An Improved Tabular Technique for Presenting the Various Algorithms for Finding Initial Basic Feasible Solutions to Transportation Problems

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Abstract

The special structure of the transportation problem allows securing a non-artificial starting basic solution using one of the three methods: Northwest-corner Method, Least-cost Method and Vogel Approximation Method. The difference among the three methods is the "quality" of the starting basic solution they produce, in the sense that a good starting solution yields an optimal solution in a fewer number of iterations. In general, the Vogel Approximation Method yields the best starting basic solution, and the Northwest-corner Method yields the worst. However, the Northwest-corner Method involves the least computations. Extensive and critical examination regarding the usage and improvement of these methods abound in several publications with trivial consideration given to the development of a near-ideal technique for presenting these algorithms. This paper develops and implements an all-encompassing tabular technique for presenting the algorithms for these three methods.

Keywords: Transportation problem, Improved Tabular Technique, Initial Basic Feasible Solution

1. Introduction

The transportation model is a special class of the linear programming problem. It deals with situations in which a commodity is shipped from "sources" to "destinations" [Taha, 1997; Reeb & Leavengood, 2002; Bijulal, 2013]. The objective is to determine the amounts shipped from each source to each destination that minimize the total shipping cost, while satisfying both the supply limits and the demand requirements [Taha, 1997; Sharma, 2009; Bijulal, 2013]. The model assumes that the shipping cost on a given route is directly proportional to the number of units shipped on that route [Taha, 1997; Reeb & Leavengood, 2002]. In general, the transportation model can be extended to areas other than the direct transportation of a commodity, including, among others, inventory control, employment scheduling, and personnel assignment [Taha, 1997; Reeb & Leavengood, 2002; Sharma, 2009].

The general problem is represented by the network below. In the network, there are m sources and n destinations, each represented by a "node". There are also "arcs" linking the sources and destinations. These arcs represent the routes between the sources and the destinations. Arcs $[i, j]$ joining source i to destination j carry two pieces of information: [1] the transportation cost per unit c_{ij}, and [2] the amount shipped x_{ij}. The amount of supply at source i is a_i and the amount of demand at destination j is b_j [Taha, 1997; Wang, 2008, Effanga, 2011].

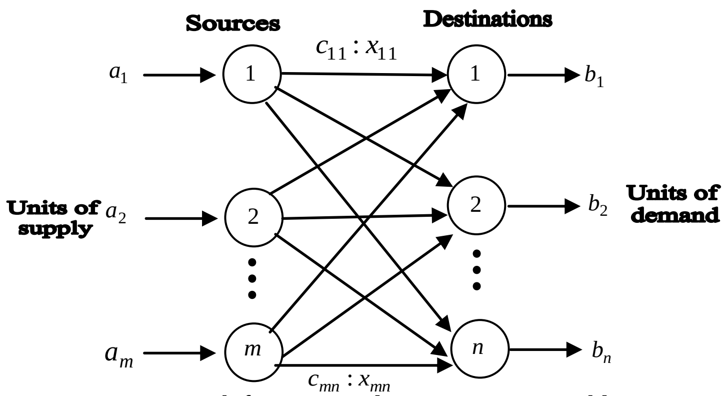

Figure. Network for a General Transportation Problem

The objective of a transportation model is to determine the unknowns x_{ij} that will minimize the total transportation cost while satisfying all the supply and demand restrictions [Taha, 1997; Sharma, 2009; Effanga, 2011]. Mathematically, the problem, in general, may be stated as follows:

$$Min\ \ C = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

$$s.t:\ \ \sum_{j=1}^{n} x_{ij} = S_i \quad i = 1, 2, 3, \cdots, m$$

$$\sum_{i=1}^{m} x_{ij} = D_j \quad j = 1, 2, 3, \cdots, n$$

$$x_{ij} \geq 0,\ \forall\ i,\ j$$

where S_i is the supply at origin i, D_j is the demand at destination j, x_{ij} is the number of units shipped from origin i to destination j, $C = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$ is the total shipping cost, and $\sum_{i=1}^{m} S_i = \sum_{j=1}^{n} D_j$.

The transportation algorithm follows the exact steps of the simplex. However, instead of using the regular simplex tableau, advantage is taken of the special structure of the transportation model to present the algorithm in a more convenient form [Taha, 1997; Reeb & Leavengood, 2002; Sharma, 2009].

There are several methods available to obtain an initial basic feasible solution. But the three major methods for a transportation problem are the Northwest-corner Method, Least-cost Method and Vogel Approximation Method [Taha, 1997; Sharma, 2009]. Each of these methods has variously been previously presented using a continuum of tables for any given transportation problem as can be seen in the publications of Taha [1997], Reeb & Leavengood [2002], Wang [2008], Sharma, [2009], Effanga [2011] and Bijulal [2013]. It is in this vain that this paper presents an improvement of the presentation technique for the three major methods which utilizes just one table for the routine with better detail and clarity than previous techniques; and a case study from Effanga [2011] has been used for the implementation.

2. Methodology

The proposed presentation technique utilizes a "well-designed grid" which extends the tabular presentation of the given problem beyond the supply rows and demand columns to as many more columns and rows that would be needed in the course of the solution process. In this work, we call the new columns and rows are called "stages". Variations of the well-designed grid for each method have been considered for the purpose of flexibility as explained below, and illustrated in the next section.

2.1 Northwest Corner Rule

For the presentation of the Northwest Corner Rule, its laid down algorithm is applied, as we iteratively extend the columns and the rows [from stage to stage] until the supplies are exhausted and demands satisfied.

2.2 Least Cost Rows Method

The presentation, using this method, adheres to the laid-down algorithm of the method. It also proceeds horizontally along and vertically down the well-designed grid in stages just like the case of the North-west corner rule except for the introduction of a set of well-defined "keys" [based on the presenter's discretion]; which are used in an ordered progression from the beginning of the solution process to the end. The keys [in their order] are used in the solution steps of the algorithms and in the tabular presentation to ensure clarity when deleting rows and columns in accordance with the laid down algorithms.

2.3 Vogel Approximation Method

In the case of the presentation technique for Vogel Approximation Method we acknowledge that the algorithm for this technique includes rows and columns differences. Hence, as a variation of the presentation technique for the Least-cost method, we have incorporated additional columns and rows just after each stage to cater for the column and row differences respectively.

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3. Illustrating The Proposed Tabular Presentation

Table 1. Case study

	Destination				
Origin	**1**	**2**	**3**	**4**	S_i
1	8	6	10	9	**35**
2	9	12	13	7	**50**
3	14	9	16	5	**40**
D_i	**45**	**20**	**30**	**30**	

In order to illustrate the use of the proposed tabular technique, we have employed the use of a case study from Effanga [2011]. This case study is a balanced transportation problem, and is presented in table 1. The problem requires one to find the initial basic feasible solution using the Northwest-corner Method, Least-cost Method and Vogel Approximation Method. We have used the proposed improved presentation technique for the three algorithms in finding the initial basic feasible solution to this problem and have obtained table 2 below. However, the illustrations of the solution process are shown in the tables 3 through 7.

Table 2. Initial basic feasible [IBF] solutions

S/N	Method	IBF Solution
1	Northwest-corner	1180
2	Least-cost	1080
3	Vogel Approximation	1020

Table 3. Northwest-corner method

	Destination										
Origin	**1**	**2**	**3**	**4**	S_i	Stag1	Stag2	Stag3	Stag4	Stag5	Stag6
1	8	6	10	9	**35**	35/35=0	0	0	0	0	0
2	9	12	13	7	**50**	50	50/10=40	40/20=20	20/20=0	0	0
3	14	9	16	5	**40**	40	40	40	40	40/10=30	30/30=0
D_i	**45**	**20**	**30**	**30**	125						
Stag1	45/35=10	20	30	30		90					
Stag2	10/10=0	20	30	30			80				
Stag3	0	20/20=0	30	30				60			
Stag4	0	0	30/20=10	30					40		
Stag5	0	0	10/10=0	30						30	
Stag6	0	0	0	30/30=0							0

Table 4. Least-cost method

	Destination										
Origin	**1**	**2**	**3**	**4**	**S_i**	Stag1	Stag2	Stag3	Stag4	Stag5	Stag6
1	8	6	10	9	**35**	35	35/20=15	15/15=0	0	0	0
2	9	12	13	7	**50**	50	50	50	50/30=20	20/20=0	0
3	14	9	16	5	**40**	40/30=10	10	10	10	10	10/10=0
D_i	**45**	**20**	**30**	**30**	125						
Stag1	45	20	30	30/30=0		95					
Stag2	45	20/20=0	30	0			75				
Stag3	45/15=30	0	30	0				60			
Stag4	30/30=0	0	30	0					30		
Stag5	0	0	30/20=10	0						10	
Stag6	0	0	10/10	0							0

Table 5. Key for least-cost method

Order of Deletion	**Name of Colour**	**Colour Description**
First – Fourth column	Red	
Second – Second column	Yellow	
Third – Cells (1, 1) & (1, 3)	Green	
Fourth – Cells (2, 1) & (3, 1)	Deep Blue	
Fifth – Cell (2, 3)	Blue	
Sixth – Cell (3, 3)	White	

Table 6. Vogel approximation method

	Destination					St1		St2		St3		St4		St5	
Origin	**1**	**2**	**3**	**4**	**S_i**	Rd1	1	Rd2	2	Rd3	3	Rd4	4	Rd5	5
1	8	6	10	9	**35**	8/6=2	35	8/6=2	35	8/6=2	35/10=25	10/8=2	25	10	25
2	9	12	13	7	**50**	9/7=2	50	12/9=3	50	12/9=3	50	13/9=4	50/45=5	13	5/5=0
3	14	9	16	5	**40**	9/5=4	40/30=10	14/9=5	10/ 10= 0	-	0	-	0	-	0
D_i	**45**	**20**	**30**	**30**	125										
Cd1	9/8=1	9/6=3	13/10=3	7/5=2											
1	45	20	30	30/30=0			95								
Cd2	9/8=1	9/6=3	13/10=3	-											
2	45	20/10=10	30	0					85						
Cd3	9/8=1	12/6=6	13/10=3	-											
3	45	10/10=0	30	0							75				
Cd4	9/8=1	-	13/10=3	-											
4	45/45=0	0	30	0									30		
Cd5	-	-	13/10=3	-											
5	0	0	30/5=25	0											25

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Cd6	-	-	10	-											
6	0	0	25/25=0	0											

St6	
Rd6	6
10	25/25=0
-	0
-	0
	0

Table 7. Key for Vogel approximation method

Order of Deletion	**Name of Colour**	**Colour Description**
First – Fourth column	Red	
Second – Second row	Yellow	
Third – Second column	Green	
Fourth – Cells (1, 1) & (2, 1)	Deep Blue	
Fifth – Cell (2, 3)	Sky Blue	
Sixth – Cell (3, 3)	White	

4. Conclusion

The proposed tabular presentation technique for presenting the three major algorithms for finding initial basic feasible solutions to transportation problems is explicit and gives better clarity to the solution process. It simplifies all previous presentation patterns used for this same purpose in an all-encompassing manner. Hence, tendencies are that it could extend its implementation to variants of these three major algorithms that may be developed by researchers in research directions based on improving the existing algorithms. We therefore recommend that this proposed presentation technique should be incorporated into the available tool-kits for doing this routine.

References

Bijulal, D. (2013), "Transportation Problem", *MBA 202 – Operations Research*, College of Engineering, Trivaddium.

Effanga, E. O. (2011), "Transportation Problem", *Lecture Notes on STA 6022 – Mathematical Programming*. Department of Statistics, University of Calabar, Nigeria

Sharma, J. K.. (2009), "Transportation Problem", *Operations Research – Theory and Applications*, Delhi: Macmillan Publishers India Limited, 259 – 312.

Reeb, J. & Leavengood, S. (2002), "Transportation Problem – A Special Case for Linear Programming Problems", *Operations Research: Performance Excellence in the Wood Products Industry*.

Taha, H. A. (1997), "Transportation Problem", *Operations Research – Theory and Applications*, Prentice-Hall International, Inc, USA, 165 – 185.

Wang, J. Y. (2008), "Chapter 8 – The Transportation and Assignment Problems", *Operations Research*, College of Management, NCTU.s