in the i^{th} security, μ_i is the expected rate of R_i, σ_{ij} is the covariance between R_i and R_j σ_{ii}

is the variance of R_i

X_i is the percentage of the investors assets which are allocated to the i^{th} security, while the yield of the portfolio as a whole will be found by using the formula $R = \sum_i R_i X_i$.

But you have to note that R_i are the random variables and X_i are fixed by the investor and are percentages so that $\sum_i X_i = 1$. The expectation of R will be $E(R) = \sum_i^n X_i \mu_i$ and the variance is computed by the formulae; $V(R) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} X_i X_j$

The minimization of $V(R)$ at the given level of $E(R)$ for the equation $V(R) = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} X_i X_j$ leads to the Markowitz's mean variance model also known as the modern portfolio theory MPT which has the following assumptions

1. There are n risk assets and there is no risk free asset
2. Price of all assets are exogenous given
3. There is a single time period for selling an asset
4. There are transaction costs and taxes
5. Markets are liquid for all assets
6. Markets are infinitely divisible i.e. you can buy any units of a certain given stock
7. There is full investment i.e. all the stocks are in total operation aiming at providing a return
8. All portfolios are selected according to the mean- variance criterion Mayanja (2011).

3.3 Mean variance analysis

Mean variance analysis is the core of the MPT. For any two assets the mean of the portfolio is given by

$$\mu_p = \sum_{i=1}^n X_i \mu_i \tag{3.6}$$

where

μ_i is the mean $(\overline{R_i})$ of the R_i $\forall_{i=1,2,3,...n}$ and the variance is given by

$$\sigma_p^2 = \sum_{i=1}^n \sum_{j=1}^n \sigma_{ij} X_i X_j \tag{3.6.1}$$

μ_p is the desired level of expected return

σ_p^2 is the desired level of variance

Now letting K to denote a covariance matrix so that

$\sigma_p^2 = X^T K X$ where $X = (X_1 X_2)^T$

For $n = 2$ the model equations becomes

$$\mu_p = \sum_{i=1}^{2} X_i \mu_i \tag{3.6.2}$$

$$= \mathrm{X}_1\mu_1 + \mathrm{X}_2\mu_2$$

$$\sigma_p^2 = (X_1 \quad X_2)\begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix}\begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$

$$= X_1^2\sigma_1^2 + X_1X_2\left(\sigma_{12} \quad \sigma_{21}\right) + X_2^2\sigma_2^2 \tag{3.6.3}$$

μ_i, σ_i and σ_{ij} for $\forall_{i=1,2,3,\ldots n}$ are assumed to be known

$$(\overline{R_1}), (\overline{R_2}), \sigma_1^2, \sigma_2^2 \text{ and } \rho \text{ are known because } \sum_{i=1}^{n} X_i = 1 \tag{3.6.4}$$

Now if the weight of the two assets is $X_2 = \alpha$ then

From $\sum_{i=1}^{n} X_i = 1$ we get $X_1 = 1 - \alpha$ and the equation $\mu_p = \sum_{i=1}^{2} X_i \mu_i$ becomes

$\mu_p = \overline{R_p} = (1-\alpha)\overline{R_1} + \alpha\overline{R_p}$ where $0 \le \alpha \le 1$ and

$X_1^2\sigma_1^2 + X_1X_2\left(\sigma_{12} \quad \sigma_{21}\right) + X_2^2\sigma_2^2$ becomes

$$\sigma_p^2 = \left(1-\alpha\right)^2 \sigma_1^2 + 2\rho\alpha\left(1-\alpha\right)\sigma_1\sigma_2 + \alpha^2\sigma_2^2$$

Hence the model for the two risk assets becomes

$$\overline{R_p} = \left(1-\alpha\right)\overline{R_1} + \alpha\overline{R_2} \text{ for } 0 \le \alpha \le 1 \tag{3.6.5}$$

$$\sigma_p^2 = \left(1-\alpha\right)^2 \sigma_1^2 + 2\rho\alpha\left(1-\alpha\right)\sigma_1\sigma_2 + \alpha^2\sigma_2^2 \text{ for } 0 \le \alpha \le 1 \tag{3.6.6}$$

3.3.1 Solution to the two asset model equation

Then solving these two equations (3.6.5) and (3.6.6) for different values of ρ in the interval $-1 \le \rho \le 1$

From equation $(3.3.6)$ if $\rho = 1$ we get $\sigma_p^2 = \left(1-\alpha\right)^2 \sigma_1^2 + 2\alpha\left(1-\alpha\right)\sigma_1\sigma_2 + \alpha^2\sigma_2^2$ solving for σ_p we get $\sigma_p = \left(1-\alpha\right)\sigma_1 + \alpha\sigma_2$ and if $\alpha = 0$ and 1 respectively we get $\sigma_p = \sigma_1$ and $\sigma_p = \sigma_2$

Again substituting the value of $\alpha = 0$ and $\alpha = 1$ in the first equation we get

$\overline{R_p} = \overline{R_1}$ for $\alpha = 0$ and $\overline{R_p} = \overline{R_2}$ for $\alpha = 1$ this implies that for $\alpha = 0$ the portfolio value is $P_0\left(\sigma_1 \quad \overline{R_1}\right)$ and for $\alpha = 1$ the portfolio value is $P_1\left(\sigma_2 \quad \overline{R_2}\right)$ for $\alpha = 0$ means all the investments are in 1 stock and for $\alpha = 1$ means all the investments are in the second stock because $-1 \le \rho \le 1$ substituting

$\rho=-1$ into the equation $\sigma_p^2=(1-\alpha)^2\sigma_1^2+2\alpha\rho(1-\alpha)\sigma_1\sigma_2+\alpha^2\sigma_2^2$

we get $\sigma_p^2=(1-\alpha)^2\sigma_1^2+2\alpha(1-\alpha)\sigma_1\sigma_2+\alpha^2\sigma_2^2$ after solving this equation we get $\sigma_p=\left|(1-\alpha)\sigma_1-\alpha\sigma_2\right|$ and again when $\alpha=0$ we get the point $P_0\left(\sigma_1 \quad \overline{R_1}\right)$ and when $\sigma_p=0$ i.e $(1-\alpha)\sigma_1-\alpha\sigma_2=0$ solving for α from this equation we get $\alpha=\dfrac{\sigma_1}{\sigma_1+\sigma_2}$ and

$\overline{R_p}=\left[1-\dfrac{\sigma_1}{\sigma_1+\sigma_2}\right]\overline{R_1}+\left(\dfrac{\sigma_1}{\sigma_1+\sigma_2}\right)\overline{R_p}$ hence the portfolio value becomes

$A\left(0,\left(1-\dfrac{\sigma_1}{\sigma_1+\sigma_2}\right)\overline{R_1}+\left(\dfrac{\sigma_1}{\sigma_1+\sigma_2}\right)\overline{R_p}\right)$ when plotting in the graph it will be easily seen that the point Ap_1 corresponds to the equation $\sigma_p=(1-\alpha)\sigma_1-\alpha\sigma_2$ and we have to note that the value of $(1-\alpha)\sigma_1-\alpha\sigma_2$ remain positive until when $\alpha=\dfrac{\sigma_1}{\sigma_1+\sigma_2}$ and when $\alpha>\dfrac{\sigma_1}{\sigma_1+\sigma_2}$ the quantity $(1-\alpha)\sigma_1-\alpha\sigma_2$ becomes negative leading to $(\alpha-1)\sigma_1+\alpha\sigma_2$ and as α approaches to 1, $\sigma_1=\sigma_2$ which gives the corresponding point close to $P_2\left(\sigma_2 \quad \overline{R_2}\right)$ and the locus will trace out the line AP_1

Lastly we have to consider the interval $-1<\rho<1$ (this is the case of real data as those that will be obtained at DSE).

The minimum variance point are obtained by solving the equation $\dfrac{\partial\sigma_p{}^2}{\partial\alpha}=0$

$$\frac{\partial\sigma_p{}^2}{\partial\alpha}=\frac{\partial}{\partial\alpha}\left((1-\alpha)^2\sigma_1^2+2\alpha\rho(1-\alpha)\sigma_1\sigma_2+\alpha^2\sigma_2^2\right)$$
$$=2(1-\alpha)\sigma_1^2+2\rho(1-\alpha)\sigma_1\sigma_2-2\rho\alpha\sigma_1\sigma_2+2\alpha\sigma_2^2$$

But $\dfrac{\partial\sigma_p{}^2}{\partial\alpha}=0 \Rightarrow -2(1-\alpha)\sigma_1^2+2\rho(1-\alpha)\sigma_1\sigma_2-2\rho\alpha\sigma_1\sigma_2+2\alpha\sigma_2^2=0$ now solving for α we get $\alpha=\dfrac{\sigma_1{}^2-\rho\sigma_1\sigma_2}{\left(\sigma_1{}^2-2\rho\sigma_1\sigma_2+\sigma_2{}^2\right)}$ and if we substitute the value of α in the equations $(3.3.5)$ And $(3.3.6)$ it provides the minimum variance point $\mathrm{p}_{\min}\left(\sigma_{p_{\min}},\overline{R_{p_{\min}}}\right)$ hence the solution to the two assets model in a mean standard deviation $\left(\sigma_p-\overline{R_p}\right)$ can be presented in the plane as below

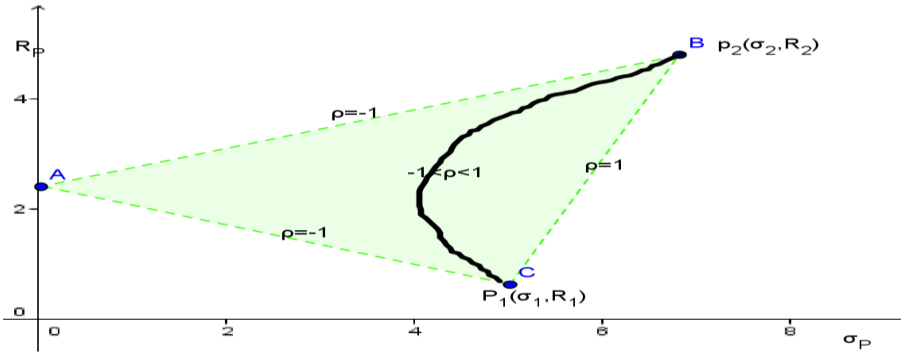

Figure 1

In this diagram we determine the minimum variance so that we can represent the real data because real data have $-1<\rho<1$ that is in the extremes of the interval it can never be achieved for example the point with $\sigma_p = 0$ cannot be achieved because it correspond to the value of $\rho = -1$

Hence the region CAB defines the strict boundaries for the portfolio and so the inside region is the feasible region that can be achieved.

3.3.2 Conclusion for the 2 asset model analysis

First as α (weight proportion) varies the locus of $\left(\sigma_p, \overline{R_p}\right)$ traces out a hyperbola curve in the $\left(\sigma_p - \overline{R_p}\right)$ plane.

Secondly when $\rho = -1$ it is possible to have $\sigma_p = 0$ for some choices of $\alpha = \dfrac{\sigma_1}{\sigma_1 + \sigma_2}$ however since for real data it is not possible to have $\rho = -1$ we can say in general that putting two assets whose returns are negatively correlated has a desirable effect of lowering the portfolio risk at a given level of return. The validity of these theoretical conclusions are going to be tested with the real data from DSE and its results will be shown

4. Results and Discussion

The data was sorted so as to arrange them for each stock and from this the log returns of each stock was calculated. The range was calculated by using the maximum and minimum values of the log returns, and this helped us to ascertain the intervals for the log returns of each stock, using these intervals we determined the frequencies of the log returns using the excel in built function 'FREQUENCY'. Using these frequencies the cumulative frequencies were calculated using the formula;

$$cumulative\ frequence, cf_i = \frac{frequence,\ f_i}{total\ frequence} x100\%$$,and this provides the actual cumulative frequencies for the historical data of the stock. Then using the same mean and standard deviation for each of the stock we simulated the cumulative frequencies for a normally distributed data set. To produce cumulative frequencies that are normally distributed for the given mean and standard deviation of the data set excel's 'NORMDIST' function was used. We then plotted the actual cumulative frequencies of the historical data and the simulated normal distribution frequencies on the same graph, for each stock. The resulting graphs are as shown in figures below

The TTCL cumulative and simulated cumulative frequency curves

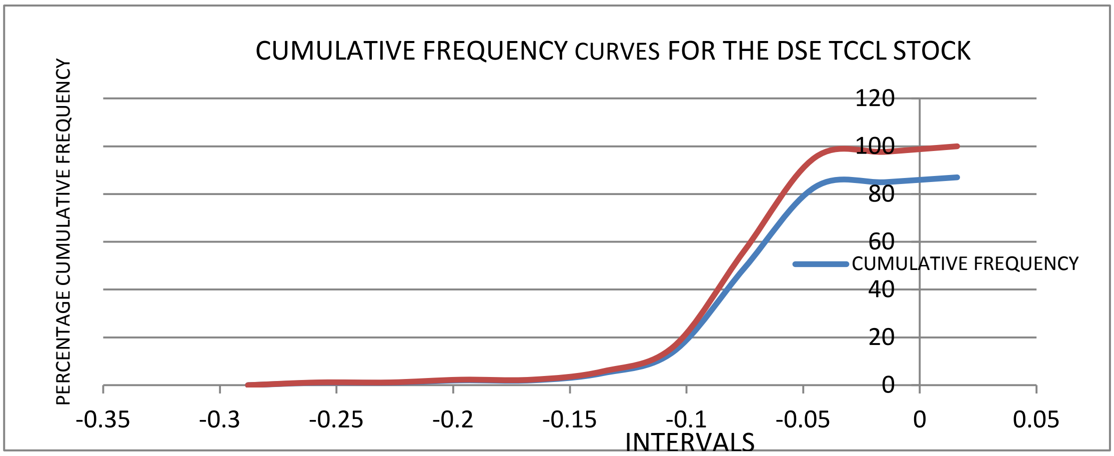

Figure 2

The red line represents the cumulative frequencies and the blue line represents the simulated normal distribution of the cumulative frequency values. As seen from the graph the simulated normal distribution graph seem to be superior to the cumulative frequencies graph.

The TBL cumulative frequency and the simulated cumulative frequencies graphs

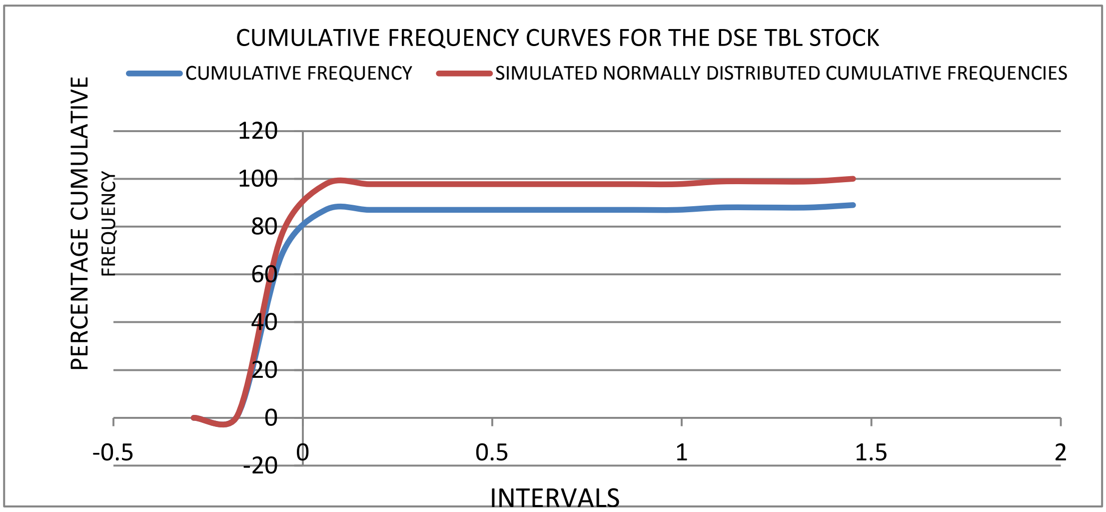

Figure 3

If we observe between these two graphs for the TTCL and the TBL we can see that the TTCL graph is more concentrated to the left of zero and this is because we used the log returns in calculating the intervals for which most of them were negative and for the TBL most of them were positive.

For the TCC Company the cumulative frequency and simulated normal distribution cumulative graphs are as below

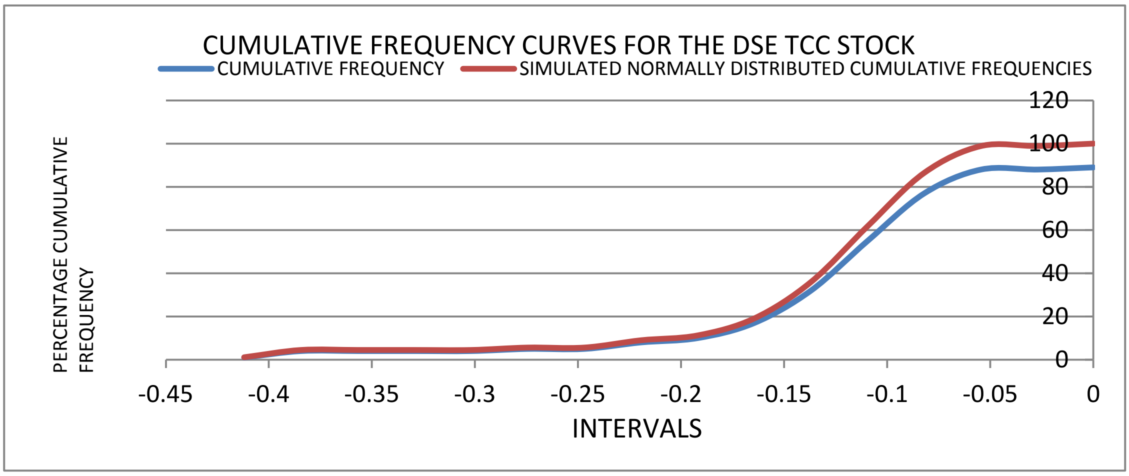

Figure 4

The cumulative and simulated normal distribution graphs for the TOL stocks

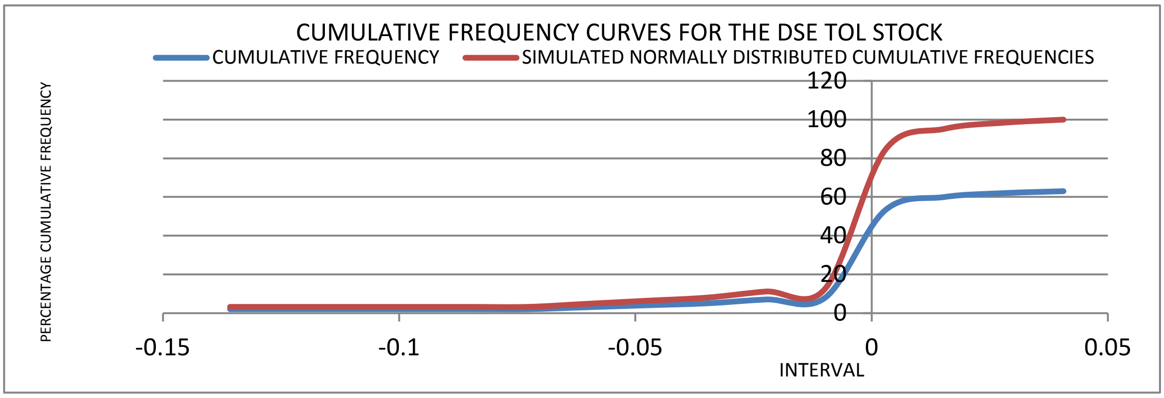

Figure 5

Also for the TOL and TCC companies we observed that the graphs concentrated more on the left this is caused by the fluctuation in the prices causing the average monthly return to be negative. Also we can see that there is abruptly rise in graphs which is caused by abrupt increase in prices.

The TPCC cumulative frequency and simulated normally distributed graphs are as below

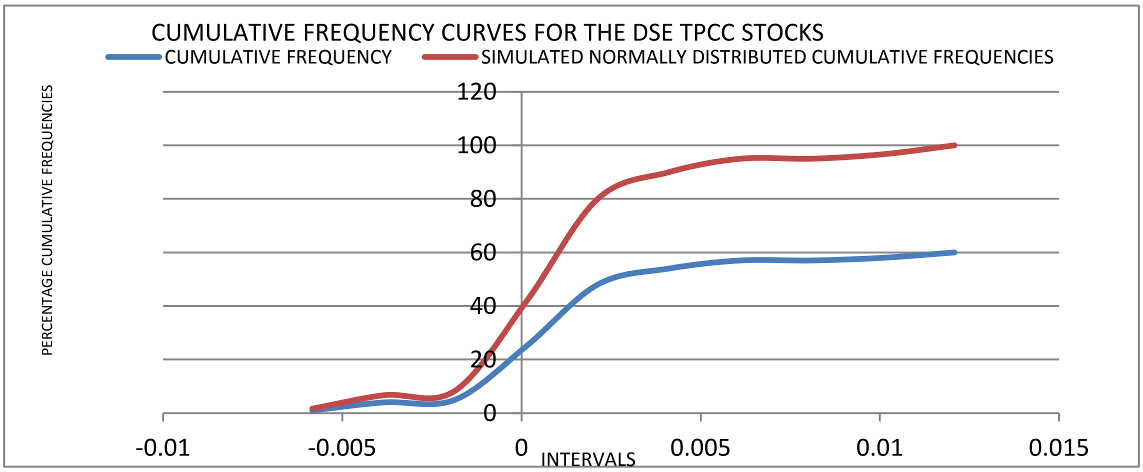

Figure 6

The TTP cumulative frequency graphs as well as the simulated normal distribution graphs. are shown in the figure below.

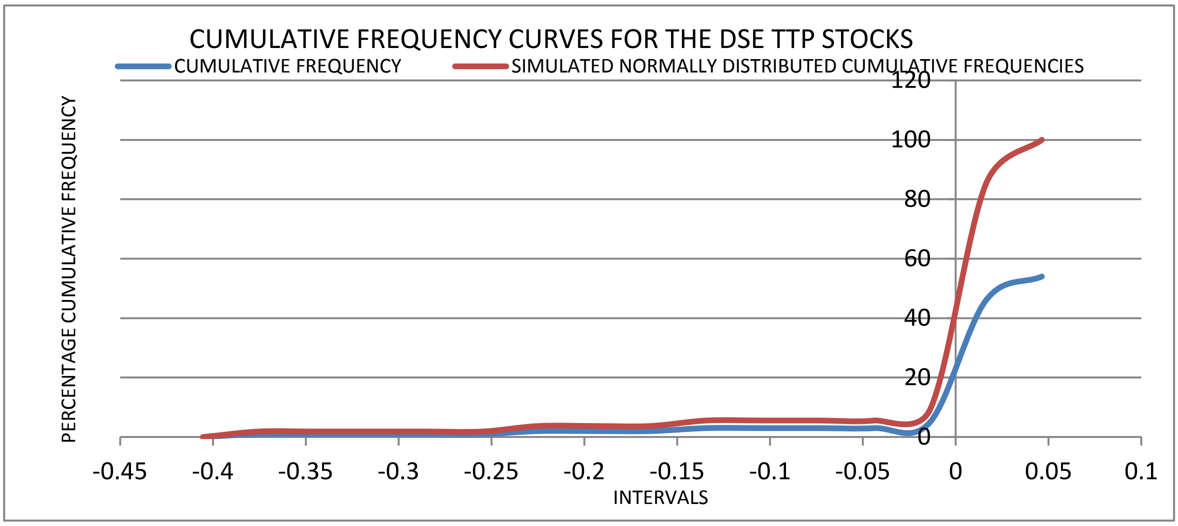

Figure 7

TPCC and TPP companies also behave in the same way as other companies. That is the greater part of the graph is concentrated to the left showing that the average monthly returns were negative and it were caused by the change in prices.

For the CRDB Company stock the graphs for the cumulative frequencies and the simulated normal distribution cumulative frequency are also as below

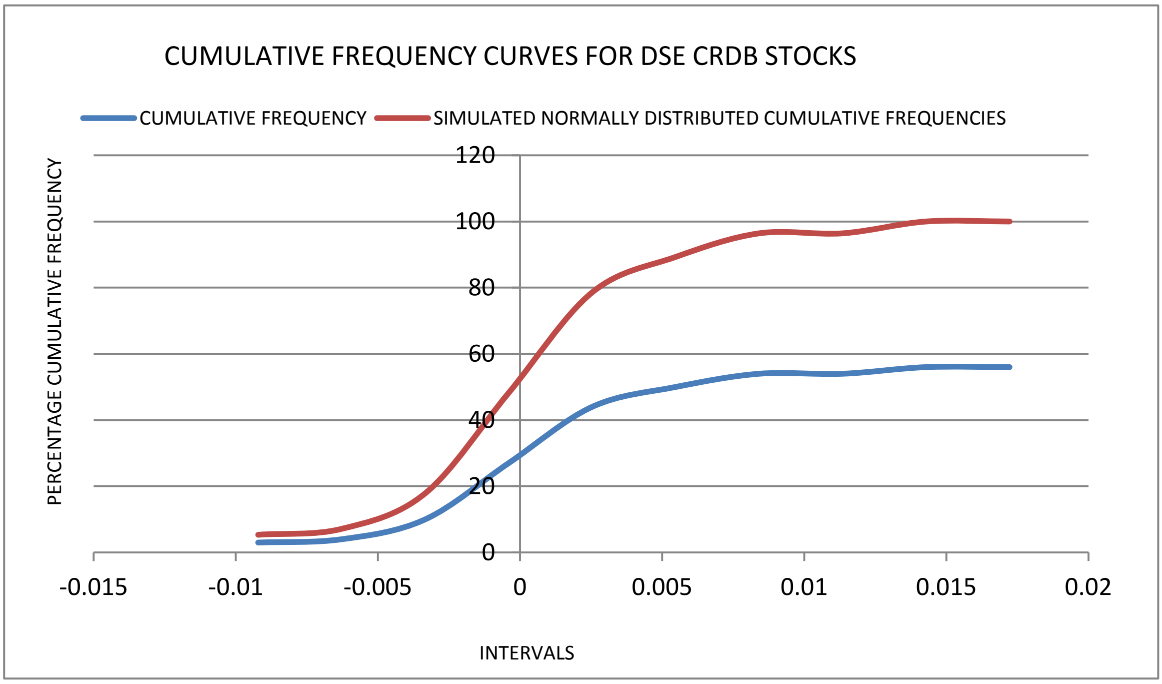

Figure 8

For DCB stock the graphs for the cumulative frequencies and the simulated normal distribution cumulative frequency are also as below

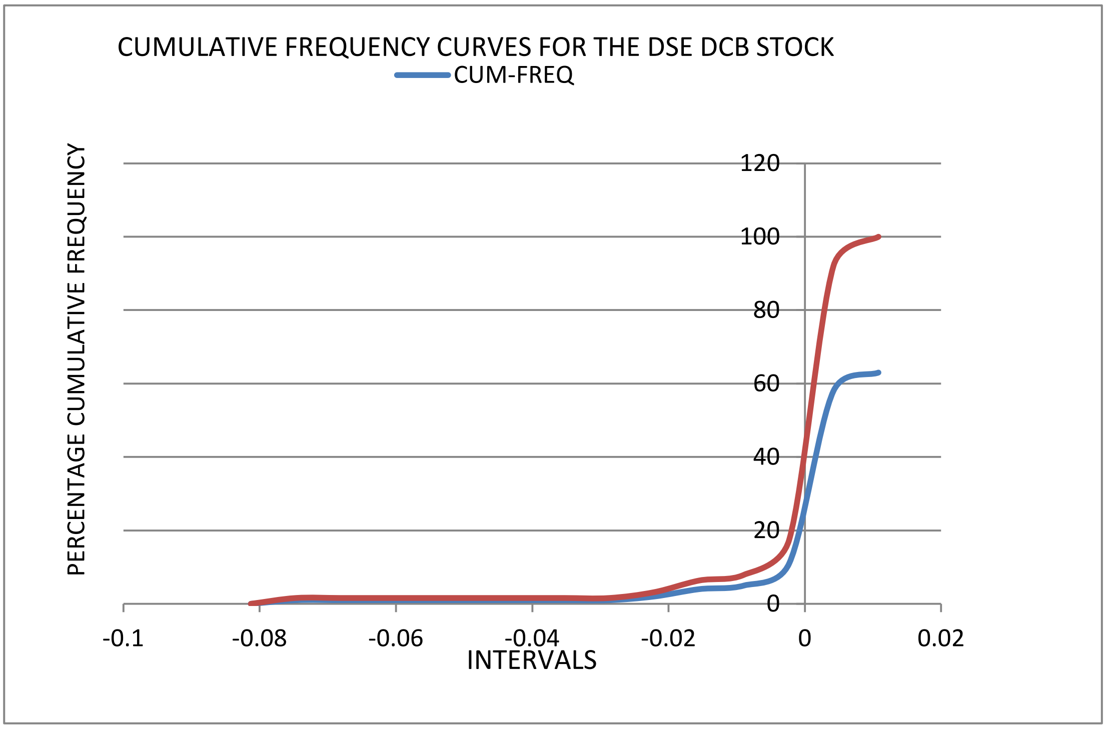

Figure 9

Comparing the CRDB and the DCB stock graphs we can observe that the graph for the CRDB is more concentrated to the right of zero showing that there was an increase in prices from time to time and for the DCB the increase was just abrupt showing that at one time the prices rises significantly while in most of the time was almost constant.

For the GOV the graphs for the cumulative frequencies and the simulated normal distribution cumulative frequency are also as below

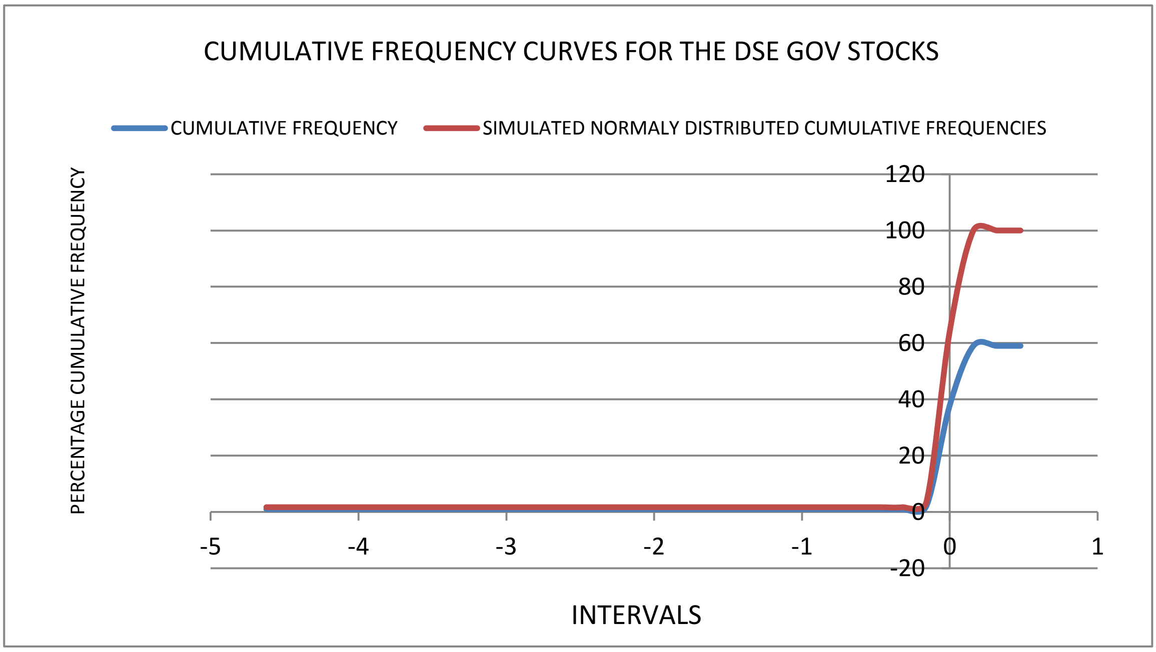

Figure 10

For the NMB the graphs for the cumulative frequencies and the simulated normal distribution cumulative frequency are also as below

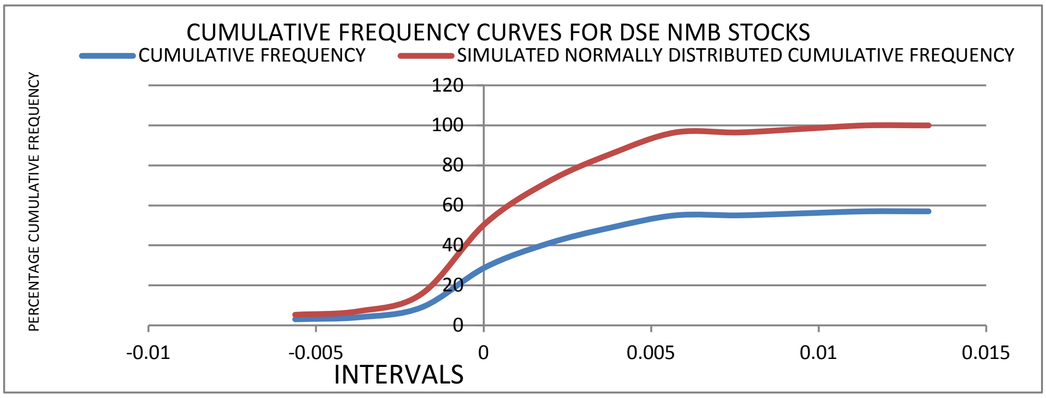

Figure 11

For the SWISS the graphs for the cumulative frequencies and the simulated normal distribution cumulative frequency are also as below

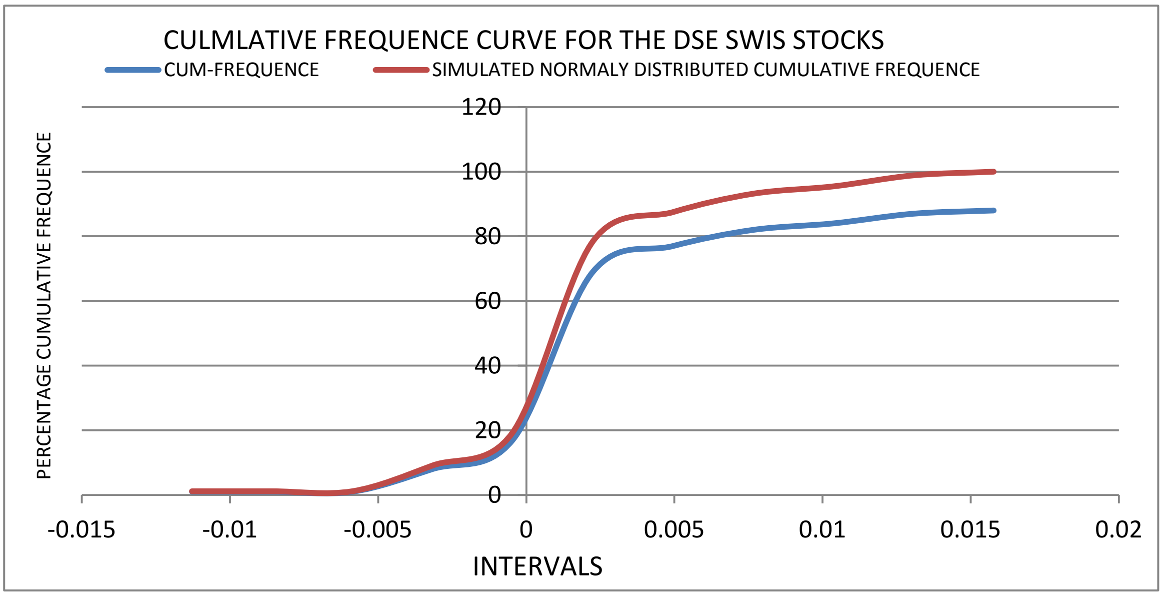

Figure 12

As we observe from all the graphs it is well noted that the simulated normal distribution graphs are superior to the cumulative frequency graphs. Also, it can be noted that there are some deviations from the graphs which can be caused by the stock selling's which are at higher level in some time and at lower level at another time. For example for the NMB it was observed that there was a high change in price which can be caused by different market conditions, consumer interest, change in consumer's income, inflation rates and other factors that prevail at the market place. Also we can observe that for the GOV stock the prices was almost constant and there was a high increase in price and causing the greater change in the nature of the graph. All these factors do show that markets are not static and do change according to some factors. To confirm that there was a high deviation in some stocks prices we plotted the normal stock values as below.

The normal stock values plot

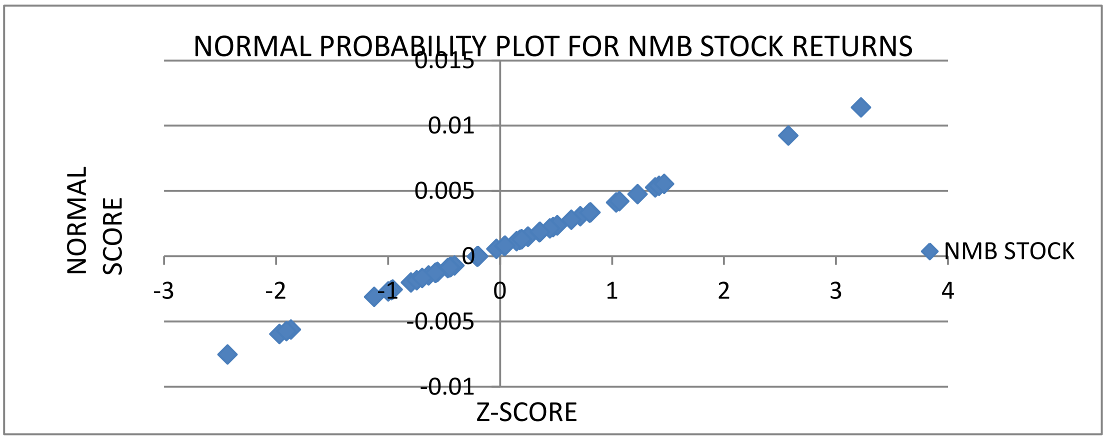

Figure 13

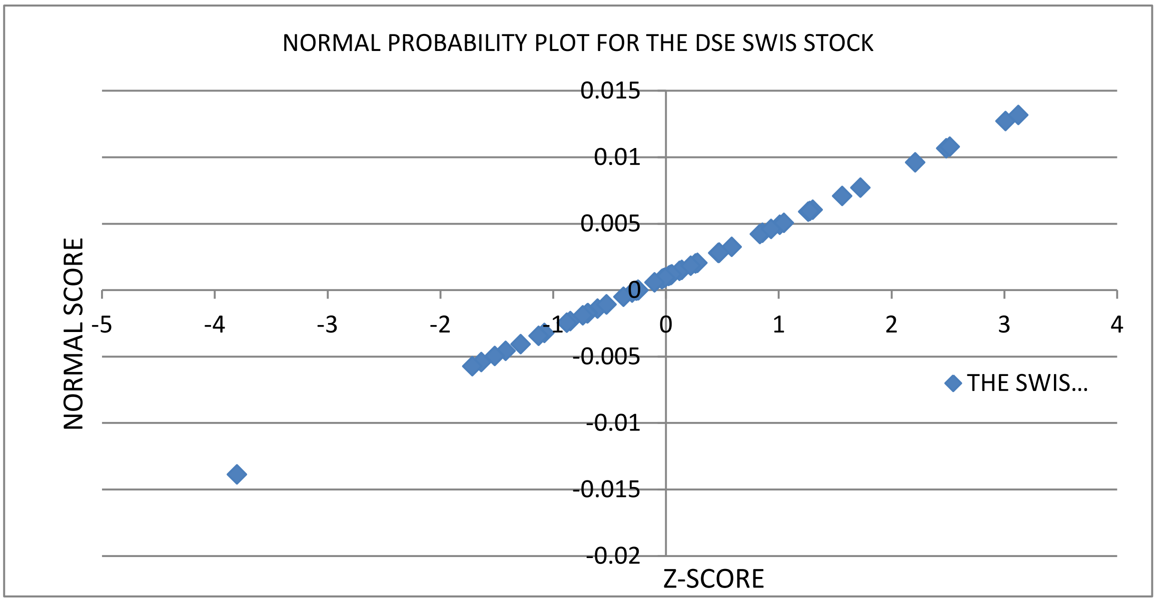

Figure 14

From these normal plot we observed that both of the two graphs approximate to a straight line with high concentration in some parts of the graph showing that the changing in prices was minimum for some months and there was a high changer in some of the months but in general conclusion we can see that the high concentration is along the center showing that most of the average monthly returns are along the mean.

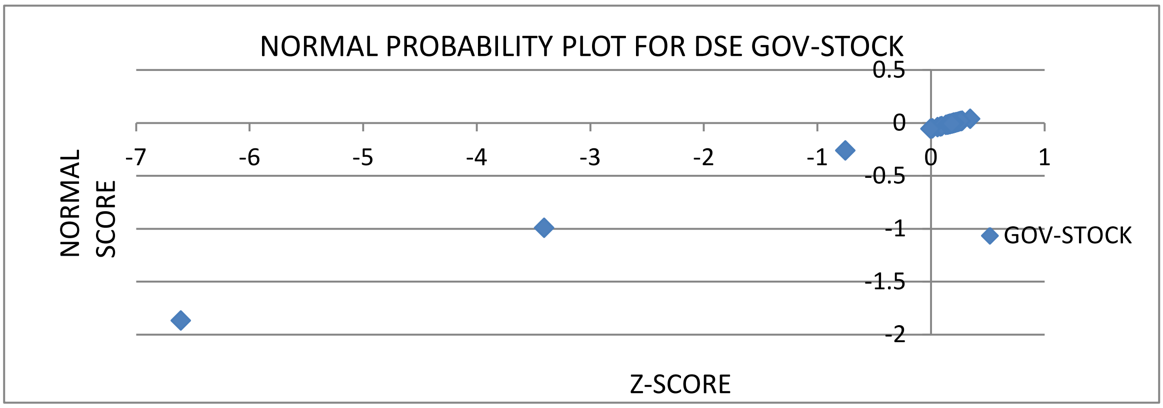

Figure 15

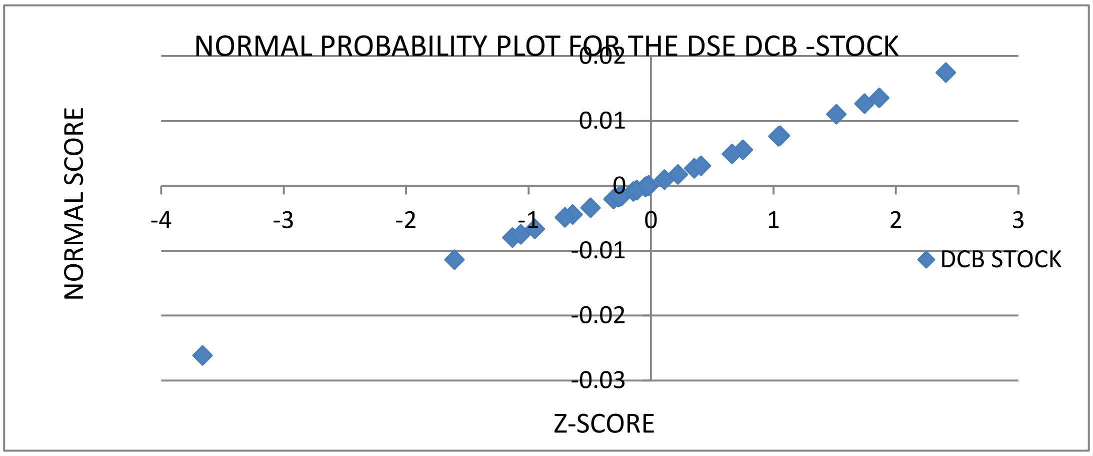

Figure 16

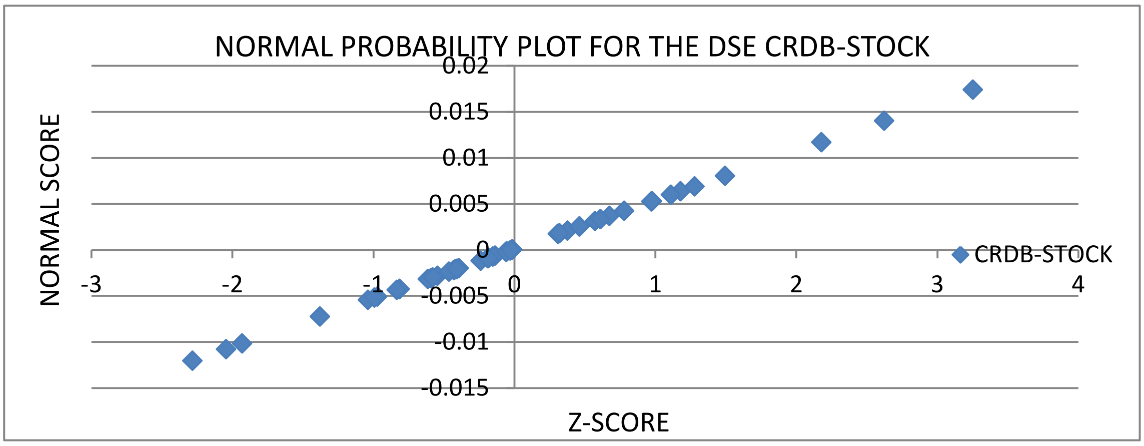

Figure 17

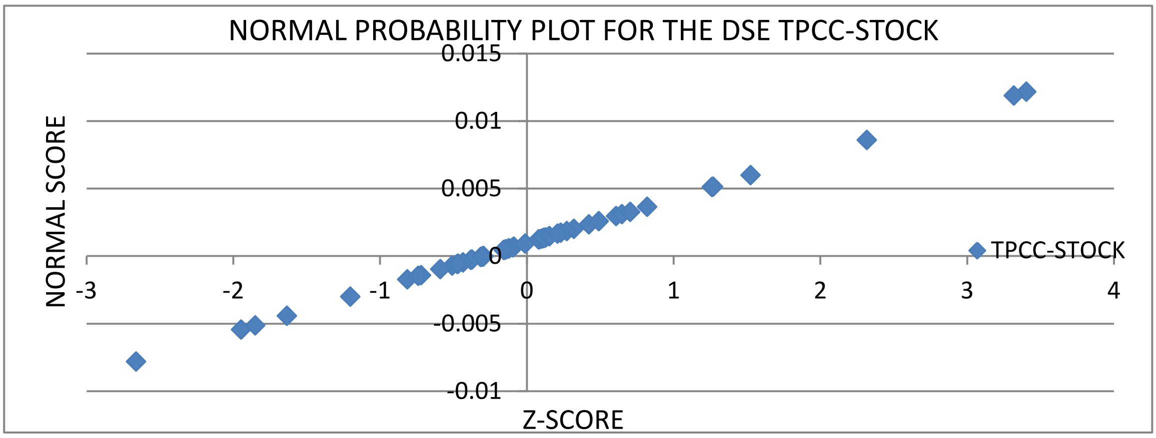

Figure 18

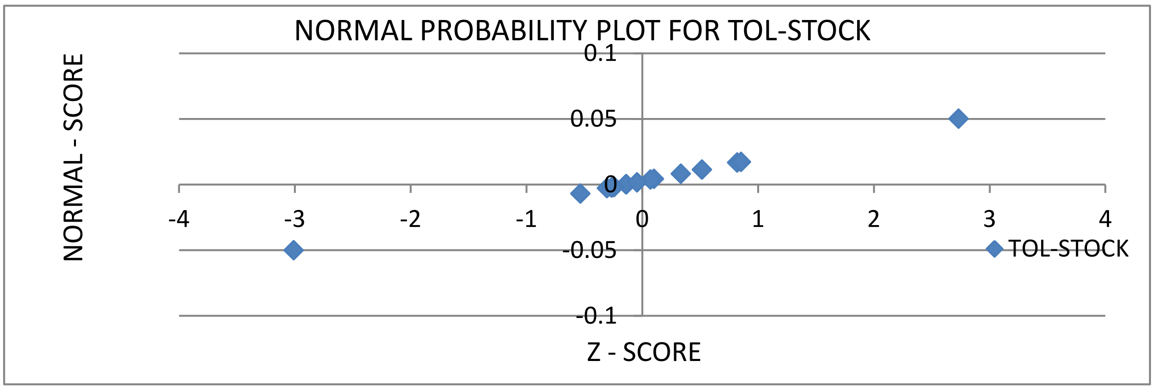

Figure 19

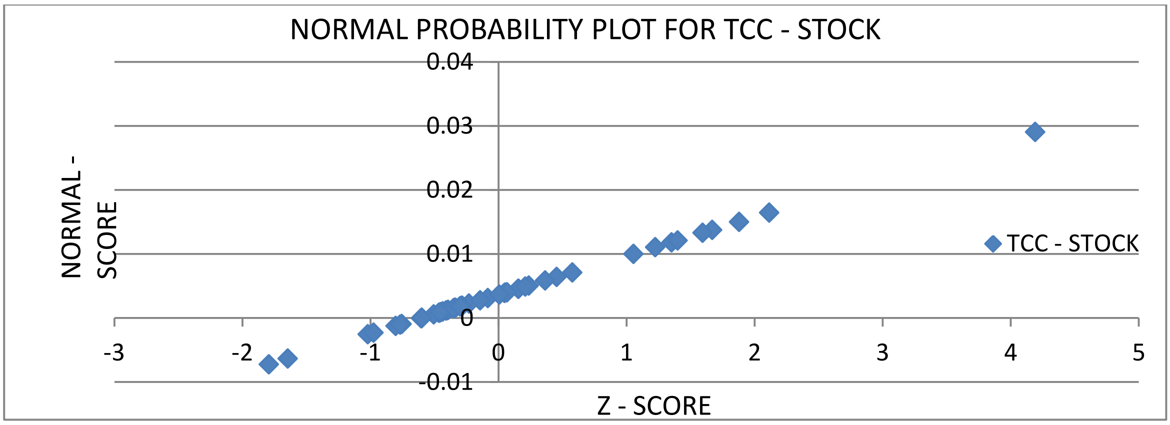

Figure 20

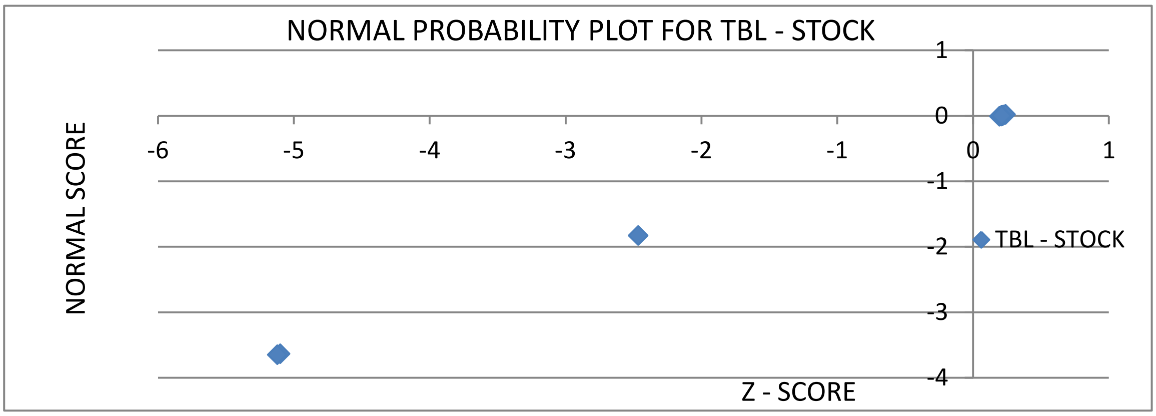

Figure 21

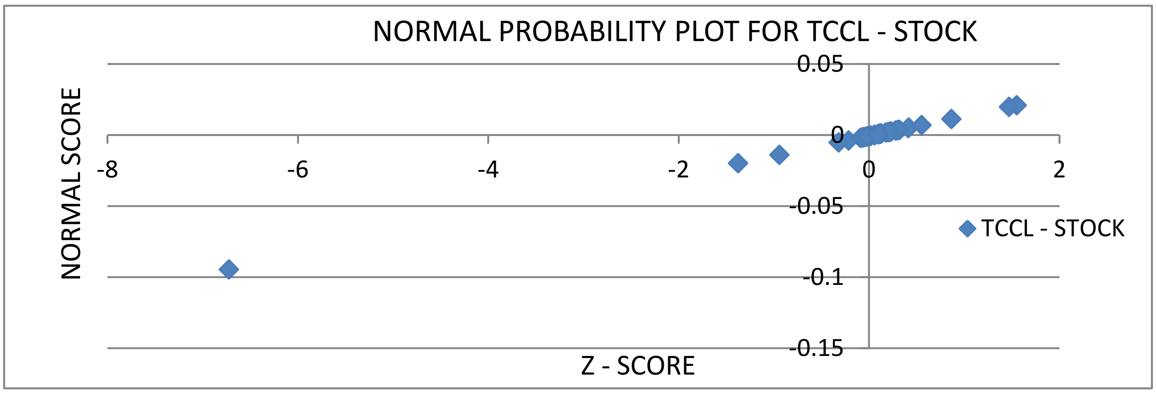

Figure 22

Also from these normal plots you can easily see that they are approximately straight lines which do correspond to the theoretical conclusions that the graphs of the normal distribution are a straight line with positive slop. Also most of the dots are concentrated along the zero x-axis with little variation in the y-axis showing that the average monthly returns are near to the mean of the whole stock. This also shows that the extreme deviations of the plots of some stocks are due to the fluctuation of the prices. To prove this we plotted the stock log returns graphs for each stock for the period the stock was in operation to see how the stock was performing and the resulting graphs are as follows.

The DSE stock performance

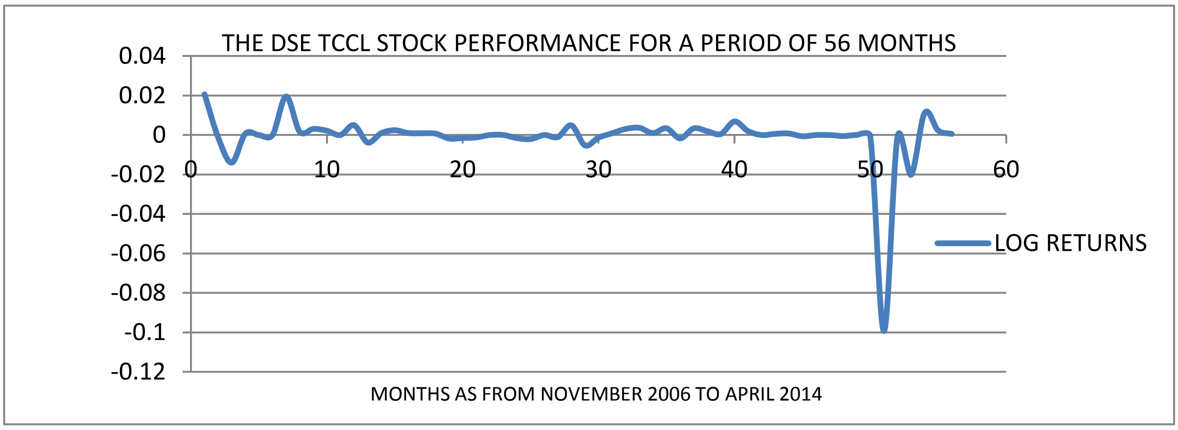

Figure 23

As you can observe there is a month with which its average performance was high and others its performance was low and hence resulting to these small, small deviations.

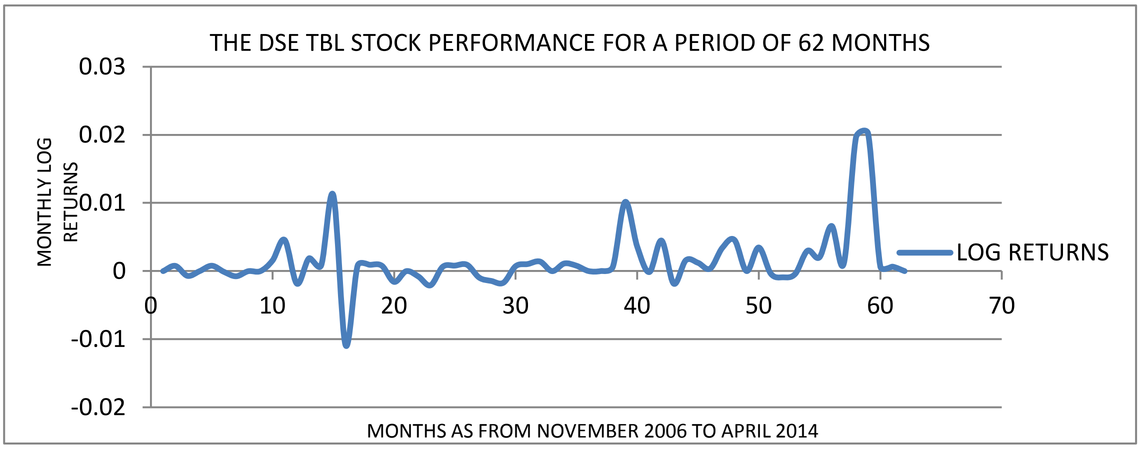

Figure 24

As we can observe for the TTCL and TBL stock, the TTCL has approximately one high deviation and the TBL has almost two extreme deviations one is caused by high increase in price.

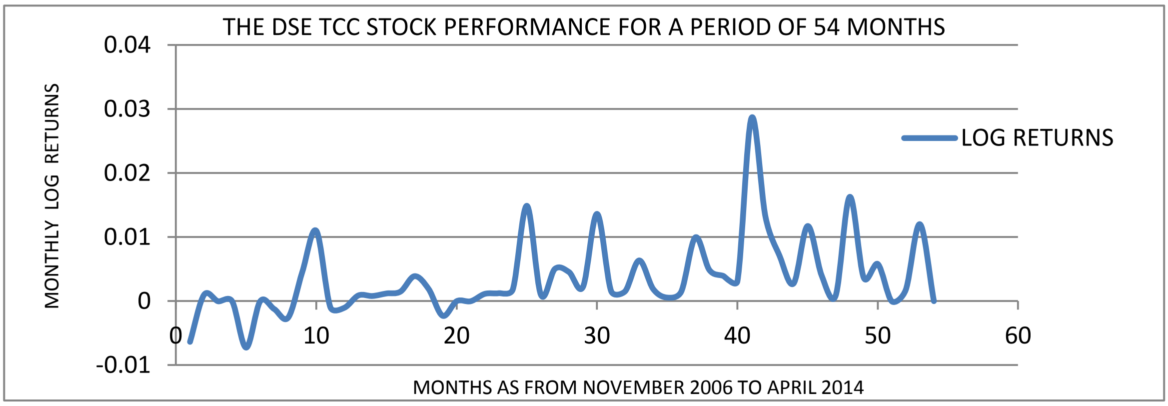

Figure 25

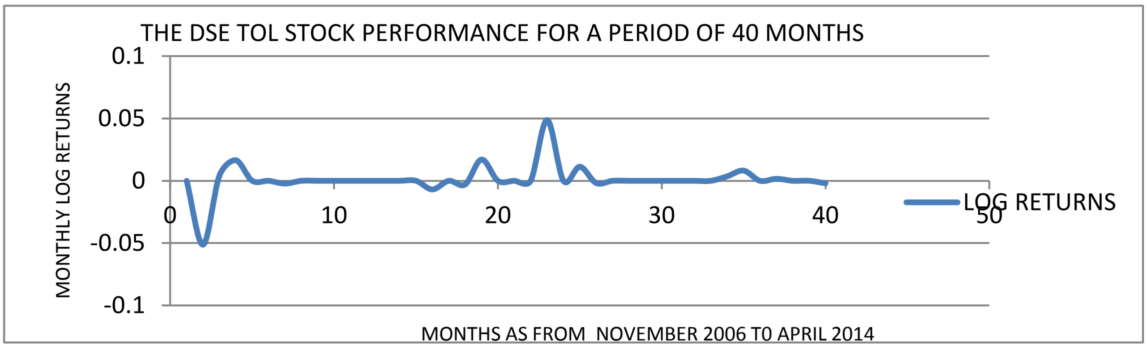

Figure 26

As we can observe for the TCC and TBL stock, the TOL has approximately one high deviation showing that there was a month with high average return than the normal and the TBL has almost two extreme deviations one is is on the positive side showing that there was a month with higher average return compared to others and the other to the negative side showing that there was a month with lower average return compared to others, and all these are caused by price fluctuations.

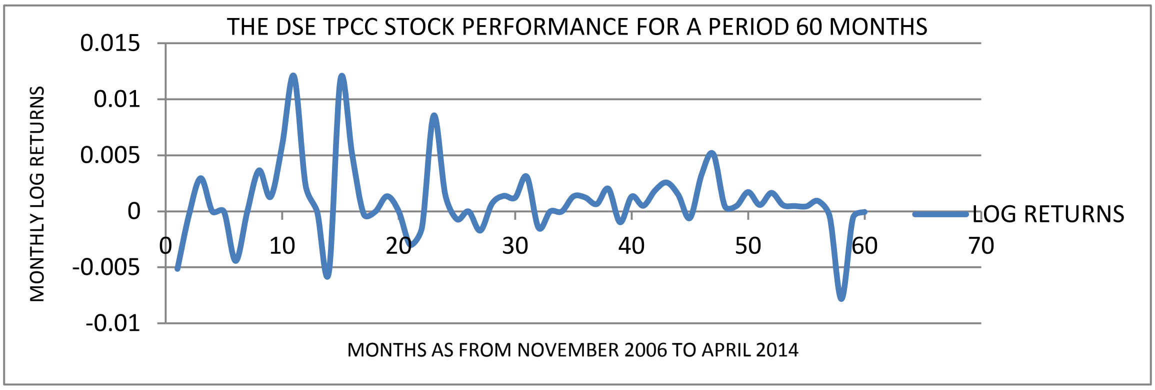

Figure 27

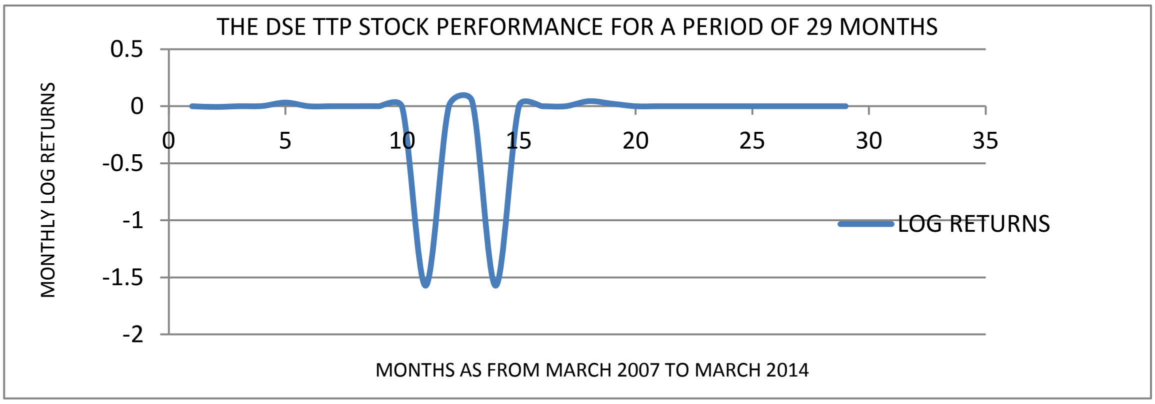

Figure 28

As we can observe for the TPCC and TBL stock, the TTP has approximately four high deviations showing that there was four months with high average return than the normal two months were having high prices and the other two were having low prices. The TBL has almost two extreme deviations on the negative side showing that there was two months with lower average return compared to others, and all these are caused by price fluctuations.

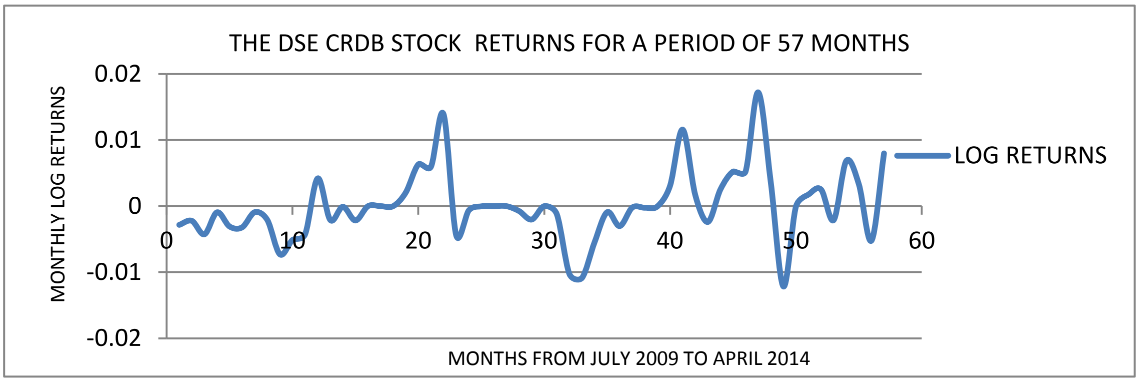

Figure 29

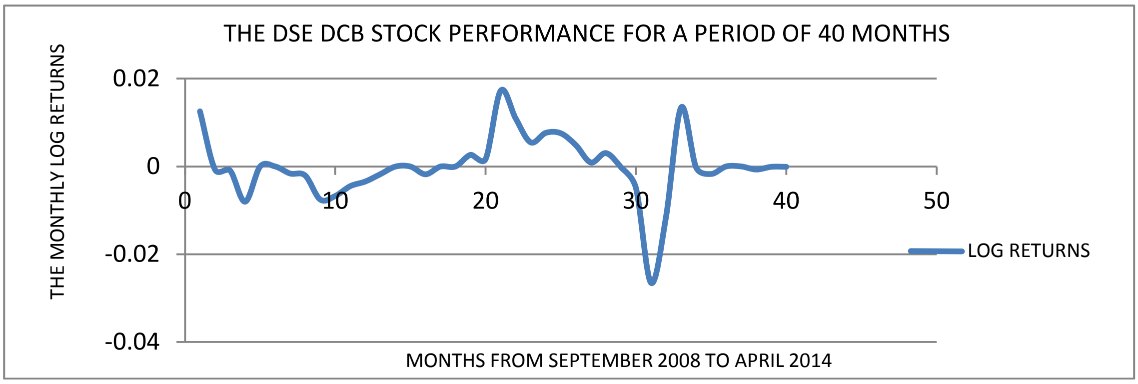

Figure 30

As we can observe for the CRDB and DCB stock, the CRDB has approximately three high deviations showing that there was two month with high average return than the normal and one month with low average return than the normal and the general outlook of the stock seem to be fluctuating more than other stocks. For the TBL has almost three extreme deviations one on the negative side showing that there was a month with low average return compared to others and others to the negative side showing that there was a month with higher average return compared to others, and all these are caused by price fluctuations.

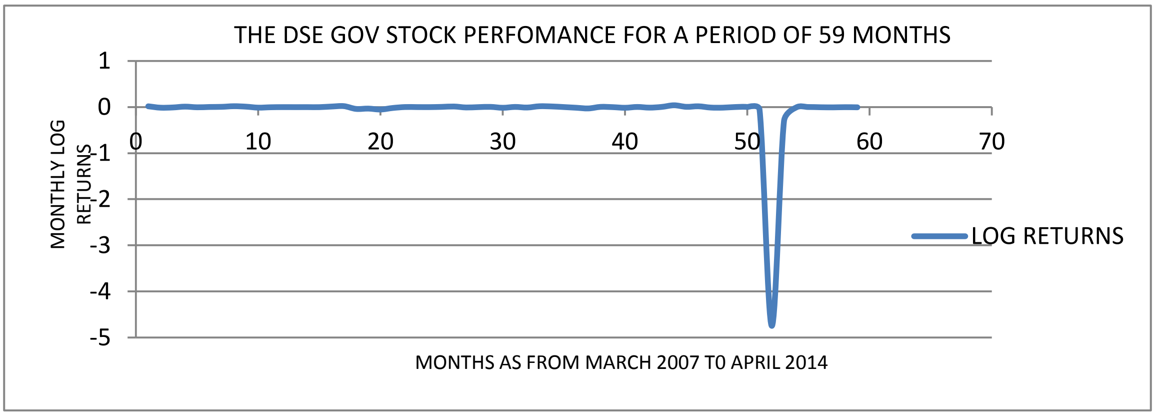

Figure 31

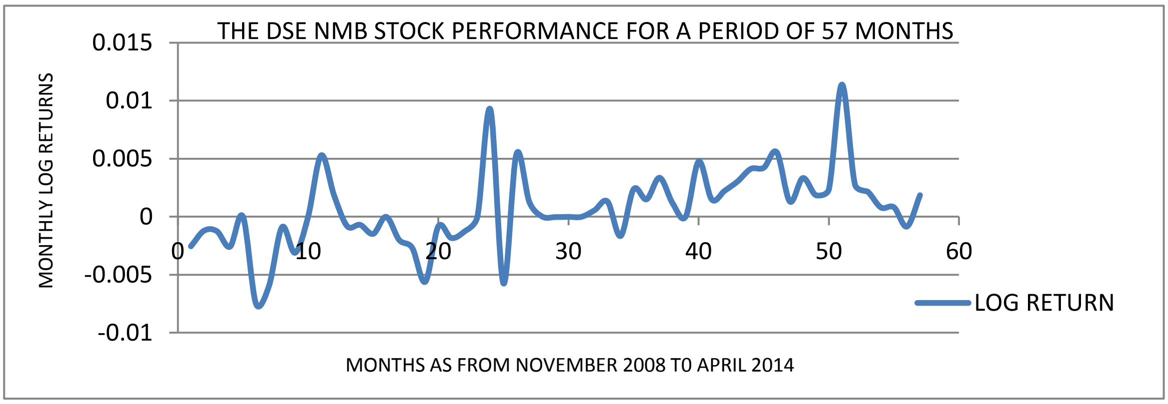

Figure 32

For the GOV and NMB it is found that the GOV stock has one extremely which was caused by high increase in price in one month and on the other months the prices are almost constant. On the other hand the NMB stock has fluctuating behavior which shows that the prices are not stable they increase in one month and decrease in the other causing almost three positive deviation and three negative deviation

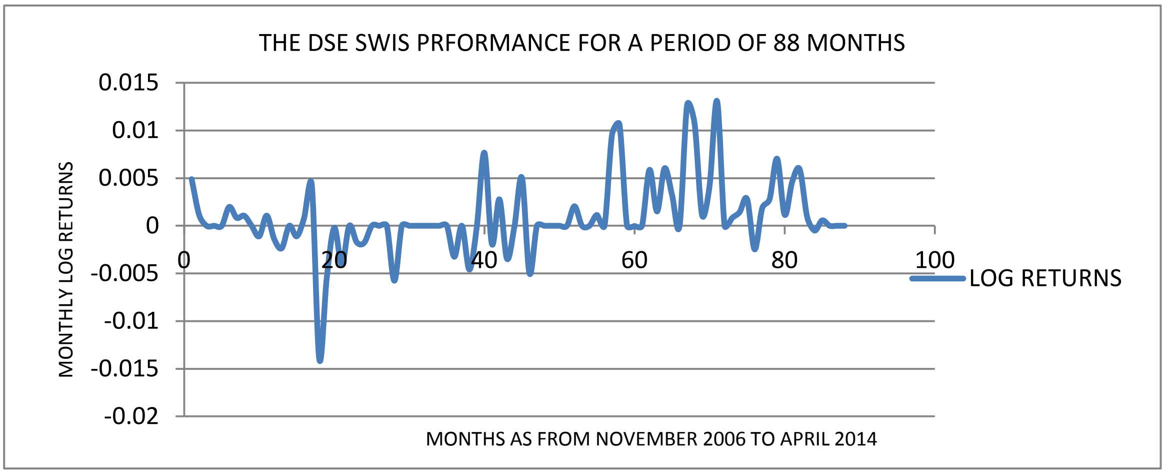

Figure 33

The DSE SWISS stock is the most fluctuating stock with increase and decrease in prices.

Generally as well shown from the graphs it is observed that there are some noticeable changes in stock performance which results to changes in the normal plots and hence we can confirm that the DSE log returns of stocks are normally distributed. Also we can prove this by checking and calculating the skewness and the kurtosis effect but according to our case it cannot be a good means because as in Mayanja (2011) these parameters do provide answers according to the nature of the data you have used, even for the same dada choosing different samples it can provide different values hence if we use these values for our data it cannot provide a certain answer for the variations of the stock parameters.

4. Conclusion and recommendations

As observed from the graphs it shows that the log returns of the DSE stocks are normally distributed. Also we can observe that in every graph from figure 23 to figure 33 there is extremely high deviations, this means that every stock has some months with high prices and low prices. These deviations cause the deviation in the normality plots. Also we have to note that for the real data like these from DSE deviations are expected. From this reason we can comfortably conclude that the log returns of the DSE stocks are intelligibly normally distributed. This result corresponds to the literatures that log returns are normally distributed Elton, Graber, (1974).

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