

Numerical Solution of Poisson Equation Using Fuzzy Data by finite Difference

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ABSTRACT

In this paper, we have discussed fuzzification of elliptic partial differential equation taking Poisson Equation in two dimensions are discussed. The interval of fuzzy interval can be determined. Finite difference method applied of two different grids using five points, first for initial values and the second to solve Poisson equation numerically.

Keywords: Fuzzy membership function (f.m.f.), interval of confidence, triangular fuzzy number (t.f.n.), $\alpha -$ cuts, five points finite difference, Poisson Equation.

1 – Introduction

The concept of Fuzzy differential equation was first introduced by Chang Zadeh [10]. Dubois and Prade[5] has given extension principle. Raphel and Mhassin [8,9], used five points in regular domain. Here implementing five-points for finite difference method to solve Poisson equation in two variables numerically, then fuzzified.

2- Definitions

A triangular Fuzzy number μ is defined by three real numbers with base as the interval $[a,c]$ and b as the vertex of triangle. The membership function are defined as follows [8, ,]:

$$\mu(x)(=)\begin{cases} \dfrac{x-a}{b-a} & ; \ where \quad a \le x \le b \\ \dfrac{x-c}{b-c} & ; \ where \quad b \le x \le c \\ 0 & ; \quad otherwise \end{cases}$$

The $\alpha -$ cuts are defined by $\Delta_L(\alpha)(=)a+\alpha(b-a)$ and $\Delta_R(\alpha)(=)c+\alpha(b-c)$.

A triangular Fuzzy number μ_f is defined by three real numbers with base as the interval $[f_a, f_c]$ and f_b as the vertex of triangle. The membership function are defined as follows

$$\mu_f(x)(=)\begin{cases} \dfrac{x-f_a}{f_b-f_a} & ; \ where \quad f_a \le x \le f_b \\ \dfrac{x-f_c}{f_b-f_c} & ; \ where \quad f_b \le x \le f_c \\ 0 & ; \quad otherwise \end{cases}$$

The $\alpha -$ cuts are for the function defined by $\Delta_L(\alpha)(=)f_a+\alpha(f_b-f_a)$ and $\Delta_R(\alpha)(=)f_c+\alpha(f_b-f_c)$.

2.1 Finite difference using to solve Poisson equation

Poisson equation in two variables is defined by

$$u_{xx}(x,y)+u_{yy}(x,y)=f(x,y) \qquad (1)$$

This equation is encountered in many application, fluid mechanics, study state , electrostatics, mass transfer, and for other areas of mechanics and physics. Replacing u_{xx} and u_{yy} by the central difference formula the value of $u(x_i, x_j)$ at any mesh point is the arithmetic mean of the values at four neighboring mesh to the left, right, above and below which is called standard five points formula Fig.1, use for finding the initial data respectively as follows, replacing u_{xx} and u_{yy} by finite difference method

$$u_{xx}=\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}+u_{i,j-1}+u_{i,j+1}}{h^2} \text{ and } u_{yy}=\frac{u_{i,j-1}-2u_{i,j}+u_{i,j+1}+u_{i,j-1}}{k^2}.$$

Then (1) becomes

$$\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{h^2}+\frac{u_{i,j-1}-2u_{i,j}+u_{i,j+1}}{k^2}=f_{i\,j} \qquad (2)$$

Here we will take $h=k$ in the square mesh, the value of $u_{i,j}$ at any point is the arithmetic mean of its values at the four neighboring mesh points to the left, right, above, and below, then

$$u_{i,j}=\frac{1}{4}\left[u_{i-1,j}+u_{i+1,j}+u_{i,j-1}+u_{i,j+1}-h^2 f_{i,j}\right] \qquad (3)$$

Which is called Standard Five Points Formula (SFPF) as in Fig.1, Or

$$u_{i,j}=\frac{1}{4}\left[u_{i+1,j+1}+u_{i+1,j-1}+u_{i-1,j+1}+u_{i-1,j-1}-2h^2 f_{i,j}\right] \qquad (4)$$

Which is called Diagonally Five Points Formula (DFPF) as in Fig.2.[young], we will use (4) wherever necessary.

3. Application of Fuzzy interval in Poisson Equation

From c_1 to c_{16} represents the boundary conditions of the square mesh with fuzzy interval as in table 1.

Table –1-

$c_1(=)[l_{1,1};l_{1,2};l_{1,3}]$	$c_2(=)[l_{2,1};l_{2,2};l_{2,3}]$	$c_3(=)[l_{3,1};l_{3,2};l_{3,3}]$	$c_4(=)[l_{4,1};l_{4,2};l_{4,3}]$
$c_5(=)[l_{5,1};l_{5,2};l_{5,3}]$	$c_6(=)[l_{6,1};l_{6,2};l_{6,3}]$	$c_7(=)[l_{7,1};l_{7,2};l_{7,3}]$	$c_8(=)[l_{8,1};l_{8,2};l_{8,3}]$
$c_9(=)[l_{9,1};l_{9,2};l_{9,3}]$	$c_{10}(=)[l_{10,1};l_{10,2};l_{10,3}]$	$c_{11}(=)[l_{11,1};l_{11,2};l_{11,3}]$	$c_{12}(=)[l_{12,1};l_{12,2};l_{12,3}]$
$c_{13}(=)[l_{13,1};l_{13,2};l_{13,3}]$	$c_{14}(=)[l_{14,1};l_{14,2};l_{14,3}]$	$c_{15}(=)[l_{15,1};l_{15,2};l_{15,3}]$	$c_{16}(=)[l_{16,1};l_{16,2};l_{16,3}]$

The interior points due to the square grid are u_1 to u_9.

Now to find the initial value of $u_5^{(0)}$ using standard five-points formula (4) as

$$u_5^{(0)}(=)\frac{1}{4}\left[c_3(+)c_7(+)c_{11}(+)c_{15}(-)h^2(f_5)\right] \tag{5}$$

We can write equation (4) in some details as

$$\left[u_5^{(0)}\right]^{(\alpha)}(=)\left[\frac{l_{3,1}+l_{7,1}+l_{11,1}+l_{15,1}-h^2 f_{5,1}}{4},\frac{l_{3,2}+l_{7,2}+l_{11,2}+l_{15,2}-h^2 f_{5,2}}{4},\frac{l_{3,3}+l_{7,3}+l_{11,3}+l_{15,3}-h^2 f_{5,3}}{4}\right] \tag{6}$$

Fuzzy membership functions (f.m.f) are respective α -cuts of $c_1, c_{3,} c_{5,} c_{7,} c_{9,} c_{11,} c_{13}$ *and* c_{15} are respectively as

$$\mu_{C_i}(x)(=)\left\{\begin{array}{ll} \frac{x-l_{i,1}}{l_{i,2}-l_{i,1}} & ;\ where\ \ l_{i,1}\le x\le l_{i,2} \\ \frac{x-l_{i,3}}{l_{i,2}-l_{i,3}} & ;\ where\ \ l_{i,2}\le x\le l_{i,3} \\ 0 & ;\ otherwise \end{array}\right\}$$ Hence the $\alpha-$cuts of c_i is given by

$[c_i]^{(\alpha)}(=)\left[l_{i,1}+\alpha(l_{i,2}-l_{i,1}),\quad l_{i,3}+\alpha(l_{i,2}-l_{i,3})\right]$ Where $i=1,2,\cdots,16$. Then from equation (6) we have

$$\mu_{u_5^{(0)}}(x)(=)\left\{\begin{aligned}&\frac{(l_{3,2}-l_{3,1})+(l_{7,2}-l_{7,1})+(l_{11,2}-l_{11,1})+(l_{15,2}-l_{15,1})-h^2(f_{5,2}-h^2 f_{5,1})}{4}\alpha+\\&\frac{\left(l_{3,1}+l_{7,1}+l_{11,1}+l_{15,1}-h^2 f_{5,1}\right)}{4}\quad ,\\&\frac{(l_{3,2}-l_{3,3})+(l_{7,2}-l_{7,3})+(l_{11,2}-l_{11,3})+(l_{15,2}-l_{15,3})-h^2(f_{5,2}-f_{5,3})}{4}\alpha+\\&\frac{(l_{3,3}+l_{7,3}+l_{11,3}+l_{15,3}-h^2 f_{5,3})}{4}\end{aligned}\right\}$$

or

$$\mu_{u_5^{(0)}}(x)(=)\left\{\begin{aligned}&\frac{H_{5,2}-H_{5,1}}{4}\alpha+\frac{H_{5,1}}{4}\\&\frac{H_{5,2}-H_{5,3}}{4}\alpha+\frac{H_{5,3}}{4}\end{aligned}\right\}\quad \text{Where}\quad \left\{\begin{aligned}&H_{5,1}=l_{3,1}+l_{7,1}+l_{11,1}+l_{15,1}-h^2 f_{5,1}\\&H_{5,2}=l_{3,2}+l_{7,2}+l_{11,2}+l_{15,2}-h^2 f_{5,2}\\&H_{5,3}=l_{3,3}+l_{7,3}+l_{11,3}+l_{15,3}-h^2 f_{5,3}\end{aligned}\right\}$$

Fig.1

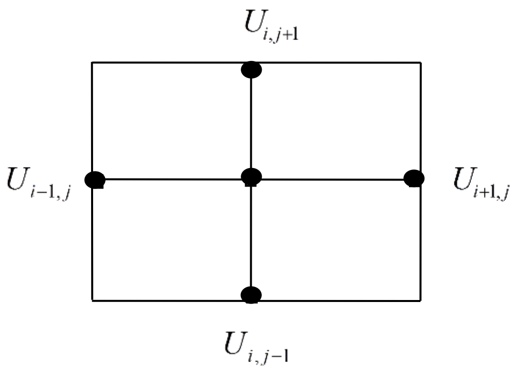


Fig.2

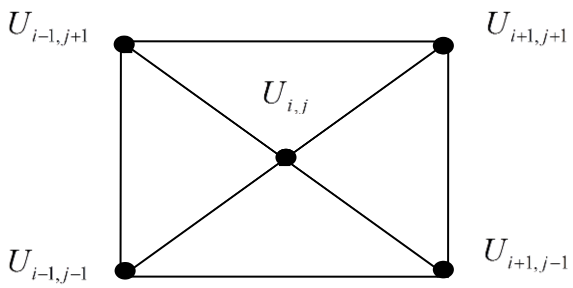
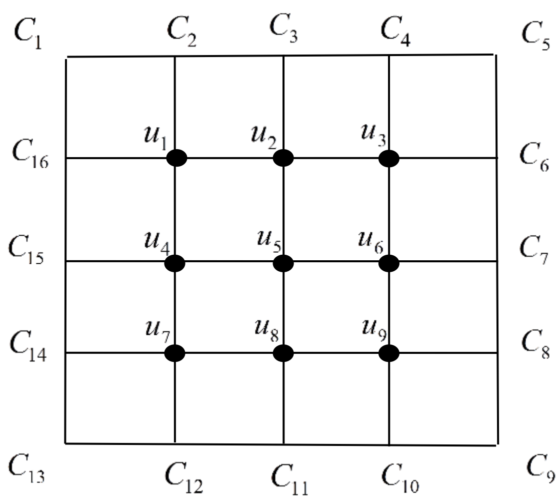


Fig.3



let

$x_1 = \frac{H_{5,2} - H_{5,1}}{4}\alpha + \frac{H_{5,1}}{4}$ and $x_2 = \frac{H_{5,2} - H_{5,3}}{4}\alpha + \frac{H_{5,3}}{4}$. Solving for α , we have $\alpha = \frac{4x_1 - H_{5,1}}{H_{5,2} - H_{5,1}}$

and $\alpha = \frac{4x_2 - H_{5,3}}{H_{5,2} - H_{5,3}}$. Hence f.m.f. for $u_5^{(0)}$ is

$$\mu_{u_5^{(0)}}(x)(=)\begin{cases} \frac{4x - H_{5,1}}{H_{5,2} - H_{5,1}}; & \frac{1}{4}H_{5,1} \le x \le \frac{1}{4}H_{5,2} \\ \frac{4x - H_{5,3}}{H_{5,2} - H_{5,3}}; & \frac{1}{4}H_{5,2} \le x \le \frac{1}{4}H_{5,3} \\ 0 & ;otherwise \end{cases} \tag{7}$$

Where $\alpha \in [0,1]$.

But to find the initial values of u_1, u_3 , u_9 and u_7 using five points diagonally (DFPF) i.e. equation (4), and to find The initial values of u_2 , u_6 , u_8 and u_4 by (SFPF) i.e. equation (3).The initial value of $u_1^{(0)}$ we use equation (4) the interval of confidence

$$u_1^{(0)}(=)\left[\frac{l_{1,1} + l_{3,1} + u_{5,1}^{(0)} + l_{15,1} - 2h^2 f_{1,1}}{4}, \frac{l_{1,2} + l_{3,2} + u_{5,2}^{(0)} + l_{15,2} - 2h^2 f_{1,2}}{4}, \frac{l_{1,3} + l_{3,3} + u_{5,3}^{(0)} + l_{15,3} - 2h^2 f_{1,3}}{4}\right] \tag{8}$$

Hence f.m.f. for c_1 is

$$\mu_{C_1}(x)(=)\begin{cases} \frac{x - l_{1,1}}{l_{1,2} - l_{1,1}} & ;\ where\ \ l_{1,1} \le x \le l_{1,2} \\ \frac{x - l_{1,3}}{l_{1,2} - l_{1,3}} & ;\ where\ \ l_{1,2} \le x \le l_{1,3} \\ 0 & ;\ \ otherwise \end{cases}$$

, Hence $\alpha - cuts$ of c_1

$$[c_1]^{(\alpha)}(=)\left[l_{1,1} + \alpha(l_{1,2} - l_{1,1}), \quad l_{1,3} + \alpha(l_{1,2} - l_{1,3})\right].$$

As well, $\alpha - cuts$ of c_3, c_{15}, and $u_5^{(0)}$ are $[c_3]^{(\alpha)}(=)\left[l_{3,1} + \alpha(l_{3,2} - l_{3,1}), \quad l_{3,3} + \alpha(l_{3,2} - l_{3,3})\right]$,

$[c_{15}]^{(\alpha)}(=)\left[l_{15,1} + \alpha(l_{15,2} - l_{15,1}), \quad l_{15,3} + \alpha(l_{15,2} - l_{15,3})\right]$ and $\left[u_5^{(0)}\right]^{(\alpha)}(=)\left[u_{5,1}^{(0)} + \alpha(u_{5,2}^{(0)} - u_{5,1}^{(0)}), \quad u_{5,3}^{(0)} + \alpha(u_{5,2}^{(0)} - u_{5,3}^{(0)})\right]$.

So the interval of confidence of $u_1^{(0)}$ is

$$u_1^{(0)}(=)\left\{\begin{matrix} \dfrac{X_{1,2}-X_{1,1}}{4}\alpha+\dfrac{X_{1,1}}{4} \\ \\ \dfrac{X_{1,2}-X_{1,3}}{4}\alpha+\dfrac{X_{1,3}}{4}\end{matrix}\right\} \quad \text{where} \quad \left\{\begin{matrix} X_{1,1}=l_{1,1}+l_{3,1}+l_{15,1}+u_{5,1}^{(0)}-2h^2 f_{1,1} \\ X_{1,2}=l_{1,2}+l_{3,2}+l_{15,2}+u_{5,2}^{(0)}-2h^2 f_{1,2} \\ X_{1,3}=l_{1,3}+l_{3,3}+l_{15,3}+u_{5,3}^{(0)}-2h^2 f_{1,3}\end{matrix}\right\}.$$

Le t $x_1=\dfrac{X_{1,2}-X_{1,1}}{4}\alpha+\dfrac{X_{1,1}}{4}$ and $x_2=\dfrac{X_{1,2}-X_{1,3}}{4}\alpha+\dfrac{X_{1,3}}{4}$, solve for α

$\alpha=\dfrac{4x_1-X_{1,1}}{X_{1,2}-X_{1,1}}$ and $\alpha=\dfrac{4x_2-X_{1,3}}{X_{1,2}-X_{1,3}}$.

Hence f.m.f. for $u_1^{(0)}$ is

$$\mu_{u_1^{(0)}}(x)(=)\left\{\begin{matrix} \dfrac{4x-X_{1,1}}{X_{1,2}-X_{1,1}}\ where\ \dfrac{1}{4}X_{1,1}\le x\le\dfrac{1}{4}X_{1,2} \\ \dfrac{4x-X_{1,3}}{X_{1,3}-X_{1,2}}\ where\ \dfrac{1}{4}X_{1,2}\le x\le\dfrac{1}{4}X_{1,3} \\ 0 \quad ;\quad otherwise\end{matrix}\right\} \tag{9}$$

This process also for u_3, u_9 and u_7. In a similar way we evaluate u_2, u_6, u_8 and u_4, for $u_2^{(0)}$ we find the $\alpha-cuts$ of $c_3, u_3^{(0)}, u_5^{(0)}$ and $u_1^{(0)}$ we get

$$u_2^{(0)}(=)\left[\frac{l_{3,1}+u_{3,1}^{(0)}+u_{5,1}^{(0)}+u_{1,1}^{(0)}-h^2 f_{2,1}}{4},\frac{l_{3,2}+u_{3,2}^{(0)}+u_{5,2}^{(0)}+u_{1,2}^{(0)}-h^2 f_{2,2}}{4},\frac{l_{3,3}+u_{3,3}^{(0)}+u_{5,3}^{(0)}+u_{1,3}^{(0)}-h^2 f_{2,3}}{4}\right] \tag{10}$$

$$u_2^{(0)}(=)\left\{\begin{matrix} \dfrac{H_{2.2}-H_{2.1}}{4}\alpha+\dfrac{H_{2,1}}{4} \\ \\ \dfrac{H_{2.2}-H_{2.3}}{4}\alpha+\dfrac{H_{2,3}}{4}\end{matrix}\right\} \quad \text{where} \quad \left\{\begin{matrix} H_{2,1}=l_{3,1}+u_{3,1}^{(0)}+u_{5,1}^{(0)}+u_{1,1}^{(0)}-h^2 f_{2,1} \\ H_{2,2}=l_{3,2}+u_{3,2}^{(0)}+u_{5,2}^{(0)}+u_{1,2}^{(0)}-h^2 f_{2,2} \\ H_{2,3}=l_{3,3}+u_{3,3}^{(0)}+u_{5,3}^{(0)}+u_{1,3}^{(0)}-h^2 f_{2,3}\end{matrix}\right\}$$

Next successive approximations with their f.m.f. as required be obtain from previous approximations and specified boundary conditions.

4. Numerical example

Let us consider the Poisson equation

$$u_{xx}(+)u_{yy}(=)2(x^2+y^2)e^{-4} \tag{11}$$

In the domain $0 \le x \le 4, 0 \le y \le 4$ with boundary conditions corresponding to the points shown in table -2-.

Table -2-

$c_1 = 0$	$c_2 = 0.293$	$c_3 = 1.172$	$c_4 = 2.637$	$c_5 = 4.688$
$c_{16} = 0$	u_1	u_2	u_3	$c_6 = 2.637$
$c_{15} = 0$	u_4	u_5	u_6	$c_7 = 1.172$
$c_{14} = 0$	u_7	u_8	u_9	$c_8 = 0.293$
$c_{13} = 0$	$c_{12} = 0$	$c_{11} = 0$	$c_{10} = 0$	$c_9 = 0$

Leibmann's process will be applied to solve equation (12).

Solution:

The boundary conditions are given, the initial values of $u_i = 1,2,3,...,9$ may be calculated with the help of standard five points and diagonal five points formulas, to get the approximate solution. From the equations (5) and (7) we have

$u_5^{(0)} (=) \frac{1}{4}\left[c_3 (+) c_7 (+) c_{11} (+) c_{15} (-) h^2 (f_5)\right]$ and

$$\mu_{u_5^{(0)}}(x) (=) \begin{cases} \dfrac{4x - H_{5,1}}{H_{5,2} - H_{5,1}}; & \dfrac{1}{4}H_{5,1} \le x \le \dfrac{1}{4}H_{5,2} \\ \dfrac{4x - H_{5,3}}{H_{5,2} - H_{5,3}}; & \dfrac{1}{4}H_{5,2} \le x \le \dfrac{1}{4}H_{5,3} \\ 0 & ; otherwise \end{cases} \quad \text{Where} \quad \begin{cases} H_{5,1} = l_{3,1} + l_{7,1} + l_{11,1} + l_{15,1} - h^2 f_{5,1} \\ H_{5,2} = l_{3,2} + l_{7,2} + l_{11,2} + l_{15,2} - h^2 f_{5,2} \\ H_{5,3} = l_{3,3} + l_{7,3} + l_{11,3} + l_{15,3} - h^2 f_{5,3} \end{cases}$$

$$u_5^{(0)} (=) [0.292, 0.293, 0.294] \qquad (12)$$

To find f.m.f. and respective interval of confidence these eight c_i's as follows:

$$\mu_{C_i}(x) (=) \begin{cases} \dfrac{x + 0.001}{0 + 0.001} \; ; \; where \;\; 0.001 \le x \le 0 \\ \dfrac{x - 0.001}{0 - 0.001} \; ; \; where \;\; 0 \le x \le 0.001 \\ 0 \quad ; \quad otherwise \end{cases}, \; [c_i]^{(\alpha)} (=) [0.001\alpha + 0.001, -0.001\alpha + 0.001]$$

for $i = 1,9,10,11,12,13,14,15$ and c_{16} are the same f.m.f. .

$$\mu_{C_2}(x)(=)\begin{cases} \dfrac{x-0.292}{0.293-0.292} & ;\ where\ \ 0.292 \le x \le 0.293 \\ \dfrac{x-0.294}{0.293-0.294} & ;\ where\ \ 0.293 \le x \le 00.294 \\ 0 & ;\ otherwise \end{cases},\ [c_2]^{(\alpha)}(=)[0.001\alpha+0.292,-0.001\alpha+0.294],$$

$$\mu_{C_3}(x)(=)\begin{cases} \dfrac{x-0.171}{0.172-0.171} & ;\ where\ \ 0.171 \le x \le 0.172 \\ \dfrac{x-0.173}{0.172-0.173} & ;\ where\ \ 0.172 \le x \le 0.173 \\ 0 & ;\ otherwise \end{cases},\ [c_3]^{(\alpha)}(=)[0.001\alpha+0.171,-0.001\alpha+0.173],$$

$$\mu_{C_4}(x)(=)\begin{cases} \dfrac{x-2.636}{2.637-2.636} & ;\ where\ \ 2.636 \le x \le 2.637 \\ \dfrac{x-2.638}{2.637-2.638} & ;\ where\ \ 2.637 \le x \le 2.638 \\ 0 & ;\ otherwise \end{cases},\ [c_4]^{(\alpha)}(=)[0.001\alpha+2.636,-0.001\alpha+2.638],$$

$$\mu_{C_5}(x)(=)\begin{cases} \dfrac{x-4.688}{4.689-4.688} & ;\ where\ \ 4.688 \le x \le 4.689 \\ \dfrac{x-4.690}{4.690-4.960} & ;\ where\ \ 4.689 \le x \le 4.690 \\ 0 & ;\ otherwise \end{cases},\ [c_5]^{(\alpha)}(=)[0.001\alpha+4.688,-0.001\alpha+4.690],$$

$[c_6]^{(\alpha)}(=)[0.001\alpha+2.636,-0.001\alpha+2.638]$, $[c_7]^{(\alpha)}(=)[0.001\alpha+0.171,-0.001\alpha+0.173]$ and $[c_8]^{(\alpha)}(=)[0.001\alpha+0.292,-0.001\alpha+0.294]$, with $\alpha\in[0,1]$. To find $u_5^{(0)}$ using equation (7)

Let $0.001\alpha+0.292=x_1$ and $-0.001\alpha+0.294=x_2$, then solving for α we get

$\alpha=\dfrac{x_1-0.292}{0.001}$ and $\alpha=\dfrac{x_2-0.294}{-0.001}$, hence f.m.f. for $u_5^{(0)}$ is

$$\mu_{u_5^{(0)}}(x)(=)\begin{cases} \dfrac{x-0.292}{0.293-0.292} & ;\ where\ \ 0.292 \le x \le 0.293 \\ \dfrac{x-0.294}{0.293-0.294} & ;\ where\ \ 0.293 \le x \le 0.294 \\ 0 & ;\ otherwise \end{cases}$$

In the same way we find f.m.f. for $u_1^{(0)}, u_3^{(0)}, u_9^{(0)}, u_7^{(0)}, u_2^{(0)}, u_6^{(0)}, u_8^{(0)}$ and $u_4^{(0)}$ are respectively.

$$\mu_{u_1^{(0)}}(x)(=)\begin{cases} \dfrac{x-0.182}{0.183-0.182} & ; \ where \ \ 0.182 \le x \le 0.183 \\ \dfrac{x-0.184}{0.183-0.184} & ; \ where \ \ 0.183 \le x \le 0.184 \\ 0 \ \ ; \ \ otherwise \end{cases},$$

$$\mu_{u_3^{(0)}}(x)(=)\begin{cases} \dfrac{x-1.50}{1.501-1.50} & ; \ where \ \ 1.500 \le x \le 1.501 \\ \dfrac{x-1.502}{1.501-1.502} & ; \ where \ \ 1.501 \le x \le 1.502 \\ 0 \ \ ; \ \ otherwise \end{cases}$$

$$\mu_{u_7^{(0)}}(x)(=)\begin{cases} \dfrac{x-0.035}{0.036-0.035} & ; \ where \ \ 0.035 \le x \le 0.036 \\ \dfrac{x-0.037}{0.036-0.037} & ; \ where \ \ 0.036 \le x \le 0.037 \\ 0 \ \ ; \ \ otherwise \end{cases},$$

$$\mu_{u_9^{(0)}}(x)(=)\begin{cases} \dfrac{x-0.183}{0.184-0.183} & ; \ where \ \ 0.183 \le x \le 0.184 \\ \dfrac{x-0.185}{0.184-0.185} & ; \ where \ \ 0.184 \le x \le 0.185 \\ 0 \ \ ; \ \ otherwise \end{cases}$$

$$\mu_{u_2^{(0)}}(x)(=)\begin{cases} \dfrac{x-0.667}{0.668-0.667} & ; \ where \ \ 0.667 \le x \le 0.668 \\ \dfrac{x-0.668}{0.667-0.668} & ; \ where \ \ 0.667 \le x \le 0.668 \\ 0 \ \ ; \ \ otherwise \end{cases},$$

$$\mu_{u_4^{(0)}}(x)(=)\begin{cases} \dfrac{x-0.081}{0.082-0.081} & ; \ where \ \ 0.081 \le x \le 0.082 \\ \dfrac{x-0.083}{0.082-0.083} & ; \ where \ \ 0.082 \le x \le 0.083 \\ 0 \ \ ; \ \ otherwise \end{cases} \quad \mu_{u_6^{(0)}}(x)(=)\begin{cases} \dfrac{x-0.667}{0.668-0.667} & ; \ where \ \ 0.667 \le x \le 0.668 \\ \dfrac{x-0.668}{0.667-0.668} & ; \ where \ \ 0.667 \le x \le 0.668 \\ 0 \ \ ; \ \ otherwise \end{cases}$$

and
$$\mu_{u_8^{(0)}}(x)(=)\begin{cases} \dfrac{x-0.081}{0.082-0.081} & ; \ where \ \ 0.081 \le x \le 0.082 \\ \dfrac{x-0.083}{0.082-0.083} & ; \ where \ \ 0.082 \le x \le 0.083 \\ 0 \ \ ; \ \ otherwise \end{cases}$$

In the following there are f.m.f of the fifth approximations using nine-points by the method of Lebmann's iteration process applied to equation (4) have been found as

$$\mu_{u_1^{(5)}}(x)(=)\begin{cases} \dfrac{x-0.1654236621554}{0.1655236621554-0.1654236621554} & ;where\ \ 0.1654236621554 \le x \le 0.1655236621554 \\ \dfrac{x-0.1656236621554}{0.1655236621554-2.06693458557129} & ;where\ \ 0.1655236621554 \le x \le 0.1656236621554 \\ 0 \quad ; \quad otherwise \end{cases}$$

$$\mu_{u_2^{(5)}}(x)(=)\begin{cases} \dfrac{x-0.6600579802214}{0.6599579802214-0.659957980221554} & ;\ where\ \ 0.6599579802214 \le x \le 0.6600579802214 \\ \dfrac{x-0.6601579802214}{0.6600579802214-0.6601579802214} & ;\ where\ \ 0.6600579802214 \le x \le 0.6601579802214 \\ 0 \quad ; \quad otherwise \end{cases}$$

$$\mu_{u_3^{(5)}}(x)(=)\begin{cases} \dfrac{x-1.4839162514350}{1.4838162514350-1.4839162514350} & ;\ where\ \ 1.4838162514350 \le x \le 1.4839162514350 \\ \dfrac{x-1.4840162514350}{0.6600579802214-1.4840162514350} & ;\ where\ \ 1.4839162514350 \le x \le 1.4840162514350 \\ 0 \quad ; \quad otherwise \end{cases}$$

$$\mu_{u_4^{(5)}}(x)(=)\begin{cases} \dfrac{x-0.0738575356564}{0.0739575356564-0.0738575356564} & ;where\ \ 0.0738575356564 \le x \le 0.0739575356564 \\ \dfrac{x-0.0740575356564}{0.0738575356564-0.0740575356564} & ;where\ \ 0.0739575356564 \le x \le 0.0740575356564 \\ 0 \quad ; \quad otherwise \end{cases}$$

$$\mu_{u_5^{(5)}}(x)(=)\begin{cases} \dfrac{x-0.293749250375}{0.2936492250375-0.293749250375} & ;where\ \ 0.0.2936492250375 \le x \le 0.293749250375 \\ \dfrac{x-0.2938492250375}{0.0738575356564-0.2938492250375} & ;where\ \ 0.293749250375 \le x \le 0.2938492250375 \\ 0 \quad ; \quad otherwise \end{cases}$$

$$\mu_{u_6^{(5)}}(x)(=)\begin{cases} \dfrac{x-0.6597130042601}{0.6597130042601-0.6596130042601} & ;where\ \ 0.6596130042601 \le x \le 0.6597130042601 \\ \dfrac{x-0.6598130042601}{0.6597130042601-0.6598130042601} & ;where\ \ 0.6597130042601 \le x \le 0.6598130042601 \\ 0 \quad ; \quad otherwise \end{cases}$$

$$\mu_{u_7^{(5)}}(x)(=)\begin{cases} \dfrac{x-0.0186651402525}{0.0185651402525-0.0186651402525} & ;where\ \ 0.0185651402525 \le x \le 0.0186651402525 \\ \dfrac{x-0.0187651402525}{0.0185651402525-0.0187651402525} & ;where\ \ 0.0186651402525 \le x \le 0.0187651402525 \\ 0 \quad ; \quad otherwise \end{cases}$$

$$\mu_{u_8^{(4)}}(x)(=)\begin{cases} \dfrac{x-0.0736125597645}{0.0735125597645-0.0736125597645} & ;where \quad 0.0735125597645 \le x \le 0.0736125597645 \\ \dfrac{x-0.0737125597645}{0.0735125597645-0.0737125597645} & ;where \quad 0.0736125597645 \le x \le 0.0737125597645 \\ 0 \quad ; \quad otherwise & \end{cases}$$

$$\mu_{u_9^{(5)}}(x)(=)\begin{cases} \dfrac{x-0.1650157520562}{0.1649157520562-0.1650157520562} & ; \quad where \quad 0.1649157520562 \le x \le 0.1650157520562 \\ \dfrac{x-0.1651157520562}{0.1649157520562-0.1651157520562} & ; \quad where \quad 0.1650157520562 \le x \le 0.1651157520562 \\ 0 \quad ; \quad otherwise & \end{cases}$$

5. CONCLUSION

For the given initial values the fourth approximations to solve the above example numerically is very significant results in comparison with example solved in [8] using five points only. However may increased the accuracy as desired if we take more iterations. As well, using nine-points is more accurate than five points.

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