Common Fixed Point Theorem in Intuitionistic Fuzzy 2- Metric Spaces for Integral Inequality

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Abstract

In this paper, we prove a common fixed point theorem for weakly compatible maps in intuitionistic
Fuzzy 2- metric space for integral type inequality.

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Introduction and Preliminaries

The first result on fixed points for contractive type mapping was the much celebrated Banach's contraction principle by S. Banach [20] in 1922. In the general setting of complete metric space, this theorem runs as the follows,

Theorem 1(Banach's contraction principle) Let (X, d) be a complete metric space, c∈ (0, 1) and f: X→X be a mapping such that for each x, y ∈X, d (fx, fy) ≤ c d(x, y) Then f has a unique fixed point a ∈X, such that for each x ∈ X, $\lim_{n\to\infty} f^n x = a$. After the classical result, R.Kannan [18] gave a subsequently new contractive mapping to prove the fixed point theorem. Since then a number of mathematicians have been worked on fixed point theory dealing with mappings satisfying various type of contractive conditions. In 2002, A. Branciari [1] analyzed the existence of fixed point for mapping f defined on a complete metric space (X,d) satisfying a general contractive condition of integral type.

Theorem 2(Branciari) Let (X, d) be a complete metric space, c∈ (0, 1) and let f: X→X be a mapping such that for each x , y ∈ X, $\int_0^{d(fx,fy)} \varphi(t)dt \le c \int_0^{d(x,y)} \varphi(t)dt$. Where φ: [0,+∞)

→[0,+∞) is a Lebesgue integrable mapping which is summable on each compact subset of [0,+∞) , non negative, and such that for each ε >o , $\int_0^{\varepsilon} \varphi(t)dt,$ then f has a unique fixed point a ∈ X such that for each x ∈ X, $\lim_{n\to\infty} f^n x = a$ After the paper of Branciari, a lot of a research works have been carried out on generalizing contractive conditions of integral type for a different contractive mapping satisfying various known properties. A fine work has been done by Rhoades [2] extending the result of Branciari by replacing the condition [1] by the following

$\int_0^{d(fx,fy)} \varphi(t)dt \le \int_0^{max\left\{d(x,y),d(x,fx),d(y,fy),\frac{d(x,fy)+d(y,fx)}{2}\right\}} \varphi(t)dt$ .The aim of this paper is to generalize some fixed type of contractive conditions to the mapping and then a pair of mappings, satisfying general contractive mappings such as R. Kannan type [18], S.K. Chatrterjee type [19], T. Zamfirescu type [25], etc.

It proved a turning point in the development of fuzzy mathematics when the notion of fuzzy set was

introduced by Zadeh [15]. Atanassov [14] introduced and studied the concept of intuitionistic fuzzy sets. Coker [6] introduced the concept of intuitionistic fuzzy topological spaces. Jungck's [13] common fixed point theorem in the setting of intuitionistic fuzzy metric space. Turkoglu et al. [7] further formulated the notions of weakly commuting and R weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [16]. Gregori et al. [26], Saadati and Park [21] studied the concept of intuitionistic fuzzy metric space and its applications. Recently, many authors have also studied the fixed point theory in fuzzy and intuitionistic fuzzy metric spaces as (Dimri et.al.[8], Grabiec [10], Imdad et. al.[11]).On the same way we established a common fixed point theorem in this space for integral type mappings.

Definition 1.1[3] A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions:

(1) * is associative and commutative,

(2) * is continuous,

(3) a * 1 = a for all a ∈ [0, 1],

(4) a * b ≤ c * d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0, 1],

Two typical examples of continuous t-norm are a * b = ab and a * b = min (a, b).

Definition 1.2[3]. A binary operation $\diamondsuit$: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if it satisfies the following conditions:

(1) $\diamondsuit$ is associative and commutative,

(2) $\diamondsuit$ is continuous,

(3) a $\diamondsuit$1 = a for all a ∈ [0, 1],

(4) a $\diamondsuit$ b ≤ c $\diamondsuit$ d whenever a ≤ c and b ≤ d for all a, b, c, d ∈ [0, 1],

Two typical examples of continuous t-co norm are a $\diamondsuit$ b = ab and a $\diamondsuit$ b = min (a, b).

Definition 1.3[4]. A 5-tuple (X, M, N, ∗, $\diamondsuit$) is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, ∗ is a continuous t-norm, $\diamondsuit$ is a continuous t-co norm and M,N are fuzzy sets on $X^2 \times [0,\infty)$ satisfying the following conditions:

(i) $M(x,y,t) + N(x,y,t) \leq 1 \, for \, all \, x,y \in X \, and \, t > 0;$

(ii) $M(x,y,0) = 0 \, for \, all \, x,y \in X;$

(iii) $M(x,y,t) = 1 \, for \, all \, x,y \in X \, and \, t > 0 \, if \, and \, only \, if \, x = y;$

(iv) $M(x,y,t) = M(y,x,t) \, for \, all \, x,y \in X \, and \, t > 0;$

(v) $M(x,y,t) * M(y,z,s) \leq M(x,z,t+s) for \, all \, x,y \in X \, and \, s,t > 0;$

(vi) $for \, all \, x,y \in X, M(x,y,\,.): [0,\infty) \to [0,1] \, is \, left \, continuous;$

(vii) $\lim_{t\to\infty} M(x,y,t) = 1 \, for \, all \, x,y \in X \, and \, t > 0;$

$(viii)$ $N(x,y,0) = 1 \, for \, all \, x,y \in X;$

(ix) $M(x,y,t) = 1 \, for \, all \, x,y \in X \, and \, t > 0 \, if \, and \, only \, if \, x = y;$

(x) $N(x,y,t) = N(y,x,t) \, for \, all \, x,y \in X \, and \, t > 0;$

(xi) $N(x,y,t) \diamondsuit N(y,z,s) \geq N(x,z,t+s) for \, all \, x,y \in X \, and \, s,t > 0;$

(xii) $for \, all \, x,y \in X, N(x,y,\,.): [0,\infty) \to [0,1] \, is \, right \, continuous;$

(xiii) $\lim_{t\to\infty} N(x,y,t) = 0 \, for \, all \, x,y \in X \, and \, t > 0.$

Then (M, N) is called an intuitionistic fuzzy metric space on X. The functions M(x, y, t) and N(x, y, t) denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark1.1[4]. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \diamond)$ such that t-norm $*$ and t-co norm $\diamond$ defined by $a * a \geq a, a \in [0,1]$ &
$(1-a) \diamond (1-a) \leq (1-a)$ $for\ all\ x, y \in X$, In intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, $M(x, y, *)$ is non-decreasing and $N(x, y, \diamond)$ is non-increasing.

Remark1.2[17]. Let (X, d) be a metric space. Define t-norm a * b = min (a, b) and t-co norm
a $\diamond$ b = max (a, b), $for\ all\ x, y \in X$ & $t > 0$. $M_d(x, y, t) = \frac{t}{t + d(x,y)}$, $N_d(x, y, t) = \frac{d(x,y)}{t + d(x,y)}$
Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space induced by the metric. It is obvious that
$N(x, y, t) = 1 - M(x, y, t)$

Alaca, Turkoglu and Yildiz [4] introduced the following notions:

Definition 1.4. Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is called Cauchy-sequence if, for all t > 0 and P > 0, $\lim_{n\to\infty} M(x_{n+p}, x_n, t) = 1$

and $\lim_{n\to\infty} N(x_{n+p}, x_n, t) = 0$,

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if , for all $t > 0$,

$\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$.

Definition 1.5. Let (X, M, N, $*$, $\diamond$) be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X if there exists a number $k \in (0,1)$ such that:
1. $M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t)$,
2. $N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$

for all t > 0 and n = 1 ,2 ,3……then $\{y_n\}$ is a Cauchy sequence in X.

Definition 1.6. A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space (X, M, N, $*$, $\diamond$) is said to be compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) = 1$ & $\lim_{n\to\infty} N(fgx_n, gfx_n, t) = 0$ for every t > 0, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some. $z \in X$.

Definition 1.7. A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space (X, M, N, $*$, $\diamond$) is said to be non-compatible if $\lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$ & $\lim_{n\to\infty} N(fgx_n, gfx_n, t) \neq 0$ for every t > 0, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = z$, for some. $z \in X$.

Definition 1.8. An intuitionistic fuzzy metric space (X, M, N, $*$, $\diamond$) is said to be complete if and only if every Cauchy sequence in X is convergent.

In 1998, Jungck [13] introduced the notion of weakly compatible maps as follows:

Definition 1.9[13]. A pair of self mappings (f, g) of a metric space is said to be weakly compatible if they commute at the coincidence points i.e. $fu = gu\ for\ some\ u \in X$, then $fgu = gfu$.

Sharma, Sharma and Iseki [23] studied for the first time contraction type mappings in 2-metic space. Sharma[24] defined fuzzy 2- metric space.On the same way we are taking the results for intuitionistic fuzzy 2-metric space motivated by Sharma[24]

2. Main results

Theorem 2.1. Let A, B, S and T be self maps of intuitionistic fuzzy 2-metric spaces (X, M, N, $*$, $\diamond$) with continuous t-norm * and continuous t-co norm $\diamond$ defined by $t * t \geq t$ and $(1-t) \diamond (1-t) \leq (1-t)$ for all $t \in [0, 1]$ satisfying the following condition:

(2.1) A(X) $\subset$ S(X) and B(X) $\subset$ T(X),

(2.2) If one of the A ,B ,S and T is a complete subspace of X then {A ,T}and{B ,S}have a coincidence point,

(2.3) The pairs (A, T) and (B, S) are weakly compatible,

(2.4)

$$\int_0^{M(Ax,\ By,a,t)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\begin{pmatrix} M(Tx,Sy,a,\ t)*M(Tx,Ax,t)*M(Ax,Sy,a,t), \\ M(Sy,\ Tx,a,t)*M(Bx,Ty,a,t)*M(Bx,Sx,a,\ t)\end{pmatrix}\right\}} \zeta(t)dt$$

and

$$\int_0^{N(Ax,\ By,a,t)} \zeta(t)dt \leq \int_0^{\varphi\left\{\max\begin{pmatrix} N(Tx,Sy,a,t)\diamond N(Tx,Ax,a,\ t)\diamond N(Ax,Sy,a,t), \\ N(Sy,\ Tx,a,\ t)\diamond N(Bx,Ty,a,\ t)\diamond N(Bx,Sx,a,\ t)\end{pmatrix}\right\}} \zeta(t)dt$$

$\forall x, y \in X$ & > 0 , where $\emptyset, \varphi: [0,1] \to [0,1]$ is a continuous function such that $\emptyset(t) > t$ & $\varphi(t) < t$ for each $0 < t < 1$ and $\emptyset(1) = 1$ and $\varphi(0) = 0$ with $M(x, y, a, t) > 0, a > 0$ *is real* Then A, B, S and T have a unique common fixed point in X.

Proof : Since A(X) ⊂ S(X), therefore for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Sx_1$ and

for the point x_1 , we can choose a point $x_2 \in X$ such that $Bx_1 = Tx_2$ as B(X) ⊂ S(X). Inductively, we get

sequence $\{y_n\}$ in x as follows $y_{2n+1} = Bx_{2n+1} = Tx_{2n+2}$ and $y_{2n} = Ax_{2n} = Sx_{2n+1}$ for n = 0,1,2…..

putting $x = x_{2n}$, $y = x_{2n+1}$ in (2.4) we have,

$$\int_0^{M(Ax_{2n}, Bx_{2n+1}, a, t)} \zeta(t)dt \ge \int_0^{\phi\left\{\min\begin{pmatrix} M(Tx_{2n}, Sx_{2n+1}, a, t) * M(Tx_{2n}, Ax_{2n}, a, t) \\ * M(Ax_{2n}, Sx_{2n+1}, a, t) * M(Sx_{2n+1}, Tx_{2n}, a, t) \\ * M(Bx_{2n}, Tx_{2n+1}, a, t) * M(Bx_{2n}, Sx_{2n}, a, t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{M(y_{2n}, y_{2n+1}, a, t)} \zeta(t)dt \ge \int_0^{\phi\left\{\min\begin{pmatrix} M(y_{2n-1}, y_{2n}, a, t) * M(y_{2n-1}, y_{2n}, a, t) \\ * M(y_{2n}, y_{2n}, a, t) * M(y_{2n}, y_{2n-1}, a, t) \\ * M(y_{2n-1}, y_{2n}, a, t) * M(y_{2n}, y_{2n-1}, a, t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{M(y_{2n}, y_{2n+1}, a, t)} \zeta(t)dt \ge \int_0^{\phi\left\{\min\begin{pmatrix} M(y_{2n-1}, y_{2n}, t) * M(y_{2n-1}, y_{2n}, a, t) * 1 \\ * M(y_{2n}, y_{2n-1}, a, t) * M(y_{2n-1}, y_{2n}, a, t) \\ * M(y_{2n}, y_{2n-1}, a, t) \end{pmatrix}\right\}} \zeta(t)dt$$

i.e.

$$\int_0^{M(y_{2n}, y_{2n+1}, a, t)} \zeta(t)dt \ge \int_0^{\phi\{M(y_{2n-1}, y_{2n}, a, t)\}} \zeta(t)dt > \int_0^{M(y_{2n-1}, y_{2n}, a, t)} \zeta(t)dt$$

as $\emptyset(t) > t$ for each 0 < t < 1 and

$$\int_0^{N(Ax_{2n}, Bx_{2n+1}, a, t)} \zeta(t)dt \le \int_0^{\varphi\left\{\max\begin{pmatrix} N(Tx_{2n}, Sx_{2n+1}, t) \Diamond N(Tx_{2n}, Ax_{2n}, a, t) \\ \Diamond N(Ax_{2n}, Sx_{2n+1}, a, t) \Diamond N(Sx_{2n+1}, Tx_{2n}, a, t) \\ \Diamond N(Bx_{2n}, Tx_{2n+1}, a, t) \Diamond N(Bx_{2n}, Sx_{2n}, a, t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{N(y_{2n},\, y_{2n+1},a,\, t)} \zeta(t)dt \le \int_0^{\varphi\left\{\max\begin{pmatrix} N(y_{2n-1},y_{2n},a,t)\Diamond N(y_{2n-1},y_{2n},a,\, t) \\ \Diamond N(y_{2n},y_{2n},a,\, t)\Diamond N(y_{2n},y_{2n-1},a,t) \\ \Diamond N(y_{2n-1},y_{2n},a,t)\Diamond N(y_{2n},y_{2n-1},a,\, t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{N(y_{2n},\, y_{2n+1},a,\, t)} \zeta(t)dt \le \int_0^{\varphi\left\{\max\begin{pmatrix} N(y_{2n-1},y_{2n},a,\, t)\Diamond N(y_{2n-1},y_{2n},a,t)\Diamond 0 \\ \Diamond N(y_{2n},y_{2n-1},a,\, t)\Diamond N(y_{2n-1},y_{2n},a,\, t) \\ \Diamond N(y_{2n},y_{2n-1},t) \end{pmatrix}\right\}} \zeta(t)dt$$

i.e.

$$\int_0^{N(y_{2n},\, y_{2n+1},a,\, t)} \zeta(t)dt \le \int_0^{\varphi\left\{N(y_{2n-1},y_{2n},a,\, t)\right\}} \zeta(t)dt < \int_0^{N(y_{2n-1},y_{2n},a,t)} \zeta(t)dt$$

as $\varphi(t) < t$ for each 0 < t < 1. Thus $\{M(y_{2n}, y_{2n+1}, a, t), n \ge 0\}$ is an increasing sequence of positive real numbers in [0 ,1]which tends to a limit $l \le 1$, also $\{N(y_{2n+1}, y_{2n+2}, t), n \ge 0\}$ is an decreasing sequence of positive real numbers in [0 ,1]which tends to a limit $k = 0$. Therefore for every $n \in I^+$

$M(y_n, y_{n+1}, a, t) > M(y_{n-1}, y_n, a, t)$, $\lim_{n\to\infty} M(y_n, y_{n+1}, a, t) = 1$, $N(y_n, y_{n+1}, a, t) < N(y_{n-1}, y_n, a, t)$ and $\lim_{n\to\infty} N(y_n, y_{n+1}, t) = 0$. Now any positive integer p,we obtain $\lim_{n\to\infty} M(y_n, y_{n+p}, a, t) = 1$ and

$\lim_{n\to\infty} N(y_n, y_{n+p}, a, t) = 0$. Which shows that $\{y_n\}$ is a Cauchy sequence in X. Let $w \in S^{-1}u$ then $Sw = u$. we shall use the fact that subsequence $\{y_{2n+1}\}$ also converges to u.

Now by putting $x = x_{2n}$, $y = w$ in (2.4) and taking $n \to \infty$

$$\int_0^{M(Ax_{2n},\, Bw,a,\, t)} \zeta(t)dt \ge \int_0^{\phi\left\{\min\begin{pmatrix} M(Tx_{2n},Sw,a,t)*M(Tx_{2n},Ax_{2n},a,\, t) \\ *M(Ax_{2n},Sw,a,\, t)*M(Sw,\, Tx_{2n},a,\, t) \\ *M(Bx_{2n},Tw,a,\, t)*M(Bx_{2n},Sx_{2n},a,\, t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{M(u,\, Bw,t)} \zeta(t)dt \ge \int_0^{\phi\left\{\min\begin{pmatrix} M(u,u,a,\, t)*M(u,ua,,t)*M(u,ua,,t) \\ *M(u,u,a,\, t)*M(u,u,a,\, t)*M(u,ua,,t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{M(u,\, Bw,a,\, t)} \zeta(t)dt \ge \int_0^{\phi(1)} \zeta(t)dt$$

$i.e. M(u, Bw, a, t) \geq 1,$

…………(3)

Also

$$\int_0^{N(Ax_{2n}, Bw, a, t)} \zeta(t)dt \leq \int_0^{\varphi\left\{\max\begin{pmatrix} N(Tx_{2n}, Sw, a, t)\lozenge N(Tx_{2n}, Ax_{2n}, a, t) \\ \lozenge N(Ax_{2n} Sw, a, t)\lozenge N(Sw, Tx_{2n}, a, t) \\ \lozenge N(Bx_{2n}, Tw, t)\lozenge N(Bx_{2n}, Sx_{2n}, t)\end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{N(u, Bw, a, t)} \zeta(t)dt \leq \int_0^{\varphi\left\{\max\begin{pmatrix} N(u, u, a, t)\lozenge N(u, u, a, t)\lozenge N(u, u, a, t) \\ \lozenge N(u, u, a, t)\lozenge N(u, u, a, t)\lozenge N(u, u, a, t)\end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{N(u, Bw, a, t)} \zeta(t)dt \leq \int_0^{\varphi(0)} \zeta(t)dt$$

$i.e. N(u, Bw, a, t) \leq 0$
…………(4)

From (3) and (4) Let $u = Bw$. Since $Sw = u$ we have $Sw = Bw = u$ i.e. w is the coincidence point of B and S. As B(X) ⊂ T(X), $= Bw \to u \epsilon T(X)$. Let $v \in T^{-1}u$ then $Tv = u$. Now by putting

$x = v, \; y = x_{2n+1}$ in (2.4)

$$\int_0^{M(Av, Bx_{2n+1}, a, t)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\begin{pmatrix} M(Tv, Sx_{2n+1}, a, t) * M(Tv, Av, a, t) \\ * M(Av, Sx_{2n+1}, a, t) * M(Sx_{2n+1}, Tv, a, t) \\ * M(Bv, Tx_{2n+1}, a, t) * M(Bv, Sv, a, t)\end{pmatrix}\right\}} \zeta(t)dt$$

taking $n \to \infty$

$$\int_0^{M(Av, u, a, t)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\begin{pmatrix} M(u, u, a, t) * M(u, Av, a, t) * M(Av, u, a, t) \\ * M(u, u, a, t) * M(u, u, a, t) * M(u, u, a, t)\end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{M(Av, u, a, t)} \zeta(t)dt \geq \int_0^{\phi\{\min(1 * M(u, Av, a, t) * M(Av, u, a, t) * 1 * 1 * 1)\}} \zeta(t)dt$$

$$\int_0^{M(Av, u, a, t)} \zeta(t)dt \geq \int_0^{\phi(M(u, Av, a, t))} \zeta(t)dt > \int_0^{M(u, Av, a, t)} \zeta(t)dt$$

And

$$\int_0^{N(Av,Bx_{2n+1},a,t)} \zeta(t)dt \le \int_0^{\varphi\left\{\max\begin{pmatrix} N(Tv,Sx_{2n+1},a,t)\Diamond N(Tv,Av,a,t) \\ \Diamond N(Av,Sx_{2n+1},a,t)\Diamond N(Sx_{2n+1},Tv,a,t) \\ \Diamond N(Bv,Tx_{2n+1},a,t)\Diamond N(Bv,Sv,a,t) \end{pmatrix}\right\}} \zeta(t)dt$$

taking $n \to \infty$

$$\int_0^{N(Av,u,a,t)} \zeta(t)dt \le \int_0^{\varphi\left\{\max\begin{pmatrix} N(u,u,a,t)\Diamond N(u,Av,a,t)\Diamond N(Av,u,a,t) \\ \Diamond N(u,u,a,t)\Diamond N(u,u,a,t)\Diamond N(u,u,a,t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{N(Av,u,a,t)} \zeta(t)dt \le \int_0^{\varphi\left\{\max\left(0\Diamond N(u,Av,a,t)\Diamond N(Av,u,a,t)\Diamond 0\Diamond 0\Diamond 0\right)\right\}} \zeta(t)dt$$

$$\int_0^{N(Av,u,a,t)} \zeta(t)dt \le \int_0^{\varphi\left(N(u,Av,a,t)\right)} \zeta(t)dt < \int_0^{N(u,Av,a,t)} \zeta(t)dt$$

Therefore, we get $Av = u$. we have $Tv = Av = u$. Thus v is a coincidence point of A and T.

Since the pairs $\{A, T\}$ and $\{B, S\}$ are weakly compatible i.e. $B(Sw) = S(Bw) \to Bu = Su$ and

$A(Tv) = T(Av) \to Au = Tu.$

Now by putting $x = u$, $y = x_{2n+1}$ in (2.4)

$$\int_0^{M(Au,Bx_{2n+1},a,t)} \zeta(t)dt \ge \int_0^{\phi\left\{\min\begin{pmatrix} M(Tu,Sx_{2n+1},a,t)*M(Tu,Au,a,t) \\ *M(Au,Sx_{2n+1},a,t)*M(Sx_{2n+1},Tu,a,t) \\ *M(Bu,Tx_{2n+1},a,t)*M(Bu,Su,a,t) \end{pmatrix}\right\}} \zeta(t)dt$$

taking $n \to \infty$

$$\int_0^{M(Au,u,a,t)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\left(\begin{array}{l} M(Au,u,a,t)*M(Au,Au,a,t)*M(Au,u,a,t) \\ *M(u,Au,a,t)*M(u,u,a,t)*M(u,u,a,t) \end{array}\right)\right\}} \zeta(t)dt$$

$$\int_0^{M(Au,u,a,t)} \zeta(t)dt \geq \int_0^{\phi\{\min(M(Au,u,a,t)*1*M(Au,u,a,t)*M(u,Au,a,t)*1*1)\}} \zeta(t)dt$$

$$\int_0^{M(Au,u,a,t)} \zeta(t)dt \geq \int_0^{\phi(M(Au,u,a,t))} \zeta(t)dt > \int_0^{M(Au,u,a,t)} \zeta(t)dt$$

And

$$\int_0^{N(Au,Bx_{2n+1},a,t)} \zeta(t)dt \leq \int_0^{\varphi\left\{\max\left(\begin{array}{l} N(Tu,Sx_{2n+1},a,t)\Diamond N(Tu,Au,a,t) \\ \Diamond N(Au,Sx_{2n+1},a,t)\Diamond N(Sx_{2n+1},Tu,a,t) \\ \Diamond N(Bu,Tx_{2n+1},a,t)\Diamond N(Bu,Su,a,t) \end{array}\right)\right\}} \zeta(t)dt$$

taking $n \to \infty$

$$\int_0^{N(Au,u,a,t)} \zeta(t)dt \leq \int_0^{\varphi\left\{\max\left(\begin{array}{l} N(Au,u,a,t)\Diamond N(Au,Au,a,t)\Diamond N(Au,u,a,t) \\ \Diamond N(u,Au,a,t)\Diamond N(u,u,a,t)\Diamond N(u,u,a,t) \end{array}\right)\right\}} \zeta(t)dt$$

$$\int_0^{N(Au,u,a,t)} \zeta(t)dt \leq \int_0^{\varphi\{\max(N(Au,u,a,t)\Diamond 0\Diamond N(Au,u,a,t)\Diamond N(u,Au,a,t)\Diamond 0\Diamond 0)\}} \zeta(t)dt$$

$$\int_0^{N(Au,u,a,t)} \zeta(t)dt \leq \int_0^{\varphi(N(Au,u,a,t))} \zeta(t)dt < \int_0^{N(Au,u,a,t)} \zeta(t)dt$$

Therefore, we get $Au = u$. So we have $Au = Tu = u$. similarly by putting $x = x_{2n}$, $y = u$ in (2.4) as $n \to \infty$ $u = Bu = Su$. Thus $Au = Bu = Su = Tu = u$ i.e. u is a common fixed point of A, B, S and T.

Uniqueness: Let $w (w \neq u)$ be another common fixed point of A, B, S and T. then by putting $x = u, y = w$
in (2.4)

$$\int_0^{M(Au,Bw,a,t)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\begin{pmatrix} M(Tu,Sw,a,t)*M(Tu,Au,a,t)*M(Au,Sw,a,t), \\ M(Sw,Tu,t)*M(Bu,Tw,t)*M(Bu,Su,t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{M(u,w,a,t)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\begin{pmatrix} M(u,w,a,t)*M(u,u,a,t)*M(u,w,a,t) \\ *M(w,u,a,t)*M(u,w,t)*M(u,u,a,t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{M(u,w,a,t)} \zeta(t)dt \geq \int_0^{\phi\left\{\min\begin{pmatrix} M(u,w,a,t)*1*M(u,w,a,t) \\ *M(w,u,a,t)*M(u,w,a,t)*1 \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{M(u,w,a,t)} \zeta(t)dt \geq \int_0^{\phi\left(M(u,w,a,t)\right)} \zeta(t)dt > \int_0^{M(u,w,a,t)} \zeta(t)dt$$

And

$$\int_0^{N(Au,Bw,a,t)} \zeta(t)dt \leq \int_0^{\varphi\left\{\max\begin{pmatrix} N(Tu,Sw,a,t)\Diamond N(Tu,Au,a,t)\Diamond N(Au,Sw,a,t), \\ N(Sw,Tu,a,t)\Diamond N(Bu,Tw,a,t)\Diamond N(Bu,Su,a,t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{N(u,w,a,t)} \zeta(t)dt \leq \int_0^{\varphi\left\{\max\begin{pmatrix} N(u,w,a,t)\Diamond N(u,u,a,t)\Diamond N(u,w,a,t) \\ \Diamond N(w,u,a,t)\Diamond N(u,w,a,t)\Diamond N(u,u,a,t) \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{N(u,w,a,t)} \zeta(t)dt \leq \int_0^{\varphi\left\{\max\begin{pmatrix} N(u,w,a,t)\Diamond 0\Diamond N(u,w,a,t) \\ \Diamond N(w,u,a,t)\Diamond N(u,w,a,t)\Diamond 0 \end{pmatrix}\right\}} \zeta(t)dt$$

$$\int_0^{N(u,w,a,t)} \zeta(t)dt \leq \int_0^{\varphi\left(N(u,w,a,t)\right)} \zeta(t)dt < \int_0^{N(u,w,a,t)} \zeta(t)dt$$

Hence $u = w$ for all $x, y \in X$ and $t > 0$, therefore u is the unique common fixed point of A, B, S and T. This completes the proof.

References:

[1] A.Branciari. A fixed point theorem for mappings satisfying a general contractive condition of integral type. Int.J.Math.Sci. 29(9)(2002), 531 - 536.
[2]B.E. Rhoades, Two fixed point theorems for mappings satisfying a general contractive condition of integral type. International Journal of Mathematics and Mathematical Sciences, 63 (2003), 4007 - 4013.
[3] B Schweizer and A Sklar, Statistical metric spaces, Pacific J. Math., 10(1960),314–334.

[4] C. Alaca, D. Turkoglu and C. Yildiz: Fixed points in intuitionistic fuzzy metric spaces, Chaos, Solitons & Fractals, 29(5) (2006), 1073–1078.
[5] Cho. And Jung Minimization theorems for fixed point theorems in fuzzy metric spaces and applications, Fuzzy Sets and Systems 61 (1994) 199-207.
[6] D. Coker: An introduction to intuitionistic fuzzy topological spaces, Fuzzy Sets and System, 88(1997), 81–89.
[7] D. Turkoglu, C. Alaca, Y. J. Cho and C. Yildiz: Common fixed point theorems in intuitionistic fuzzy metric spaces, J. Appl. Math. & Computing, 22(2006), 411–424.
[8]Dimri R.C. and Gariya N.S, Coincidences and common fixed points in intuitionistic fuzzy metric spaces, Indian J. Math.52(3) (2010) , 479-490.
[9] G. Jungck, Commuting mappings and fixed point, Amer. Math. Monthly 83 (1976), 261-263.
[10] Grabiec M, Fixed points in fuzzy metric spaces, Fuzzy Sets and Systems, 27(1988), 385–389.
[11] Imdad M.and Ali J, Some common fixed point theorems in fuzzy metric spaces, Mathematical Communication, 11(2006), 153–163.
[12] J.S. Park, Y.C. Kwun, and J.H. Park: A fixed point theorem in the intuitionistic fuzzy metric spaces, Far East J. Math. Sci., 16(2005), 137–149.
[13] Jungck G. and Rhoades B.E, Fixed point for set valued functions without continuity, Indian J. Pure and Appl. Math., 29(3) (1998), 227-238.
[14] K. Atanassov: Intuitionistic fuzzy sets, Fuzzy Sets and System, 20(1986), 87-96.
[15] L.A. Zadeh, Fuzzy sets, Infor. and Control, 8 (1965), 338-353.
[16] Pant R.P, Common fixed points of non commuting mappings, J. Math. Anal. Appl., 188(1994), 436–440.
[17] Park J.H, Intuitionistic fuzzy metric spaces, Chaos, Solitons & Fractals,22(2004), 1039–1046.
[18] R. Kannan, Some results on fixed points, Bull. Calcutta Math. Soc., 60(1968), 71-76.
[19] S.K.Chatterjea, Fixed point theorems, C.R.Acad.Bulgare Sci. 25(1972), 727-730.
[20] S. Banach, Sur les oprations dans les ensembles abstraits et leur application aux quations intgrales, Fund. Math.3,(1922)133-181 (French).
[21] Saadati R. and Park J.H, On the intuitionistic fuzzy topological spaces, Chaos, Solitons & Fractals, 27(2006), 331–344.
[23] Sharma. P.L., Sharma. B.K. and Iseki, K. “Contractive type mapping on 2-metric spaces” Math. Japonica 21 (1976) 67-70.
[24]Sharma, S. “On fuzzy Metric space” Southeast Asian Bulletin of Mathematics 26 (2002) 133-145.
[25] T.Zamfrescu, Fixed point theorems in metric spaces, Arch. Math. (Basel) 23(1972), 292-298.
[26] V. Gregori, S. Ramaguera and P. Veeramani: A note on intuitionistic fuzzy metric Spaces, Chaos, Solitons & Fractals, 28(2006), 902–905.