An Inventory Model For Non – Instantaneous Deteriorating Products With Price And Time Dependent Demand

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ABSTRACT

Many inventory models have been developed by many researchers for deteriorating products. But certain products do not deteriorate immediately. In fact the deterioration starts after certain period of time. This kind of deterioration is known as non – instantaneous deterioration. In the present paper an inventory model for non – instantaneous deteriorating products with price and time dependent demand have been given. Shortages are allowed and completely backlogged. Numerical examples have been considered to demonstrate the effectiveness of the model.

Keywords: Price and time dependent demand, non – instantaneous deterioration, inventory, shortages

1. INTRODUCTION

Many inventory models have been study by the researchers in past. Most of the products deteriorate when stored for certain times. Ghare and Schrader [8] proposed an economic order quantity (EOQ) model for products which decays exponentially. Covert and Philip [4] extended their model by introducing variable rate of deterioration. Shah [15] further extended the above models by considering the complete backlogging of the inventory. Many researchers have developed different models to control the inventory of deteriorating products under various situations namely Dave [6], Dave and Patel [7], Hollier and Mak [9], Kang and Kim [11], Sachan [14], Datta and Pal [5], Abad [1], Aggarwal and Jaggi [2] and others.

Cohen [3] developed a model for joint pricing and ordering policy for exponentially decaying inventory with known demand and constant decay rate. Mukherjee [13] extended it by considering time varying decay rate. Further, Kaur and Sharma [12] developed an inventory model of deteriorating products with price and time dependent demand.

Many authors have developed their models by assuming that the products delivered are non – deteriorating or the deterioration of the product is instantaneous. Product starts deteriorating as soon as the retailer gets the product delivered. But, in real life there are very limited products which do not deteriorate at all or starts deteriorating immediately. Generally products start deteriorating after certain time. This kind of deterioration is known as non – instantaneous deterioration. Recently Wu, Ouyang and Yang [16] and Jaggi C. K. [10] have developed the inventory models for non – instantaneous deteriorating products.

In this paper, an inventory model for non – instantaneous deteriorating products with demand depends on both price and time is considered. There are certain products such as milk, vegetables, fruits etc whose deterioration starts after certain period or the deterioration is non – instantaneous. Further, demand is considered as function of selling price as well as time.

2. ASSUMPTIONS AND NOTATION

The following notations are used here.

P	Selling price per unit
θ	A constant governing the decreasing rate of demand
$D(t, p)$	The demand rate which is a decreasing function of time and selling price p where demand rate is given by $D(t,p) = d(p)e^{-\theta t}$ where $d(p) = a - p$ is function of selling price
Q	The Replenishment quantity
t_d	The length of time during which the product has no deterioration
t_1	The time at which the inventory level becomes zero
c	Purchasing cost per unit item
T	The length of replenishment cycle
c_1	Shortage cost per unit per unit time

Q_1	The inventory level at time t where $t \in (0, t_d)$
Q_2	The inventory level at time t where $t \in (t_d, t_1)$
Q_3	The inventory level at time t where $t \in (t_d, T)$
TP	Total profit
λ	Deteriorating rate and $0 < \lambda < 1$
h	Holding cost (excluding interest charges) per unit per unit time
I	The inventory size after clearing the backlogs.
S	Shortage of inventory
A	Ordering cost per cycle

The following assumptions are used for this model.

1. Demand is a function of time as well as selling price.
2. Replenishment is instantaneous with known constant leading time.
3. The time horizon of the inventory is infinite.
4. Shortages are allowed and completely backlogged.
5. Holding cost is constant.
6. There is no replacement or repair of decayed units during the period of consideration.
7. A single product is considered.

3. MATHEMATICAL MODELLING AND ANALYSIS

Initially, a lot size of Q units enters the system. Out of which some units are used to fulfill the backlogs. After meeting the backorders only I units enter the inventory system at time $t = 0$. As the deterioration is non – instantaneous so for the time period $0 \le t \le t_d$, no product gets deteriorated. Deterioration of inventory starts after time $t = t_d$.

So initially, inventory reduces only due to demand. Further, during the time interval (t_d, t_1) inventory becomes zero due to the combined effect of demand and deterioration. Now, shortages occur for the time interval of (t_d, T). At the time of replenishment total shortage is S. The following figure describes the behavior of the inventory system.

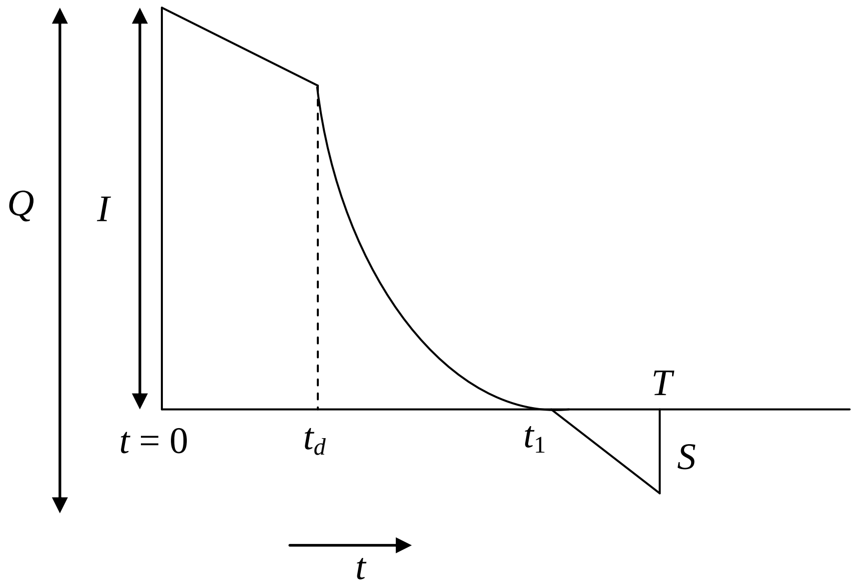

Figure 1 Graphical representation of inventory system

The differential equations describing the inventory level over the time interval (0,T) are given by

$$\frac{dQ_1}{dt} = -d(p)e^{-\theta t}, 0 \le t \le t_d \qquad (1)$$

$$\frac{dQ_2}{dt} + \lambda Q_2 = -d(p)e^{-\theta t}, t_d \le t \le t_1 \quad (2)$$

$$\frac{dQ_3}{dt} = -d(p)e^{-\theta t}, t_1 \le t \le T \quad (3)$$

With the boundary conditions

1. $Q_1(0) = I$ (4)
2. $Q_2(t_1) = 0$ (5)
3. $Q_3(t_1) = 0$ (6)

Solutions of the above differential equations [(1), (2), (3)] along with the boundary conditions [(4), (5), (6)] are given by

$$Q_1 = I - \frac{d(p)}{\theta}\left(1 - e^{-\theta t}\right) \quad (7)$$

$$Q_2 = \frac{d(p)e^{-\theta t}}{(\lambda - \theta)}\left[e^{(\lambda-\theta)(t_1 - t)} - 1\right] \quad (8)$$

$$Q_3 = \frac{d(p)e^{-\theta t_1}}{\theta}\left[e^{-\theta(t-t_1)} - 1\right] \quad (9)$$

Continuity of inventory at $t = t_d$ or $Q_1(t_d) = Q_2(t_d)$ gives

$$I - \frac{d(p)}{\theta}\left(1 - e^{-\theta t_d}\right) = \frac{d(p)e^{-\theta t_d}}{(\lambda - \theta)}\left[e^{(\lambda-\theta)(t_1 - t_d)} - 1\right] \quad (10)$$

This shows that maximum inventory level during each replenishment cycle is given by

$$I = \frac{d(p)}{\theta} + d(p)e^{-\theta t_d}\left[\frac{e^{(\lambda-\theta)(t_1 - t_d)} - 1}{\lambda - \theta} - \frac{1}{\theta}\right] \quad (11)$$

Now at $t = T$, $Q_3(T) = -S$, equation (9) gives the maximum amount of demand backlogged per cycle as

$$S = \frac{d(p)e^{-\theta t_1}}{\theta}\left[1 - e^{-\theta(T-t_1)}\right] \quad (12)$$

Thus inventory order can be evaluated as

$$Q = I + S = \frac{d(p)}{\theta} + d(p)e^{-\theta t_d}\left[\frac{e^{(\lambda-\theta)(t_1 - t_d)} - 1}{\lambda - \theta} - \frac{1}{\theta}\right] + \frac{d(p)e^{-\theta t_1}}{\theta}\left[1 - e^{-\theta(T-t_1)}\right] \quad (13)$$

Retailer's total profit per unit time during a cycle is obtained as

$$TP = \frac{1}{T}\begin{bmatrix} \text{Sales Revenue} - \text{Ordering Cost} - \text{Purchasing Cost} \\ -\text{Holding Cost} - \text{Shortage Cost} - \text{Deterioration Cost} \end{bmatrix} \quad (14)$$

Individual costs are now evaluated.

a) Sales Revenue = Selling price × Total demand over the cycle

$$= p \times \int_0^T d(p)e^{-\theta t}dt$$

$$= \frac{pd(p)}{\theta}\left[1 - e^{-\theta T}\right] \tag{15}$$

b) Ordering Cost = A

c) Purchasing Cost = $c \times Q$

$$d)\text{ Holding Cost } = h\int_0^{t_d} Q_1 dt + h\int_{t_d}^{t_1} Q_2 dt$$

$$= h\int_0^{t_d}\left[I - \frac{d(p)}{\theta}\left(1 - e^{-\theta t}\right)\right]dt + h\int_{t_d}^{t_1}\left[\frac{d(p)e^{-\theta t}}{(\lambda-\theta)}\left[e^{(\lambda-\theta)(t_1-t)} - 1\right]\right]dt \tag{16}$$

$$= hd(p)\left[\frac{e^{(\lambda-\theta)t_1}e^{-\lambda t_d}}{\lambda-\theta}\left(t_d + \frac{1}{\lambda}\right) - \frac{\lambda e^{-\theta t_d}}{\theta(\lambda-\theta)}\left(t_d + \frac{1}{\theta}\right) + \frac{1}{\theta^2} + \frac{1}{\lambda\theta}e^{-\theta t_1}\right] \tag{17}$$

$$e)\text{ Shortage Cost } = -c_1\int_{t_1}^{T} Q_3 dt = -c_1\int_{t_1}^{T}\frac{d(p)e^{-\theta t_1}}{\theta}\left[e^{-\theta(t-t_1)} - 1\right]dt$$

$$= c_1 d(p)\left[\frac{1}{\theta^2}\left(e^{-\theta T} - e^{-\theta t_1}\right) + \frac{(T-t_1)e^{-\theta t_1}}{\theta}\right] \tag{18}$$

$$f)\text{ Deterioration Cost} = c\int_{t_d}^{t_1}\lambda Q_2 dt = c\int_{t_d}^{t_1}\lambda\frac{d(p)e^{-\theta t}}{(\lambda-\theta)}\left[e^{(\lambda-\theta)(t_1-t)} - 1\right]dt$$

$$= \frac{c\lambda d(p)}{(\lambda-\theta)}\left[\frac{e^{-\theta t_1} - e^{-\theta t_d}}{\theta} + \frac{e^{-\theta t_1}}{\lambda}\left(e^{\lambda(t_1-t_d)} - 1\right)\right] \tag{19}$$

Therefore Total profit per unit time is

$TP(t_1, T, p)$

$$= \frac{1}{T}\left[\begin{array}{c} \frac{pd(p)}{\theta}\left[1-e^{-\theta T}\right] - A - cQ \\ -hd(p)\left[\frac{e^{(\lambda-\theta)t_1}e^{-\lambda t_d}}{\lambda-\theta}\left(t_d+\frac{1}{\lambda}\right) - \frac{\lambda e^{-\theta t_d}}{\theta(\lambda-\theta)}\left(t_d+\frac{1}{\theta}\right) + \frac{1}{\theta^2} + \frac{1}{\lambda\theta}e^{-\theta t_1}\right] \\ -c_1 d(p)\left[\frac{1}{\theta^2}\left(e^{-\theta T}-e^{-\theta t_1}\right) + \frac{(T-t_1)e^{-\theta t_1}}{\theta}\right] - \frac{c\lambda d(p)}{(\lambda-\theta)}\left[\frac{e^{-\theta t_1}-e^{-\theta t_d}}{\theta} + \frac{e^{-\theta t_1}}{\lambda}\left(e^{\lambda(t_1-t_d)}-1\right)\right] \end{array}\right] \tag{20}$$

$$= \frac{1}{T}\left[\begin{array}{c} \frac{pd(p)}{\theta}\left[1-e^{-\theta T}\right] - A \\ -cd(p)\left[\frac{1}{\theta} + e^{-\theta t_d}\left[\frac{e^{(\lambda-\theta)(t_1-t_d)}-1}{\lambda-\theta} - \frac{1}{\theta}\right] + \frac{e^{-\theta t_1}}{\theta}\left[1-e^{-\theta(T-t_1)}\right]\right] \\ -hd(p)\left[\frac{e^{(\lambda-\theta)t_1}e^{-\lambda t_d}}{\lambda-\theta}\left(t_d+\frac{1}{\lambda}\right) - \frac{\lambda e^{-\theta t_d}}{\theta(\lambda-\theta)}\left(t_d+\frac{1}{\theta}\right) + \frac{1}{\theta^2} + \frac{1}{\lambda\theta}e^{-\theta t_1}\right] \\ -c_1 d(p)\left[\frac{1}{\theta^2}\left(e^{-\theta T}-e^{-\theta t_1}\right) + \frac{(T-t_1)e^{-\theta t_1}}{\theta}\right] - \frac{c\lambda d(p)}{(\lambda-\theta)}\left[\frac{e^{-\theta t_1}-e^{-\theta t_d}}{\theta} + \frac{e^{-\theta t_1}}{\lambda}\left(e^{\lambda(t_1-t_d)}-1\right)\right] \end{array}\right] \tag{21}$$

Thus necessary conditions for the total profit to be maximum is

$$\frac{\partial TP}{\partial t_1} = 0, \quad \frac{\partial TP}{\partial T} = 0, \quad \frac{\partial TP}{\partial p} = 0$$

$$\frac{\partial TP}{\partial t_1} = \frac{d(p)}{T}\left[e^{(\lambda-\theta)t_1}\left\{\frac{-h}{\lambda} - e^{-\lambda t_d}(2c + ht_d)\right\} + e^{-\theta t_1}\left(2c + \frac{h}{\lambda}\right)\right] = 0 \tag{22}$$

$$\frac{\partial TP}{\partial T} = \frac{1}{T}\left[d(p)e^{-\theta T}(p-c) + \frac{c_1 d(p)}{\theta}\left(e^{-\theta T} - e^{-\theta t_1}\right)\right] - \frac{1}{T^2}\left[\begin{array}{c} \frac{pd(p)}{\theta}\left[1-e^{-\theta T}\right] - A \\ -cd(p)\left[\frac{1}{\theta} + e^{-\theta t_d}\left[\frac{e^{(\lambda-\theta)(t_1-t_d)}-1}{\lambda-\theta} - \frac{1}{\theta}\right] + \frac{e^{-\theta t_1}}{\theta}\left[1-e^{-\theta(T-t_1)}\right]\right] \\ -hd(p)\left[\frac{e^{(\lambda-\theta)t_1}e^{-\lambda t_d}}{\lambda-\theta}\left(t_d+\frac{1}{\lambda}\right) - \frac{\lambda e^{-\theta t_d}}{\theta(\lambda-\theta)}\left(t_d+\frac{1}{\theta}\right) + \frac{1}{\theta^2} + \frac{1}{\lambda\theta}e^{-\theta t_1}\right] \\ -c_1 d(p)\left[\frac{1}{\theta^2}\left(e^{-\theta T} - e^{-\theta t_1}\right) + \frac{(T-t_1)e^{-\theta t_1}}{\theta}\right] - \frac{c\lambda d(p)}{(\lambda-\theta)}\left[\frac{e^{-\theta t_1} - e^{-\theta t_d}}{\theta} + \frac{e^{-\theta t_1}}{\lambda}\left(e^{\lambda(t_1-t_d)}-1\right)\right] \end{array}\right] = 0 \tag{23}$$

$$\frac{\partial TP}{\partial p} = \frac{1}{T}\left[\begin{array}{c} \frac{d(p)}{\theta}\left[1-e^{-\theta T}\right] + \frac{pd'(p)}{\theta}\left[1-e^{-\theta T}\right] \\ -cd'(p)\left[\frac{1}{\theta} + e^{-\theta t_d}\left[\frac{e^{(\lambda-\theta)(t_1-t_d)}-1}{\lambda-\theta} - \frac{1}{\theta}\right] + \frac{e^{-\theta t_1}}{\theta}\left[1-e^{-\theta(T-t_1)}\right]\right] \\ -hd'(p)\left[\frac{e^{(\lambda-\theta)t_1}e^{-\lambda t_d}}{\lambda-\theta}\left(t_d+\frac{1}{\lambda}\right) - \frac{\lambda e^{-\theta t_d}}{\theta(\lambda-\theta)}\left(t_d+\frac{1}{\theta}\right) + \frac{1}{\theta^2} + \frac{1}{\lambda\theta}e^{-\theta t_1}\right] \\ -c_1 d'(p)\left[\frac{1}{\theta^2}\left(e^{-\theta T} - e^{-\theta t_1}\right) + \frac{(T-t_1)e^{-\theta t_1}}{\theta}\right] \\ -\frac{c\lambda d'(p)}{(\lambda-\theta)}\left[\frac{e^{-\theta t_1} - e^{-\theta t_d}}{\theta} + \frac{e^{-\theta t_1}}{\lambda}\left(e^{\lambda(t_1-t_d)}-1\right)\right] \end{array}\right] = 0 \tag{24}$$

The sufficient conditions for the total profit to be maximum are that following values are alternatively negative or positive at the optimum value of t_1, T, p.

$$\frac{\partial^2 TP}{\partial {t_1}^2}, \qquad \begin{vmatrix} \frac{\partial^2 TP}{\partial {t_1}^2} & \frac{\partial^2 TP}{\partial t_1 \partial T} \\ \frac{\partial^2 TP}{\partial t_1 \partial T} & \frac{\partial^2 TP}{\partial T^2} \end{vmatrix}, \qquad \begin{vmatrix} \frac{\partial^2 TP}{\partial {t_1}^2} & \frac{\partial^2 TP}{\partial t_1 \partial T} & \frac{\partial^2 TP}{\partial t_1 \partial p} \\ \frac{\partial^2 TP}{\partial t_1 \partial T} & \frac{\partial^2 TP}{\partial T^2} & \frac{\partial^2 TP}{\partial T \partial p} \\ \frac{\partial^2 TP}{\partial t_1 \partial p} & \frac{\partial^2 TP}{\partial T \partial p} & \frac{\partial^2 TP}{\partial p^2} \end{vmatrix} \tag{25}$$

SPECIAL CASES

Case 1: If $\theta = 0$ then Demand = $d(p)$ then this model reduces to the model given by Jaggi C. K. [10] with constant holding cost.

Case 2: If Deterioration is instantaneous then $t_d = 0$ and shortages are not allowed ($t_1 = T$) then model reduces to Kaur and Sharma model [12] and total profit is

$$TP = \frac{1}{T}\left[\frac{pd(p)}{\theta}\left[1-e^{-\theta T}\right] - A - cd(p)\left[\frac{e^{(\lambda-\theta)T}-1}{\lambda-\theta}\right] - hd(p)\left[\frac{e^{(\lambda-\theta)T}e^{-\lambda t_d}}{\lambda(\lambda-\theta)} - \frac{1}{\theta(\lambda-\theta)} + \frac{1}{\lambda\theta}e^{-\theta T}\right] - \frac{c\lambda d(p)}{(\lambda-\theta)}\left[\frac{e^{-\theta T}-1}{\theta} + \frac{e^{-\theta T}}{\lambda}\left(e^{\lambda T}-1\right)\right]\right] \tag{26}$$

Case 3: If shortages are not allowed ($t_1 = T$) then total profit is

$$TP = \frac{1}{T}\left[\begin{array}{l} \frac{pd(p)}{\theta}\left[1-e^{-\theta T}\right] - A - cd(p)\left[\frac{1}{\theta} + e^{-\theta t_d}\left[\frac{e^{(\lambda-\theta)(T-t_d)}-1}{\lambda-\theta} - \frac{1}{\theta}\right]\right] \\ -hd(p)\left[\frac{e^{(\lambda-\theta)T}e^{-\lambda t_d}}{\lambda-\theta}\left(t_d + \frac{1}{\lambda}\right) - \frac{\lambda e^{-\theta t_d}}{\theta(\lambda-\theta)}\left(t_d + \frac{1}{\theta}\right) + \frac{1}{\theta^2} + \frac{1}{\lambda\theta}e^{-\theta T}\right] \\ -\frac{c\lambda d(p)}{(\lambda-\theta)}\left[\frac{e^{-\theta T}-e^{-\theta t_d}}{\theta} + \frac{e^{-\theta T}}{\lambda}\left(e^{\lambda(T-t_d)}-1\right)\right] \end{array}\right] \quad (27)$$

4. NUMERICAL EXAMPLES

The solution procedure is described by taking numerical examples.

Example: Consider an inventory with A = 100 per order, h = 0.75 per unit per cycle, c = 20, $d(p) = 100 - p$, c_1 = 1.5.

Considering different values of λ, for fixed θ, t_d, t_1, T and p we get deterioration cost and total profit as:

Table 1 for λ = 0.1, 0.15, 0.2, 0.25

λ	θ	t_d	t_1	T	p	Deterioration cost	TP
0.1	0.02	0.5	1.2	3	25	36.90	233.81
0.15	0.02	0.5	1.2	3	25	56.01	220.90
0.2	0.02	0.5	1.2	3	25	75.57	207.68
0.25	0.02	0.5	1.2	3	25	95.60	194.14

It is observed that if rate of deterioration increases then deterioration cost also increases and therefore total profit decreases.

Table 2 for θ = 0.02, 0.04, 0.06 and T = 2.8, 3, 3.2

λ	θ	t_d	t_1	T	P	Deterioration cost	TP
0.1	0.02	0.5	1.2	3	25	36.90	233.81
0.1	0.04	0.5	1.2	3	25	36.19	225.97
0.1	0.06	0.5	1.2	3	25	35.50	218.44
0.1	0.02	0.5	1.2	3.2	25	36.90	228.43
0.1	0.04	0.5	1.2	3.2	25	36.19	220.29
0.1	0.06	0.5	1.2	3.2	25	35.50	212.50
0.1	0.02	0.5	1.2	2.8	25	36.90	238.35
0.1	0.04	0.5	1.2	2.8	25	36.19	230.84
0.1	0.06	0.5	1.2	2.8	25	35.50	223.62

It is observed that if rate of demand increases then deterioration cost and total profit decreases. On the other hand if cycle time increases then deterioration cost remains same but total profit further decreases.

Table 3 for λ = 0.1, 0.15, 0.2, 0.25, θ = 0.02, 0.04, 0.06 , T = 2.8, 3, 3.2 and t_d = 0

λ	θ	t_d	t_1	T	p	Deterioration cost	TP
0.1	0.02	0	1.2	3	25	110.66	184.43
0.15	0.02	0	1.2	3	25	169.42	144.97
0.2	0.02	0	1.2	3	25	230.61	103.88
0.25	0.02	0	1.2	3	25	294.35	103.65
0.1	0.04	0	1.2	3	25	108.90	177.29
0.1	0.06	0	1.2	3	25	107.17	170.46
0.1	0.02	0	1.2	3.2	25	110.66	182.13
0.1	0.04	0	1.2	3.2	25	108.90	174.65
0.1	0.06	0	1.2	3.2	25	107.17	167.52
0.1	0.02	0	1.2	2.8	25	110.66	185.44
0.1	0.04	0	1.2	2.8	25	108.90	178.69
0.1	0.06	0	1.2	2.8	25	107.17	172.21

For instantaneous deterioration, it is observed that if rate of deterioration increases then deterioration cost also increases and therefore total profit decreases. Further if rate of demand increases then deterioration cost decreases but total profit also decreases. On the other hand if cycle time increases then deterioration cost remains same but total profit further decreases.

It is also observed that for non – instantaneous deteriorating products deterioration cost is less than the deterioration cost due to instantaneous deteriorating products. On the other hand for non – instantaneous deteriorating products total profit is more than the total profit due to instantaneous deteriorating products.

For non – deteriorating products, λ = 0 and $t_d = t_1$

Table 4 for λ = 0, θ = 0.02, 0.04, 0.06 and T = 2.8, 3, 3.2

λ	θ	t_d	t_1	T	p	Deterioration cost	TP
0	0.02	1.2	1.2	3	25	0	258.75
0	0.04	1.2	1.2	3	25	0	250.43
0	0.06	1.2	1.2	3	25	0	242.44
0	0.02	1.2	1.2	3.2	25	0	251.81
0	0.04	1.2	1.2	3.2	25	0	243.22
0	0.06	1.2	1.2	3.2	25	0	235.00
0	0.02	1.2	1.2	2.8	25	0	265.07
0	0.04	1.2	1.2	2.8	25	0	257.05
0	0.06	1.2	1.2	2.8	25	0	249.33

For non – deteriorating products if rate of demand increases then total profit decreases. Similarly, if cycle time increases then total profit further decreases.

When t_d increases or the duration during which there is no deterioration increases then total profit is as

Table 5 for λ = 0.1, 0.15, θ = 0.02, 0.04, 0.06, t_d = 0.5, 0.6, 0.7, 0.8 and T = 3, 3.2

λ	θ	t_d	t_1	T	p	Deterioration cost	TP
0.1	0.02	0.5	1.2	3	25	36.90	233.81
0.1	0.02	0.6	1.2	3	25	27.00	240.48
0.1	0.02	0.7	1.2	3	25	18.68	246.10
0.1	0.02	0.8	1.2	3	25	11.90	250.67
0.1	0.04	0.5	1.2	3	25	36.19	225.97
0.1	0.04	0.6	1.2	3	25	26.47	232.52
0.1	0.04	0.7	1.2	3	25	18.29	238.03
0.1	0.04	0.8	1.2	3	25	11.65	242.52
0.1	0.02	0.5	1.2	3.2	25	36.90	228.43
0.1	0.02	0.6	1.2	3.2	25	27.00	234.68
0.1	0.02	0.7	1.2	3.2	25	18.68	239.95
0.1	0.02	0.8	1.2	3.2	25	11.90	244.24
0.15	0.02	0.5	1.2	3	25	56.01	220.90
0.15	0.02	0.6	1.2	3	25	40.92	231.06
0.15	0.02	0.7	1.2	3	25	28.25	239.61
0.15	0.02	0.8	1.2	3	25	17.98	246.55

It is observed that when t_d increases, if rate of deterioration also increases then deterioration cost increases and therefore total profit decreases. Further if rate of demand increases then deterioration cost as well as total profit decreases. On the other hand if cycle time increases then deterioration cost remains same but total profit further decreases.

If shortages are not allowed then $t_1 = T$.

Table 6 for T = 1.5, 1.75, 2, 2.25

λ	θ	t_d	t_1	T	p	Deterioration cost	TP
0.1	0.02	0.5	1.5	1.5	25	75.77	158.81
0.1	0.02	0.5	1.75	1.75	25	119.00	124.96
0.1	0.02	0.5	2	2	25	172.25	87.37
0.1	0.02	0.5	2.25	2.25	25	235.67	47.13

It is observed that when shortages are not allowed then deterioration cost increases and total profit decreases as cycle time increases.

5. CONCLUSION

In this paper an inventory model for non – instantaneous deteriorating products with price and time dependent demand is considered. Shortages are allowed and completely backlogged. Numerical examples are also provided here and it is observed that it is profitable for retailer to decrease the cycle time in case of increasing demand rate and increasing rate of deterioration. This model is useful for non – instantaneous deteriorating items such as fruit, vegetables, cosmetics, medicines etc.

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