

# Presentation of Conjugacy Classes in the Partial Order - Preserving Transformation Semigroup with the aid of graph

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## Abstract

Let  $P_n$  denote the set of all partial transformation semigroup in a finite set  $X_n = \{1, 2, 3, \dots, n\}$  that is the set of all mapping from  $X_n \rightarrow X_n$ . The aim of this paper is to study the elements of partial order - preserving transformation semigroup and arranged the elements in their respective conjugacy classes with the aids of unlabelled graph.

**Keywords:** Semigroup, transformation semigroup, conjugacy, conjugacy classes, graph, partial order – preserving transformation semigroup.

## 1. Introduction

The number of conjugacy classes of transformation semigroups have been studied by Ugbene, I. J, and S. O Makanjuola [1] applied the use of path structure, that is the circuit and proper path decomposition. Indeed, the alternative approach was adopted here is to effectively use unlabelled graph to arranged the elements of partial transformation semigroup in their respective conjugacy classes.

The general study of partial order - preserving transformation semigroup was initiated by Umar [2], [3], he showed the order of partial order decreasing transformation semigroup is  $\sum_{n=0}^n \binom{n}{r} \binom{n+r-1}{r}$ . Howie J.M. [4], stated that a mapping in full transformation semigroup is called order - preserving if for all  $i, j \in \{1, 2, 3, \dots, n\}$ ,  $i \leq j \Rightarrow i\alpha \leq j\alpha$ . The semigroup of partial order – preserving transformation semigroup of  $X_n$  will be denoted by  $PO_n$ . Ricard F.P. [5], studied the elements of conjugacy class of Symmetric  $S_n$  and full transformation semigroup  $T_n$ .

## 2. Preliminaries

The following theoretical definitions and results come from [4], [6], [7], [8]. For more background in graph and semigroup the reader is referred to [9],[6],[7],[8].

Let  $X_n$  be a finite set and denote by  $T_n$  the set of all functions  $\alpha : X_n \rightarrow X_n$ . then  $T_n$  is the full transformation semigroup on  $X_n$  with operation of composition of functions.

Let  $X_n = \{1, 2, 3, \dots, n\}$ . Then a (partial) transformation semigroup is defined if  $\alpha : \text{Dom} \subseteq X_n \rightarrow \text{Im}\alpha$  is said to be full or total if  $\text{Dom}\alpha = X_n$ . otherwise it is called strictly partial. The set of all partial transformation of  $n$  object forms a semigroup under the usual composition of functions. It is denoted by  $P_n$  when it is strictly partial,  $T_n$  when it is full or total and  $I_n$  when is partial one – one,

A transformation  $\alpha : \text{Dom} \subseteq X_n \rightarrow X_n$  is said to be order - preserving if  $i \leq j$  in  $\text{Dom}\alpha$ ,  $i \leq j \Rightarrow i\alpha \leq j\alpha$ .

In any group, the elements  $a$  and  $b$  are conjugates if  $a = cbc^{-1}$ , for some  $c \in G$ .

**Result 1 ([5]), (Theorem 6.2.1).** The relation  $\alpha \sim \beta$  if  $\sigma^i(\alpha) = \sigma^j(\beta)$ , for some  $i, j$  is an equivalence relation.

## 3. Methodology

The following notations will be used

**Two line notation**

Let  $\alpha \in T_n$ , then  $\alpha = \begin{pmatrix} 1 & 2 & 3 & \dots & n \\ \pi(1) & \pi(2) & \pi(3) & \dots & \pi(n) \end{pmatrix}$ . For instance. If  $\alpha \in T_5$ , and  $\pi(1) = 5, \pi(2) = 4, \pi(3) = 1, \pi(4) = 2, \pi(5) = 3$ , then  $\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix}$

**Directed Graph notation**

Consider an element  $\alpha \in T_n$ , where  $\alpha = (i, j, k, \dots, l)$  in one – line notation. Draw  $n$  vertices and labeled then  $i, j, k, \dots, l$ . Indicate  $\pi(i) = j$  by drawing a directed line segment from  $i$  to  $j$ .

For instance, if  $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \in T_5$ , the directed graph is shown in the figure 1.

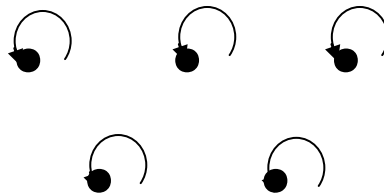


Figure 1

Since the conjugacy is equivalence relations, so the distinct conjugacy classes partitions group  $G$ . This means that  $G$  has  $n$  conjugacy classes,  $C_1, C_2, C_3, \dots, C_n$ , then  $C_i \cap C_j = \emptyset$  for  $i \neq j$

and  $\cup_i C_i = G$ . Two elements of  $PO_n$  are in the same conjugacy class if and only if they have the same graph structures.

**3.1 The conjugacy classes of  $PO_1$**

**When  $n = 1$ ,**

The conjugacy class,  $C_1 = \left\{ \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$ , consists of elements of the form;



The conjugacy class,  $C_2 = \left\{ \begin{pmatrix} 1 \\ - \end{pmatrix} \right\}$ , consists of elements of the form;



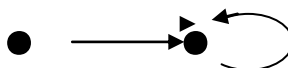
We see that  $PO_1$  has two conjugacy classes.

### 3.2 The Conjugacy Classes in $PO_2$

The conjugacy class,  $C_1 = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix} \right\}$ , consists of elements of the form;



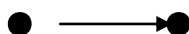
The conjugacy class,  $C_2 = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & 2 \end{pmatrix} \right\}$ , consists of elements of the form;



The conjugacy class,  $C_3 = \left\{ \begin{pmatrix} 1 & 2 \\ 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 \\ - & 2 \end{pmatrix} \right\}$ , consists of elements of the form;



The conjugacy class,  $C_4 = \left\{ \begin{pmatrix} 1 & 2 \\ - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 2 & - \end{pmatrix} \right\}$ , consists of elements of the form;



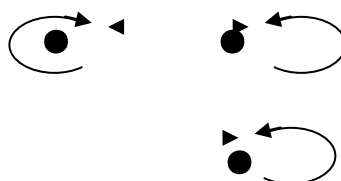
The conjugacy class,  $C_5 = \left\{ \begin{pmatrix} 1 & 2 \\ - & - \end{pmatrix} \right\}$ , consists of elements of the form;



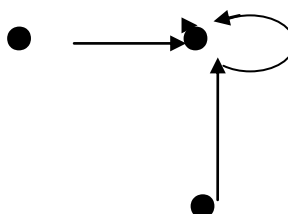
We see that  $PO_2$  has five conjugacy classes;

### 3.3 The Conjugacy Classes in $PO_3$ .

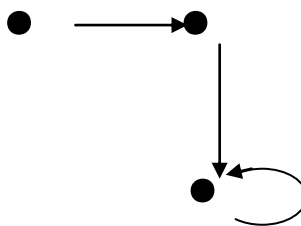
The conjugacy class,  $C_1 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \right\}$ , consists of elements of the form;



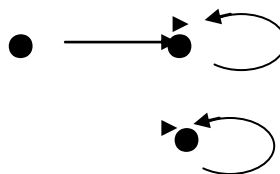
The conjugacy class,  $C_2 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & 3 \end{pmatrix} \right\}$ , consists of elements of the form;



The conjugacy class,  $C_3 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 3 \end{pmatrix} \right\}$ , consists of elements of the form;



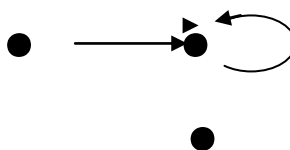
The conjugacy class,  $C_4 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & 3 \end{pmatrix} \right\}$  consists of elements of the form;



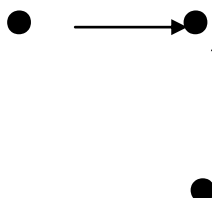
The conjugacy class,  $C_5 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ - & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ - & 3 & 3 \end{pmatrix} \right\}$$

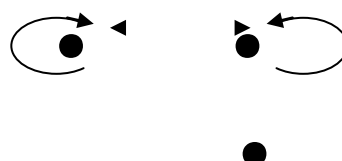
consists of elements of the form;



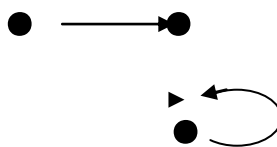
The conjugacy class,  $C_6 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ - & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 3 & - \end{pmatrix} \right\}$ , consists of elements of the form;



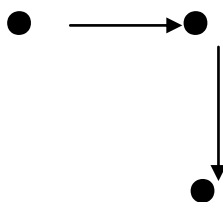
The conjugacy class,  $C_7 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & 2 & 3 \end{pmatrix} \right\}$ , consists of elements of the form;



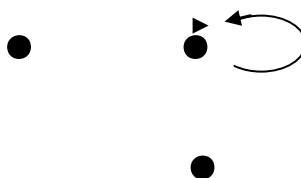
The conjugacy class,  $C_8 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & - & 3 \end{pmatrix} \right\}$ ,  
 consists of elements of the form;



The conjugacy class,  $C_9 = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ - & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & - \end{pmatrix} \right\}$ , consists of elements of the form;



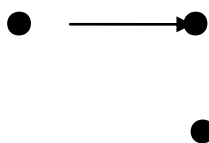
The conjugacy class,  $C_{10} = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ 1 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & - & 3 \end{pmatrix} \right\}$ , consists of elements of the form;



The conjugacy class,  $C_{11} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 \\ - & 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 3 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ - & 3 & - \end{pmatrix} \right\},$$

consists of elements of the form;



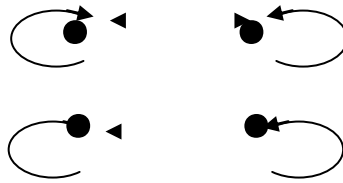
The conjugacy class,  $C_{12} = \left\{ \begin{pmatrix} 1 & 2 & 3 \\ - & - & - \end{pmatrix} \right\}$ , consists of elements of the form;



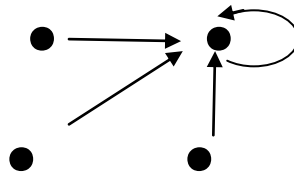
We see that  $PO_3$  has twelve conjugacy classes;

### 3.4 The Conjugacy Classes in $PO_4$ .

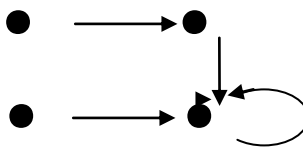
The conjugacy class,  $C_1 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \right\}$ , consists of elements of the form;



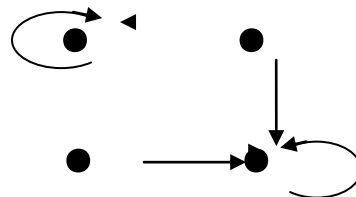
The conjugacy class,  $C_2 =$   
 $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix} \right\},$   
 consists of elements of the form;



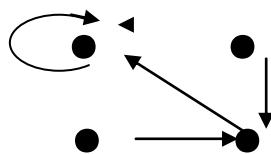
The conjugacy class,  $C_3 =$   
 $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 2 & 3 \end{pmatrix} \right.$   
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 4 & 4 \end{pmatrix} \right\},$   
 consists of elements of the form;



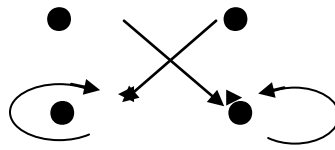
The conjugacy class,  $C_4 =$   
 $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 3 & 3 \end{pmatrix} \right.$   
 $\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & 3 & 4 \end{pmatrix} \right\},$   
 consists of elements of the form;



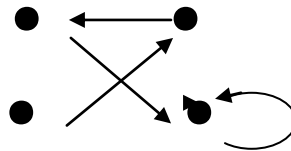
The conjugacy class,  $C_5 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & 4 & 4 \end{pmatrix} \right\},$  consists of elements of the form;



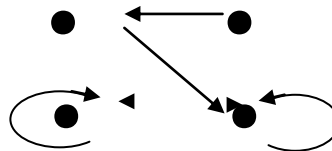
The conjugacy class,  $C_6 =$   
 $\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 4 & 4 \end{pmatrix} \right\},$   
 consists of elements of the form;



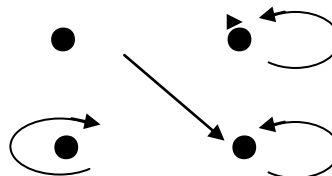
The conjugacy class,  $C_7 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 4 \end{pmatrix} \right\}$ , consists of elements of the form;



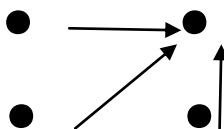
The conjugacy class,  $C_8 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 3 & 4 \end{pmatrix} \right\}$ , consists of elements of the form;



The conjugacy class,  $C_9 = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 3 & 4 \end{pmatrix} \right\}$  consists of elements of the form;

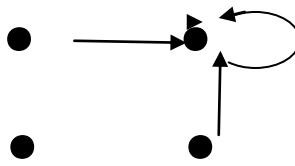


The conjugacy class,  $C_{10} = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & 4 & - \end{pmatrix} \right\}$ , consists of elements of the form;



The conjugacy class,  $C_{11} = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & - & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & - & 4 \end{pmatrix} \right\}$ , consists of elements of the form;

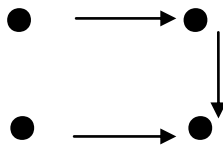
consists of elements of the form;



The conjugacy class,  $C_{12} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 2 & 3 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 4 & - \end{pmatrix} \right\},$$

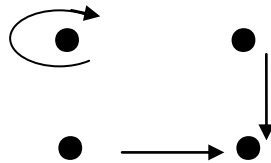
consists of elements of the form;



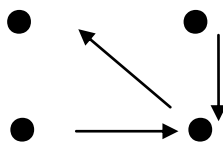
The conjugacy class,  $C_{13} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 2 & 4 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & 4 & - \end{pmatrix} \right\},$$

consists of elements of the form;



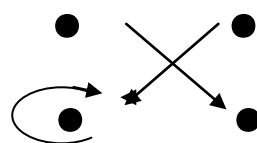
The conjugacy class,  $C_{14} = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & 4 & - \end{pmatrix} \right\}$ , consists of elements of the form;



The conjugacy class,  $C_{15} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 4 & 4 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & 4 & - \end{pmatrix} \right\},$$

consists of elements of the form;

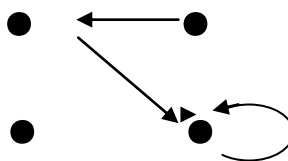


The conjugacy class,  $C_{16} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & 4 & 4 \end{pmatrix} \right\}$$



$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 3 & - \end{pmatrix}$ ,  
 consists of elements of the form;



The conjugacy class,  $C_{17} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & 3 & 4 \end{pmatrix} \right.$$

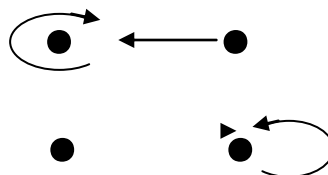
$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & 3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

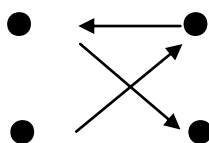
$$\begin{pmatrix} 1 & 1 & - & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & - & 2 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 4 & - & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 2 & 2 & - & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix}$$

$$\left. \begin{pmatrix} 1 & 1 & 3 & - \\ 1 & 2 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 3 & 3 & - \\ 1 & 2 & 3 & - \end{pmatrix} \begin{pmatrix} 2 & 2 & 3 & - \\ 1 & 2 & 3 & - \end{pmatrix} \right\}$$

consists of elements of the form;



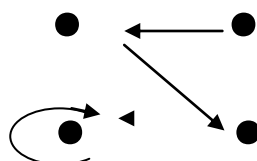
The conjugacy class,  $C_{18} = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & - \end{pmatrix} \right\}$ , consists of elements of the form;



The conjugacy class,  $C_{19} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & - & 4 \end{pmatrix} \right\},$$

consists of elements of the form;

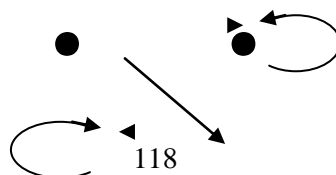


The conjugacy class,  $C_{20} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & - & 3 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 3 & 4 \end{pmatrix} \right\}$$

consists of elements of the form;

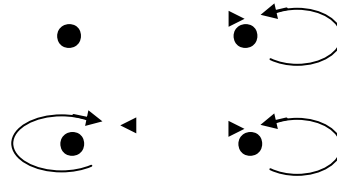




The conjugacy class,  $C_{21} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & - \end{pmatrix} \right\},$$

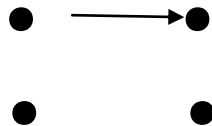
consists of elements of the form;



The conjugacy class,  $C_{22} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 2 & 2 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & - & 3 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 3 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 4 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & - & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 4 & - & - \end{pmatrix} \right\}$$

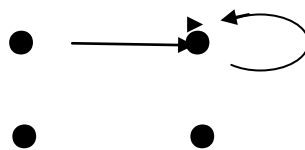
consists of elements of the form;



The conjugacy class,  $C_{23} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 1 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & - & 2 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 2 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & 3 & - \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 4 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & - & - & 4 \end{pmatrix} \right\}$$

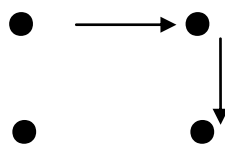
consists of elements of the form;



The conjugacy class,  $C_{24} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 2 & 3 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & 4 & - \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & - & - \end{pmatrix} \right\}$$

consists of elements of the form;



The conjugacy class,  $C_{25} =$

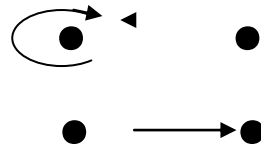
$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & - & 3 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & - & 2 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 2 & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 4 & - & - \end{pmatrix} \right\}$$

consists of elements of the form;

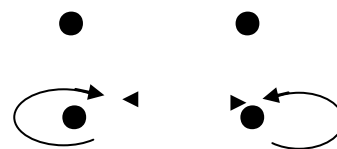


The conjugacy class,  $C_{26} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & 3 & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & - & - \end{pmatrix} \right\}$$

consists of elements of the form;

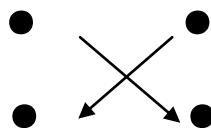


The conjugacy class,  $C_{27} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & 4 & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & - & - \end{pmatrix} \right\}$$

consists of elements of the form;



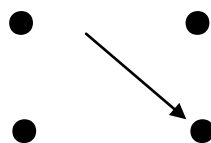
The conjugacy class,  $C_{28} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 1 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & 2 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 3 & - & - \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 4 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & - & - & - \end{pmatrix} \right\}$$

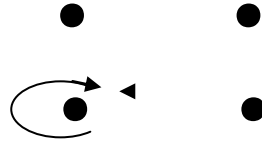
consists of elements of the form;



The conjugacy class,  $C_{29} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & - & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & 2 & - & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & 4 \end{pmatrix} \right\},$$

consists of elements of the form;



The conjugacy class,  $C_{30} = \left\{ \begin{pmatrix} 1 & 2 & 3 & 4 \\ - & - & - & - \end{pmatrix} \right\}$ , consists of elements of the form;



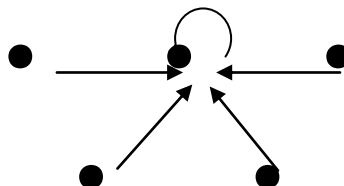
Therefore, we see that  $PO_4$  has thirty conjugacy classes.

### 3.5 The Conjugacy Classes in $PO_5$ .

The conjugacy class,  $C_1 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 4 & 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 5 & 5 & 5 & 5 \end{pmatrix} \right\}$$

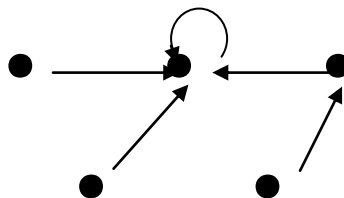
consists of elements of the form;



The conjugacy class,  $C_2 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 2 & 3 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 5 & 5 & 5 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 5 & 5 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 5 & 5 & 5 & 5 \end{pmatrix} \right\},$$

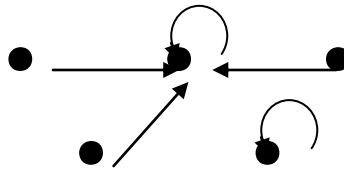
consists of elements of the form;



The conjugacy class,  $C_3 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 4 & 4 & 4 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 5 & 5 & 5 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 2 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 4 & 4 & 4 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;

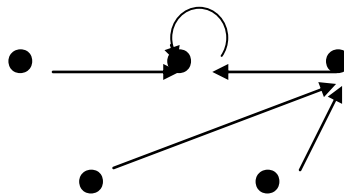


The conjugacy class,  $C_4 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 3 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 3 & 3 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 4 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 5 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 4 & 5 & 5 & 5 \end{pmatrix} \right\},$$

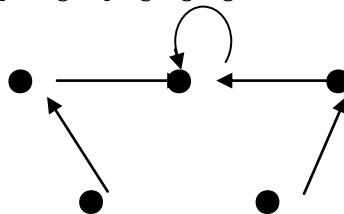
consists of elements of the form;



The conjugacy class,  $C_5 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;

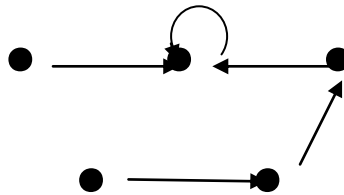


The conjugacy class,  $C_6 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 5 & 5 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 5 & 5 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;



The conjugacy class,  $C_7 =$

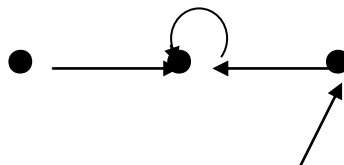
$$\left[ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 2 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 2 & 4 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 1 & 3 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 3 & 4 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 3 & 5 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 4 & 5 & 5 & 5 \end{pmatrix}, \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 3 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 4 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 4 & 4 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;

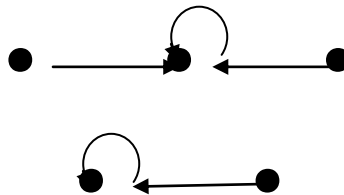




The conjugacy class,  $C_8 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 5 & 5 & 5 \\ 2 & 2 & 4 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 5 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 4 & 4 \\ 2 & 2 & 3 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 5 & 5 \\ 3 & 3 & 3 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 4 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 3 & 3 & 3 \\ 3 & 3 & 3 & 4 & 4 \end{pmatrix} \right\},$$

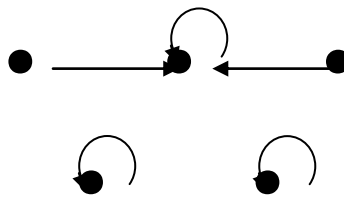
consists of elements of the form;



The conjugacy class,  $C_9 =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 5 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 2 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 3 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 3 & 3 \\ 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 4 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 5 & 5 \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 4 & 4 & 5 \end{pmatrix} \right\},$$

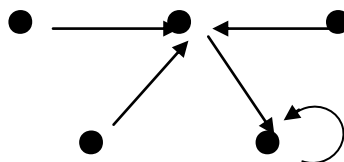
consists of elements of the form;



The conjugacy class,  $C_{10} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 2 & 2 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 4 & 4 & 5 & 5 \end{pmatrix} \right\},$$

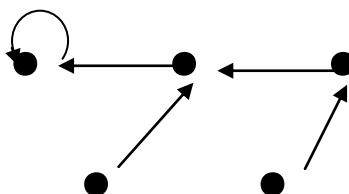
consists of elements of the form;



The conjugacy class,  $C_{11} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 2 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 4 & 5 & 5 \end{pmatrix} \right\},$$

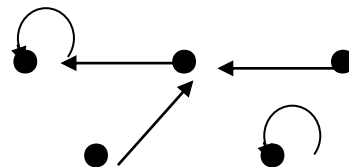
consists of elements of the form;



The conjugacy class,  $C_{12} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 2 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 4 & 5 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 4 & 4 & 5 \end{pmatrix} \right\},$$

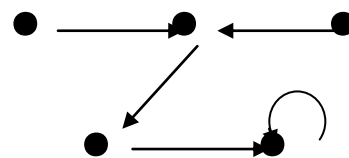
consists of elements of the form;



The conjugacy class,  $C_{13} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 4 & 5 & 5 \end{pmatrix} \right\},$$

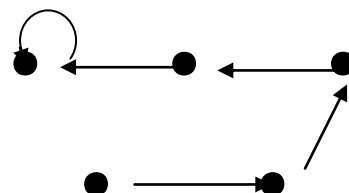
consists of elements of the form;



The conjugacy class,  $C_{14} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 5 & 5 \end{pmatrix} \right\},$$

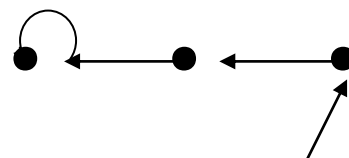
consists of elements of the form;



The conjugacy class,  $C_{15} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 5 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 4 & 4 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;

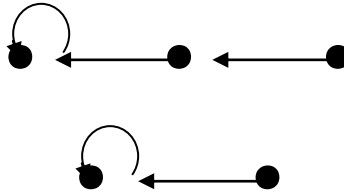




The conjugacy class,  $C_{16} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 3 & 3 & 4 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 4 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 3 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 3 & 4 & 4 \end{pmatrix} \right\},$$

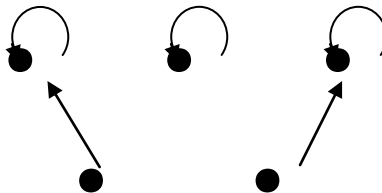
consists of elements of the form;



The conjugacy class,  $C_{17} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 3 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 3 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 3 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 4 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 4 & 4 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 3 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 3 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 4 & 4 & 5 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 4 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 3 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 3 & 4 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 3 & 5 & 5 \end{pmatrix} \right\},$$

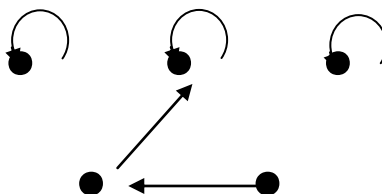
consists of elements of the form;



The conjugacy class,  $C_{18} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 4 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 3 & 4 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;

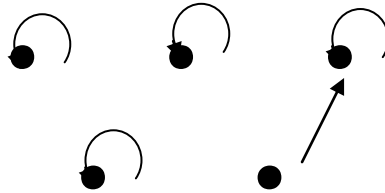


The conjugacy class,  $C_{19} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 3 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 3 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 4 \end{pmatrix}, \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 3 & 4 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 3 & 4 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;

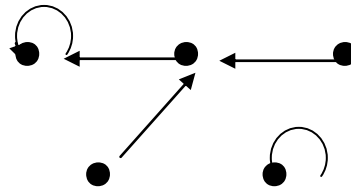




The conjugacy class,  $C_{20} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 3 & 3 \end{pmatrix} \right\},$$

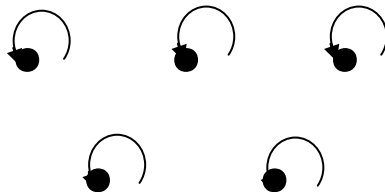
consists of elements of the form;



The conjugacy class,  $C_{21} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & 5 \end{pmatrix} \right\},$$

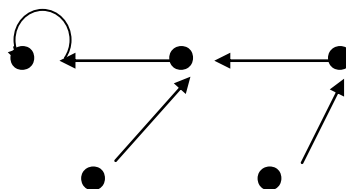
consists of elements of the form;



The conjugacy class,  $C_{22} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 4 & 5 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;



The conjugacy class,  $C_{23} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & 1 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 1 & - & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & - & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 1 & 1 & 1 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & 2 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & 2 & - & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 2 & - & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 2 & 2 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 2 & 2 & 2 \end{pmatrix} \right.$$

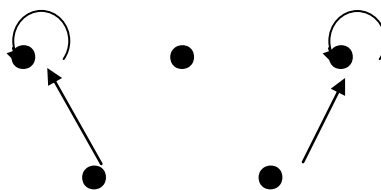
$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & 3 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & - & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & - & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & - & 3 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 3 & 3 & 3 & 3 \end{pmatrix} \right.$$

$$\left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 4 & 4 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 4 & 4 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & 4 & - & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 4 & - & 4 & 4 & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 4 & 4 & 4 & 4 \end{pmatrix} \right\}$$





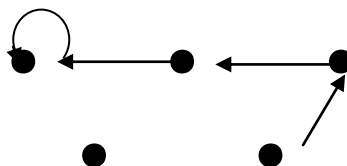




The conjugacy class,  $C_{34} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 3 & - \\ 2 & 3 & 4 & 4 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & - & 3 \\ 2 & 3 & 5 & - & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & - & 2 & 4 \\ 2 & 4 & - & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 1 & 3 & 4 \\ 3 & - & 4 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 5 \\ - & 3 & 4 & 5 & 5 \end{pmatrix} \right\},$$

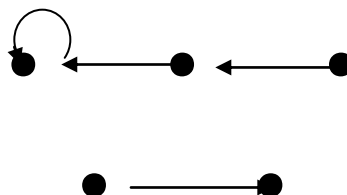
consists of elements of the form;



The conjugacy class,  $C_{35} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & - & 2 & 3 \\ 2 & 3 & 3 & 3 & 4 \\ 2 & 3 & 3 & 5 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 1 & 2 & 3 \\ 2 & - & 4 & 5 & 5 \\ 2 & - & 4 & 5 & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & - & 4 \\ 2 & 3 & 3 & - & 4 \\ 3 & 4 & 5 & - & 5 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 5 & - \\ 2 & - & 3 & 3 & 4 \\ 3 & 4 & - & 5 & 5 \end{pmatrix} \right\},$$

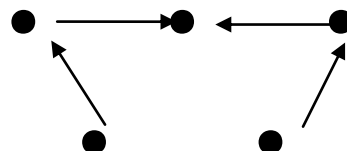
consists of elements of the form;



The conjugacy class,  $C_{36} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 1 & 2 & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & - & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 4 & 5 & 5 & - \end{pmatrix} \right\},$$

consists of elements of the form;

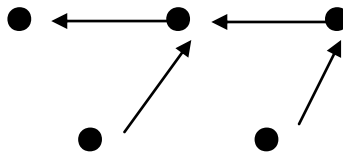


The conjugacy class,  $C_{37} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & 4 & - \\ 1 & 2 & 2 & - & 3 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & 2 & - & 5 \\ 1 & 2 & - & 2 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 1 & - & 2 & 5 \\ 1 & - & 3 & 3 & 4 \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 1 & 3 & 5 \\ 1 & 3 & 4 & 4 & - \end{pmatrix}, \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 2 & 3 & - \\ 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & 5 & - & 5 \end{pmatrix} \right\}$$



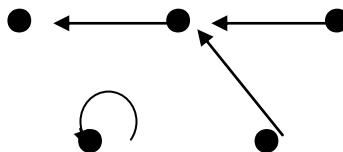




The conjugacy class,  $C_{45} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & 2 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 4 & 4 & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 4 & - & 5 \end{pmatrix} \right\},$$

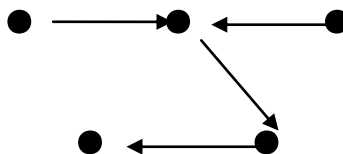
consists of elements of the form;



The conjugacy class,  $C_{46} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 2 & 3 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 4 & 5 & - \end{pmatrix} \right\},$$

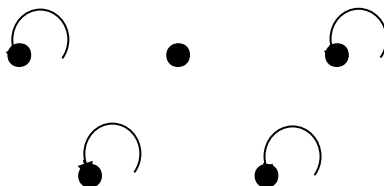
consists of elements of the form;



The conjugacy class,  $C_{47} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 4 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & - & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 2 & 3 & 4 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;



The conjugacy class,  $C_{48} =$

$$\left\{ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & - & 4 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 3 & 5 & - \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & - & 3 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 2 & 4 & - & 5 \end{pmatrix} \right. \\ \left. \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & - & 2 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 1 & 3 & - & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ - & 1 & 3 & 4 & 5 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & - & 3 & 4 & 5 \end{pmatrix} \right\},$$

consists of elements of the form;













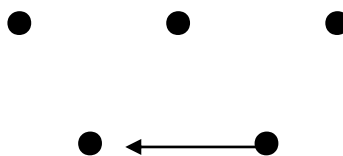








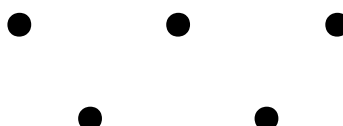




The conjugacy class,  $C_{75} =$

$$\left\{ \left( \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \\ - & - & - & - & - \end{array} \right) \right\},$$

consists of elements of the form;



We see that  $PO_5$  has seventy five conjugacy classes.

#### 4. Conclusion

The number of conjugacy classes of  $PO_n$  generate the sequences 2,5,12,30,75, ..., A00080 of the Online Encyclopedia of Integer Sequences. (QEIS) . It was discovered that the unlabelled graph can be used to determined the conjugacy class of partial order - preserving transformation semigroup.

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