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Some Fixed Point And Common Fixed Point Theorems In Usual Metric Spaces

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ABSTRACT

In the present paper we will try to find some fixed point and common fixed point theorems in usual metric spaces for rational expressions motivated by Dwivedi et.al [28]

Keywords: Common fixed point, Usual metric space, rational expression

1. Introduction and Preliminaries :

The Banch contraction principal [1] principal has been generalized by many mathematician's viz. Chu and Diza [6] , Sehgal [25] , Sharma and Rajput [11] Das and Gupta [7] , Jaggi [14] , Jaggi and Das [15] , Charrerjee [3] , Fisher [9] , Kannan [16] , Ciric [5] , Reich [22] , and others . We are introducing non-contraction mappings in usal metric spaces to extend this principal . As it is well known that a metric space (X ,d) is said to be usual metric space, if d(x ,y) = $|$x-y$|$ for all x,y $\in R$. In the present paper we find a fixed point theorem for usual metric space

2. Main Results:

Theorem2.1: Let T be mapping of usual metric space X into itself. If T satisfies the following conditions

2.1.1. T^2= I , Where I is the identity mapping .

2.1.2. $|$Tx-Ty$|$

$$\leq \alpha \frac{|x-Tx|.|x-Ty|+|x-y|^2}{|x-y|} + \beta \frac{|y-Tx|.|y-Ty|+|x-y|^2}{|x-y|} + \gamma\ |x-y|$$

$$+\delta \max\{\ |x-Tx|, |x-Ty|, |x-y|\ \}$$

For all x,y$\in X$, such that x$\neq y$ and $\alpha,\beta,\gamma,\delta > 0$ with 6$\alpha + 7\beta + 2\gamma + 8\delta < 4$ then T has a fixed point .

Futher , if $\alpha + \beta + \gamma + \delta < 1$ then T has unique fixed point.

Proof: Suppose x is a point in the usual metric space X.

Taking , y$= \frac{1}{2}$(T+I) (x), z=Ty , u=2y-z

We have, $|$z-x$|$ = $|$Ty-Ix$|$ = $|$Ty-$T^2x$$|$ = $|$Ty-TTx$|$

$$\leq \alpha \frac{|y-Ty|.|y-Ttx|+|y-Tx|^2}{|y-Tx|} + \beta \frac{|Tx-Ty|.|y-x|+|y-Tx|^2}{|y-Tx|} + \gamma\ |y-Tx|$$

$$+\delta max\{\ |y-Ty|, |y-ttx|, |y-Tx|\ \}$$

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$$= \alpha \frac{|y-Ty|.|y-x|+|y-Tx|^2}{|y-tx|} + \beta \frac{|Tx-Ty|.|Tx-x|+|y-Tx|^2}{|y-Tx|} + \gamma|y-Tx|$$

$$+\delta \max\{|y-Ty|, |y-x|, |y-Tx|\}$$

$$=\alpha \frac{|Tx-Ty|.\frac{1}{2}|Tx-x|+\frac{1}{4}|Tx-x|^2}{\frac{1}{2}|Tx-x|} + \beta \frac{[|Tx-y|.|y-Ty|].|Tx-x|+\frac{1}{4}|Tx-x|^2}{\frac{1}{2}|Tx-x|} + \gamma|Tx-x|$$

$$+\delta \max\{|y-Ty|, \frac{1}{2}|Tx-x|, \frac{1}{2}|Tx-x|\}$$

$$= \alpha[|y-Ty| + \frac{1}{2}|Tx-x|] + \beta[|Tx-x| + \frac{1}{2}|Ty-y| + \frac{1}{2}|Tx-x|]$$

$$+\gamma\frac{1}{2}|Tx-x| \, \delta \max\{|y-Ty|, \frac{1}{2}|Tx-x|\}$$

Now

Case-I

Max {a,b}=a

Where a= $|y-Ty|$

b = $\frac{1}{2}|Tx-x|$

= $\delta\{|y-Ty|\}$

$$= \alpha[|y-Ty| + \frac{1}{2}|Tx-x|] + \beta[|Tx-x| + \frac{1}{2}|y-Ty| + \frac{1}{2}|Tx-x|]$$

$$+\gamma\frac{1}{2}[|Tx-x|] + \delta[|y-Ty|]$$

$$= |Tx-x|\{\frac{\alpha}{2} + \frac{3\beta}{2} + \frac{\gamma}{2}\} + |y-Ty|\{\alpha + \frac{\beta}{2} + \delta\} \text{ - - - - - - - - -(2.1.1)}$$

Case –II

Max{a,b}=b

Where a= $|y-Ty|$

b = $\frac{1}{2}|Tx-x|$

$$= \delta\{\tfrac{1}{2}|Tx - x|\}$$

$$= \alpha[|y - Ty| + \tfrac{1}{2}|Tx - x|] + \beta[|Tx - x| + \tfrac{1}{2}|y - Ty| + \tfrac{1}{2}|Tx - x|]$$
$$+ \gamma\tfrac{1}{2}[|Tx - x|] + \delta[\tfrac{1}{2}|Tx - x|]$$

$$== |Tx-x|\{\tfrac{\alpha}{2} + \tfrac{3\beta}{2} + \tfrac{\gamma}{2} + \tfrac{\delta}{2}\} + |y-Ty|\{\alpha + \tfrac{\beta}{2}\} \text{ --------- }(2.1.2)$$

Note

$|y-x| \le |\tfrac{1}{2}(Tx + x) - x| = \tfrac{1}{2}|Tx - x|$ ----- (A)

$|y-Tx| \le |\tfrac{1}{2}(Tx + x) - Tx| = \tfrac{1}{2}|Tx - x|$ ----- (B)

Now we will calculate $|u-x|$

$|u-x| = |2y-z-x| = |(T+I)x-T-x| = |Tx-Ty|$

$$\le \alpha\frac{|x-Tx|.|x-Ty| + |x-y|^2}{|x-y|} + \beta\frac{|y-Tx|.|y-Ty| + |x-y|^2}{|x-y|} + \gamma|x - y|$$
$$+\delta\max\{|x - Tx|, |x - Ty|, |x - y|\}$$

$$\le \alpha\frac{|x-Tx|.\tfrac{1}{2}|x-Tx| + \tfrac{1}{4}|x-Tx|^2}{\tfrac{1}{2}|x-Tx|} + \beta\frac{\tfrac{1}{2}|x-Tx|.|y-Ty| + \tfrac{1}{4}|x-Tx|^2}{\tfrac{1}{2}|x-Tx|} + \gamma\frac{1}{2}|x - Tx|$$

$$+\delta\max\{|y - Ty|, \frac{1}{2}|x - Tx|, \frac{1}{2}|x - Tx|\}$$

$$= \frac{3\alpha}{2}|x-Tx| + \beta[|y - TY| + \tfrac{1}{2}|x - Tx|] + \gamma\tfrac{1}{2}|x - Tx| + \delta\max\{|y - Ty|, \tfrac{1}{2}|x - Tx|\}$$

Case –I

Max {a,b}=a

Where a= $|y - Ty|$

b = $\frac{1}{2}|Tx - x|$

= $\frac{3\alpha}{2}|x - Tx| + \beta[|y - Ty| + \frac{1}{2}|x - Tx|] + \gamma\frac{1}{2}|x - Tx| + \delta|y - Ty|$

= $[\frac{3\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2}]$ |x-Tx| + $[\beta + \delta]$ |y-Ty| - - - - - - - --- - - - (2.1.3)

Case-II

Max {a, b} =b

Where a= $|y - Ty|$

b = $\frac{1}{2}|Tx - x|$

= $\delta\{\frac{1}{2}|Tx - x|\}$

= $\frac{3\alpha}{2}|x - Tx| + \beta[|y - Ty| + \frac{1}{2}|x - Tx|] + \gamma\frac{1}{2}|x - Tx| + \delta\frac{1}{2}|x - Tx|$

= $[\frac{3\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2} + \frac{\delta}{2}]|x - Tx| + \beta|y - Ty|$ - - --- - - - - - - - - - - - - (2.1.4)

Now we will calcutate |z-u|:

|z-u| ≤ |z-x| + |x-u|

Using (2.1.1) and (2.1.3)

= $\{\frac{\alpha}{2} + \frac{3\beta}{2} + \frac{\gamma}{2}\}$ |x-Tx| + $\{\alpha + \frac{\beta}{2} + \delta\}$ |y-Ty| + $\{\frac{3\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2}\}$ |x-Tx| + $\{\beta + \delta\}$ |y-Ty|

= $\{\frac{\alpha}{2} + \frac{3\beta}{2} + \frac{\gamma}{2} + \frac{3\alpha}{2} + \frac{\beta}{2} + \frac{\gamma}{2}\}$ |x-Tx| ++ $\{\alpha + \frac{\beta}{2} + \delta + \beta + \delta\}$ |y-Ty|

= $|x - Tx|\{\frac{4\alpha + 4\beta + 2\gamma}{2}\}$ + |y-Ty| $\{\alpha + \frac{3\beta}{2} + 2\delta\}$ - - - - - - - - - - - - - (2.1.5)

Using (2.1.2) and (2.1.4)

= |x-Tx| $\{\frac{\alpha}{2}+\frac{3\beta}{2}+\frac{\gamma}{2}+\frac{\delta}{2}\}$ + |y-Ty| $\{\alpha+\frac{\beta}{2}\}$+ |x-Tx| $\{\frac{3\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2}+\frac{\delta}{2}\}$ + |y-Ty| β

= |x-Tx| $\{\frac{\alpha}{2}+\frac{3\beta}{2}+\frac{\gamma}{2}+\frac{\delta}{2}+\frac{3\alpha}{2}+\frac{\beta}{2}+\frac{\gamma}{2}+\frac{\delta}{2}\}$ +. |y-Ty| $\{\alpha+\frac{\beta}{2}+\beta\}$

= $|x-Tx|$ $\{\frac{4\alpha+4\beta+2\gamma+2\delta}{2}\}$ +. |y-Ty| $\{\alpha+\frac{3\beta}{2}\}$ ---------------- (2.1.6)

Thus |z-u| $\le$ $[2\alpha+2\beta+\gamma]$ |x-Tx| + $[\alpha+\frac{3\beta}{2}+2\delta]$. |y-Ty| --------------(2.1.7)

But

|z-u| = |Ty-(2y-z)|

= |Ty-2y+z|

= |Ty-2y+Ty|

|z-u| = 2 |Ty-y|

=2 |y-Ty| ------------- (2.1.8)

Comparing equ. (2.1.7) and (2.1.8)

=2 |y-Ty| $\le$ $[2\alpha+2\beta+\gamma]$ |x-Tx| + $[\alpha+\frac{3\beta}{2}+2\delta]$. |y-Ty|

= $[2-\alpha-\frac{3\beta}{2}-2\delta]$ |y-Ty| $\le$ $[2\alpha+2\beta+\gamma]$ |x-Tx|

= $[4-2\alpha-3\beta-4\delta]$ |y-Ty| $\le$ $[4\alpha+4\beta+2\gamma]$ |x-Tx|

= |y-Ty| $\le$ $\frac{4\alpha+4\beta+2\gamma}{4-2\alpha-3\beta-4\delta}$ |x-Tx|

= |y-Ty| $\le k_1$ |x-Tx|

Where $k_1 = \frac{4\alpha+4\beta+2\gamma}{4-2\alpha-3\beta-4\delta} < 1$

Because, $6\alpha+7\beta+2\gamma+8\delta<4$

Thus |z-u| $\le$ $[2\alpha+2\beta+\gamma+\delta]$ |x-Tx| + $[\alpha+\frac{3\beta}{2}]$. |y-Ty| ----------- (2.1.9)

Now comparing (2.1.9) and (2.1.8)

= 2 |y-Ty| $\le$ $[2\alpha+2\beta+\gamma+\delta]$ |x-Tx| + $[\alpha+\frac{3\beta}{2}]$. |y-Ty|

= $[2-\alpha-\frac{3\beta}{2}]$. |y-Ty| $\le$ $[2\alpha+2\beta+\gamma+\delta]$ |x-Tx|

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= [4-2α-3β] |y-Ty| $\leq$ [4α+4β+2$\gamma + 2\delta$] |x-Tx|

= |y-Ty| $\leq \frac{4\alpha+4\beta+2\gamma+2\delta}{4-2\alpha-3\beta}$ |x-Tx|

= |y-Ty| $\leq k_2$|x-Tx|

Where $k_2 = \frac{4\alpha+4\beta+2\gamma+2\delta}{4-2\alpha-3\beta} < 1$

Because, 6$\alpha + 7\beta + 2\gamma + 8\delta < 4$

Now , let s= $\frac{1}{2}$(T+I) then for every x$\in X$, we have

$|s^2 x - sx|$ = |ssx-sx| = |sy-y|

$|\frac{1}{2}(S + I)y - y|$ =$\frac{1}{2}$|y-Ty| =$\frac{k}{2}$|x-Tx|

By the definition of k , we claim that sequence {s^n(x)} is a Cauchy sequence inn X . By the completeness of space, we get that sequence {s^n(x)} converges to some element x_0 in X , i.e.$\log_{n\to\infty} s^n$(x) = x_0

Which implies sx_0 =x_0 . Hence, Tx_0 =x_0

So, x_0 is a fixed point of T.

Uniqueness: If possible let $y_0 \neq x_0$ be another fixed point of T . Then, T$x_0 = x_0$, $Sx_0 = x_0 , Ty_0 = y_0$ and $sy_0 = y_0$

Also

$|x_0 - y_0|$ = $|Tx_0 - Ty_0|$

$$\leq \alpha \frac{|x_0-Tx_0|.|x_0-Ty_0|+|x_0-y_0|^2}{|x_0-y_0|^2} + \beta \frac{|y_0-Tx_0|.|y_0-Ty_0|+|x_0-y_0|^2}{|x_0-0|^2} + \gamma |x_0 - y_0|$$

$$+\delta \{ |x_0 - Tx_0|, |x_0 - Ty_0|, |x_0 - y_0| \}$$

$$\leq \alpha \frac{|x_0-x_0|.|x_0-y_0|+|x_0-y_0|^2}{|x_0-y_0|^2} + \beta \frac{|y_0-x_0|.|y_0-y_0|+|x_0-y_0|^2}{|x_0-y_0|^2} + \gamma |x_0 - y_0|$$

$$+\delta \{ |x_0 - x_0|, |x_0 - y_0|, |x_0 - y_0| \}$$

= $\alpha |x_0 - y_0| + \beta |x_0 - y_0| + \gamma |x_0 - y_0| + \delta |x_0 - y_0|$

= [$\alpha + \beta + \gamma + \delta$] $|x_0 - y_0|$

= $|x_0 - y_0|$ = $[\alpha + \beta + \gamma + \delta]$ $|x_0 - y_0|$

Which is possible only when $x_0 = y_0$ because $\alpha + \beta + \gamma + \delta < 1$

Hence, x_0 Is the unique fixed point of T.

Remark: If we put δ =0 then the result of Dwivedi et.al [28], proved.

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3. References:

1. Banach , S. " Surles operation dans les ensembles abstraits et leur application aux equation integrals" Fund. Math. 3(19220 133-181 .

2. Bhardwaj , R.K. , Rajput , S.S. and Yadava , R.N. " Application of fixed point theory in metric spaces " Thai journal of Mathematics 5(2007) 253- 259 .

3.Chatterjee , S.K. " Fixed point Theorem compactes" Rend. Acad. Bulgare Sci , 25(1972) 727-730.

4.Choudhary , S. Wadhwa , K. and Bhardwaj R.K. "A fixed point theorem for continuous function" Vijnana Parishad Anushandhan Patrika. (2007) 110-113.

5.Ciric , L.B."A generalization of Banach contraction principal" Proc. Amer. Math. Soc . 25(1974) 267-273

6. Chu , S.C. and Diag, J.B. "Remarks on generalization on Banach principal of contractive mapping" J.Nath.Arab .Appli. 11 (1965)440-446.

7.Das , B.K. and Gupta , S. "An exetension of Banach contraction principal through rational expression" Indian Journal of pure and Applied Math. 6(1975) 1455-1458.

8.Dubey , R.P. and pathak , H.K "Common fixed point of mapping satisfying rational inequalities" Pure ans Applied Mathematical Science 31(1990)155-161.

9. Fisher B. "A fixed point theorem for compact matric space" Publ.Inst.Math.25(1976) 193-194 .

10. Goebel, K. " An elementary proof of the fixed point theorem of Browder and Kirk" Michigan Math.J.16(1969)381-383.

11.Iseki, K., Sharma, P.L.and Rajput S.S."An extension of Banach contraction principal through rational expression" Mathematics seminar notes kobe University 10(1982) 677-679.

12. Imdad, M. and Khan T.I. "On common fixed points of pair wise coincidently commuting non-continuous mappings satisfying a rational inequality" Bull.Ca. Math. Soc. 93(2001) 263-268.

13.IMada, M. and Khan , O.H "A common fixed point theorem for six mappings satisfying a rational inequality" Indian J. of Mathematics 44(2002) 47-57.

14. Jaggi, D.S."Some unique fixed point theorems" I.J.P.Appl. 8(1977) 223-230.

15. Jaggi, D.S.and Dsa, "An extension of Banach's fixed point theorem through rational espression" Bull.Cal. Math. Soc. 72(1980) 261-264.

16. Kannan , R. "Some results on fixed point theorems" Bull. Calcutta Math.Soc, 60(1969)71-78.

17.Kundu, A. and iwary, K.S."Acommon fixed point theorem for five mappings in metric spaces" Review Bull. Cal. Math.Soc. 182(2003)93-98.

18.Liu, Z., Feng , C. and Chun, S.A. "Fixed and periodic point theorems in 2- metric spaces" Nonlineas Funct. And Appl. 4(2003) 497-505.

19.Murthy,P.P.and Sharma, B.K.” Some uniqe common fixed point theorems” pure and Applied Mathematical Sciences 33 (1994) 105-108 .

20. Nair,S. and Shriwastava, S.”Common fixed point theorem for rational inequality” Acta Cincia Indica 32(2006) 275-278.

21. Naidu, S.V.R. “Fixed point theorems for self map on a 2-metric space” Pure and Applied Mathematic Sciences 12(1995) 73-77.

22. Reich, S.” Some remarks concerning contraction mapping” Canada.Math.Bull.14(1971)121-124.

23. Rani , D. and Chugh, R.”Some fixed point theorems on contractive type mappings” Pure and Applied Mathematic Sciences 41(1990)153-157.

24.Sahu, D.P.and Sao.G.S.” Studies on common fixed point theorem for nonlinear contraction mappings in 2-metric space” Acta Ciencia India 30 (2004)767-770.

25. Sehgal V.M. :A fixed point theorem for mapping with a contractive iterate” Proc. Amer.Math. Soc. 23(1969) 631-634.

26. Sharma.P.L/, Sharma . B.K. and Iseki, K.”Contractive type mapping on 2-metric spaces” Math Japonica 21(1976) 67-70.

27. Singh, S.L.,Kumar, A. and Hasim, A.M. “Fixed points of Contractive maps” Indian Journal of Mathematics 47(2005) 51-58.

28.Shrivastava, R. Divedi , S.K. and Bhardwaj, R.K.., “Some fixed point and common fixed point theorems in usual metric spaces” International Journal of Engineering Sciences Research-IJESR Vol 01, Issue 02, May, (2011),58-65.

29. Yadava, R. N. Rajput, S.S and Bhardwaj, R.K.”Some fixed point theorem for extension of Banach contraction principal” Acta Ciencia Indica 33, No 2 (2007) 461- 466.

30. YAdava, R.N. Rajout, S.S.,CHoudhary, S., Bhaedwaj,R.K. “Some fixed point theorems for rational inequality in 2- metric spaces” Acta Ciencia Indica 33 No 3(2007) 709-714.

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