Some Common Coupled Fixed Point Theorems for Occasionally Weakly Compatible Maps in Complex Valued Metric Spaces

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Abstract

In this paper we establish some common coupled fixed point theorems for two pair of occasionally weakly compatible mappings satisfying a generalize contractive condition in complex valued metric space. The proved results generalize and extend some of the existing results in the literature.

Keywords: Complex valued metric space, coupled fixed point, occasionally weakly compatible mappings.

1. Introduction

Azam et al. [2] introduced the concept of complex valued metric spaces and obtained sufficient conditions for the existence of common fixed points of a pair of contractive type mappings involving rational expressions. Subsequently many authors have studied the existence and uniqueness of the fixed points and common fixed points of self mapping in view of contrasting contractive conditions.

Recently, Bhaskar and Lakshmikantham [3] introduced the concepts of coupled fixed points and mixed monotone property and illustrated these results by proving the existence and uniqueness of the solution for a periodic boundary value problem. Later on these results were extended and generalized by Sedghi et al. [13], Fang [7] and Xin-Qi Hu [8] etc.

The study of fixed point theorems, involving four single-valued maps, began with the assumption that all of the maps are commuted. Sessa [14] weakened the condition of commutativity to that of pairwise weakly commuting. Jungck generalized the notion of weak commutativity to that of pairwise compatible [9] and then pairwise weakly compatible maps [10]. Jungck and Rhoades [11] introduced the concept of occasionally weakly compatible maps.

The intent of this paper is to present some coupled fixed point theorems for two pair of occasionally weakly compatible mappings satisfying a generalize contractive condition in complex valued metric space. The proved results generalize and extend some of the existing results in the literature.

2. Preliminaries

Let C be the set of complex numbers and let $z_1, z_2 \in C$. Define a partial order $\precsim$ on C as follows:

$z_1 \precsim z_2$ if and only if $Re(z_1) \le Re(z_2)$, $Im(z_1) \le Im(z_2)$. It follows that $z_1 \precsim z_2$ if one of the following conditions is satisfied:

(i) $Re(z_1) = Re(z_2)$, $Im(z_1) < Im(z_2)$,
(ii) $Re(z_1) < Re(z_2)$, $Im(z_1) = Im(z_2)$,

(iii) Re(z_1)<Re(z_2), Im(z_1)<Im(z_2),
(iv) Re(z_1)=Re(z_2), Im(z_1)=Im(z_2).

In particular, we will write $z_1 \preceq z_2$ if one of (i), (ii) and (iii) is satisfied and we will write $z_1 \prec z_2$ if only (iii) is satisfied.

Definition2.1. Let X be a non-empty set. Suppose that the mapping $d: X \times X \rightarrow C$ satisfies:

(i) $0 \preceq d(x,y)$ for all $x, y \in X$ and $d(x,y) = 0$ if and only if $x = y$;
(ii) $d(x,y) = d(y,x)$ for all $x, y \in X$;
(iii) $d(x,y) \preceq d(x,z) + d(z,y)$ for all $x, y, z \in X$.
Then d is called a complex valued metric on X and (X,d) is called a complex valued metric space.

Definition2.2. An element $(x,y) \in X \times X$ is called a

(i) Coupled fixed point of the mapping $f: X \times X \rightarrow X$ if
$f(x,y) = x,\ f(y,x) = y$.
(ii) Coupled coincidence point of the mapping $f: X \times X \rightarrow X$ and $g: X \rightarrow X$ if
$f(x,y) = g(x),\ f(y,x) = g(y)$.
(iii) Common Coupled coincidence point of the mapping $f: X \times X \rightarrow X$ and $g: X \rightarrow X$ if
$x = f(x,y) = g(x),\ y = f(y,x) = g(y)$.

Definition2.3. An element $x \in X$ is called a common fixed point of the mappings $f: X \times X \rightarrow X$ and $g: X \rightarrow X$ if

$$x = f(x,x) = g(x).$$

Definition2.4. Let $A, B: X \times X \rightarrow X$ and $S, T: X \rightarrow X$ be four mappings. Then, the pair of maps (B,S) and (A,T) are said to have Common Coupled coincidence point if there exist a, b in X such that

$B(a,b) = S(a) = T(a) = A(a,b)$ and $B(b,a) = S(b) = T(b) = A(b,a)$.

Definition2.5. The mappings $f: X \times X \rightarrow X$ and $g: X \rightarrow X$ are called occasionally weakly compatible maps iff $f(x,y) = g(x),\ f(y,x) = g(y)$

implies $gf(x,y) = f(gx,gy),\ gf(y,x) = f(gy,gx)$.

3. Main Results

Theorem 3.1. Let (X,d) be a Complex valued metric space. Let $A, B: X \times X \rightarrow X$ and $S, T: X \rightarrow X$ be four self-mappings satisfying the following conditions:

(i) $d\big(A(x,y), B(u,v)\big) \preceq$
$k\, max\{d(Sx,Tu), d(A(x,y),Sx), d(B(u,v),Tu), d\big(Sx, B(u,v)\big), d(A(x,y),Tu)\}$
for all $x, y, u, v \in X$ and $0 < k < 1$

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Proof: Since the pairs (A,S) and (B,T) are owc so there are points a, b, a', b' in X such that

$A(a,b) = Sa,\ A(b,a) = Sb$ and $B(a',b') = Ta',\ \ B(b',a') = Tb'$

We claim that $Sa = Ta'$. If not, by inequality (i) we get

$$d\big(A(a,b), B(a',b')\big) \preceq k\, max\{d(Sa,Ta'), d(A(a,b),Sa), d(B(a',b'),Ta'), d\big(Sa, B(a',b')\big), d(A(a,b),Ta')\}$$

or $d(Sa,Ta') \preceq k\, max\{d(Sa,Ta'), d(Sa,Sa), d(Ta',Ta'), d(Sa,Ta'), d(Sa,Ta')\}$
$\Rightarrow Sa = Ta'$

Therefore $A(a,b) = Sa = Ta' = B(a',b')$

Similarly $A(b,a) = Sb = Tb' = B(b',a')$

Thus the pairs (A,S) and (B,T)have common coincidence points.

Let $A(a,b) = Sa = Ta' = B(a',b') = x$

and $A(b,a) = Sb = Tb' = B(b',a') = y$

Since (A,S) and (B,T) are owc

So $Sx = SA(a,b) = A(Sa,Sb) = A(x,y)$

and $Sy = SA(b,a) = A(Sb,Sa) = A(y,x)$

Also $Tx = TB(a',b') = B(Ta',Tb') = B(x,y)$

and $Ty = TB(b',a') = B(Tb',Ta') = B(y,x)$

Next we show that $x = y$, for this

putting $= a$, $y = b$, $u = b'$, $v = a'$ in (i),

$d(x,y) = d\big(A(a,b), B(b',a')\big) \leq$
$k\, max\{d(Sa,Tb'), d(A(a,b),Sa), d(B(b',a'),Tb'), d\big(Sa,B(b',a')\big), d(A(a,b),Tb')\}$

$= k\, max\, \{d(Sa,Tb'), d(Sa,Sa), d(Tb',Tb')d(Sa,Tb'), d(Sa,Tb')\}$
$\Rightarrow x = y$

Now we prove that $Sx = Tx$

$d(Sx,Tx) = d(Sx,Ty) = d\big(A(x,y), B(y,x)\big) \leq$
$k\, max\{d(Sx,Ty), d(A(x,y),Sx), d(B(y,x),Ty)d\big(Sx,B(y,x)\big), d(A(x,y),Ty)\}$

$= k\, max\{d(Sx,Ty), d(Sx,Sx), d(Ty,Ty), d(Sx,Ty), d(Sx,Ty)\}$
$\Rightarrow Sx = Tx$

Also by condition (ii) we have,

$$x = B(x,x)$$

Thus $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Theorem: 3.2. Let (X,d) be a Complex valued metric space and $A,B: X \times X \to X$ and $S,T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $d\big(A(x,y), B(u,v)\big) \leq k\, max\{d(Sx,Tu), d(A(x,y),Sx), d(B(u,v),Tu)\}$
for all $x,y,u,v \in X$ and $0 < k < 1$

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Proof: The proof is similar as theorem 3.1.

Theorem: 3.3. Let (X,d) be a Complex valued metric space. Let $A,B: X \times X \to X$ and $S,T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $d\big(A(x,y), B(u,v)\big) \leq k\, d(Sx,Tu)$
for all $x,y,u,v \in X$ and $0 < k < 1$.

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Proof: The proof is similar as theorem 3.1.

Theorem: 3.4 Let (X,d) be a Complex valued metric space. Let $A,B: X \times X \to X$ and $S,T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $d(A(x,y),B(u,v)) \le ad(A(x,y),Tu) + bd(Sx,B(u,v)) + c\left[\frac{d(Sx,Tu)}{d(A(x,y),Sx)+d(B(u,v),Tu)+1}\right]$
for all $x,y,u,v \in X$ and $(a+b+c) \le 1$

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Proof: Since the pairs (A,S) and (B,T) are owc so there are points a,b,a',b' in X such that

$A(a,b) = Sa$, $A(b,a) = Sb$ and $B(a',b') = Ta'$, $B(b',a') = Tb'$

We claim that $Sa = Ta'$. If not, by inequality (i) we get

$$d(A(a,b),B(a',b')) \le ad(A(a,b),Ta') + bd(Sa,B(a',b')) + c\left[\frac{d(Sa,Ta')}{d(A(a,b),Sa)+d(B(a',b'),Ta')+1}\right]$$

or $d(Sa,Ta') \le ad(Sa,Ta') + bd(Sa,Ta') + c\left[\frac{d(Sa,Ta')}{d(Sa,Sa)+d(Ta',Ta')+1}\right]$

$$= ad(Sa,Ta') + bd(Sa,Ta') + cd(Sa,Ta')$$

$$= (a+b+c)d(Sa,Ta')$$

$\Rightarrow Sa = Ta'$

Therefore $A(a,b) = Sa = Ta' = B(a',b')$

Similarly $A(b,a) = Sb = Tb' = B(b',a')$

Thus the pairs (A,S) and (B,T)have common coincidence points.

Let $A(a,b) = Sa = Ta' = B(a',b') = x$

and $A(b,a) = Sb = Tb' = B(b',a') = y$

Since (A,S) and (B,T) are owc

So $Sx = SA(a,b) = A(Sa,Sb) = A(x,y)$

and $Sy = SA(b,a) = A(Sb,Sa) = A(y,x)$

Also $Tx = TB(a',b') = B(Ta',Tb') = B(x,y)$

and $Ty = TB(b',a') = B(Tb',Ta') = B(y,x)$

Next we show that $x = y$, for this

Putting $x = a$, $y = b$, $= b'$, $v = a'$ in (i),

$$d(x,y) = d\big(A(a,b), B(b',a')\big)$$
$$\leq ad(A(a,b),Tb') + bd\big(Sa, B(b',a')\big) + c\left[\frac{d(Sa,Tb')}{d(A(a,b),Sa) + d(B(b',a'),Tb') + 1}\right]$$
$$= ad(Sa,Tb') + bd(Sa,Tb') + c\left[\frac{d(Sa,Tb')}{d(Sa,Sa)+d(Tb',Tb')+1}\right]$$
$$= ad(Sa,Tb') + bd(Sa,Tb') + cd(Sa,Tb')$$
$$= (a+b+c)d(Sa,Tb')$$
$$\Longrightarrow x = y$$

Now we prove that $Sx = Tx$

$$d(Sx,Tx) = d(Sx,Ty) = d\big(A(x,y), B(y,x)\big)$$
$$\leq ad(A(x,y),Ty) + bd\big(Sx, B(y,x)\big) + c\left[\frac{d(Sx,Ty)}{d(A(x,y),Sx) + d(B(y,x),Ty) + 1}\right]$$
$$= ad(Sx,Ty) + bd(Sx,Ty) + c\left[\frac{d(Sx,Ty)}{d(Sx,Sx) + d(Ty,Ty) + 1}\right]$$
$$= ad(Sx,Ty) + bd(Sx,Ty) + c\, d(Sx,Ty)$$
$$= (a+b+c)d(Sx,Tx)$$
$$\Longrightarrow Sx = Tx$$

Also by condition (i) we have,

$$x = B(x,x)$$

Thus $A(x,x) = T(x) = B(x,x) = S(x) = x.$

Theorem: 3.5 Let (X,d) be a Complex valued metric space. Let $A,B: X \times X \to X$ and $S,T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $d\big(A(x,y), B(u,v)\big) \leq \frac{k}{3}\{d(Sx,Tu) + d(A(x,y),Tu), d\big(Sx, B(u,v)\big)\}$
for all $x,y,u,v \in X$ and $0 < k < 1$

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x.$

Proof: Since the pairs (A,S) and (B,T) are owc so there are points a,b,a',b' in X such that

$A(a,b) = Sa$, $A(b,a) = Sb$ and $B(a',b') = Ta'$, $B(b',a') = Tb'$

We claim that $Sa = Ta'$. If not, by inequality (i) we get

$$d\big(A(a,b), B(a',b')\big) \leq \frac{k}{3}\{d(Sa,Ta') + d(A(a,b),Ta') + d\big(Sa, B(a',b')\big)\}$$

or $d(Sa,Ta') \leq \frac{k}{3}\{d(Sa,Ta') + d(Sa,Ta') + d(Sa,Ta')\}$

$$= k\, d(Sa,Ta')$$

$\Rightarrow Sa = Ta'$

Therefore $A(a,b) = Sa = Ta' = B(a',b')$

Similarly $A(b,a) = Sb = Tb' = B(b',a')$

Thus the pairs (A,S) and (B,T)have common coincidence points.

Let $A(a,b) = Sa = Ta' = B(a',b') = x$

and $A(b,a) = Sb = Tb' = B(b',a') = y$

Since (A,S) and (B,T) are owc

So $Sx = SA(a,b) = A(Sa,Sb) = A(x,y)$

and $Sy = SA(b,a) = A(Sb,Sa) = A(y,x)$

Also $Tx = TB(a',b') = B(Ta',Tb') = B(x,y)$

and $Ty = TB(b',a') = B(Tb',Ta') = B(y,x)$

Next we show that $x = y$, for this

Putting $x = a\ , y = b\ , = b'\ , v = a'$ in (i),

$$d(x,y) = d\big(A(a,b), B(b',a')\big) \le \frac{k}{3}\{d(Sa,Tb') + d(A(a,b),Tb') + d\big(Sa, B(b',a')\big)\}$$

$$= \frac{k}{3}\{d(Sa,Tb') + d(Sa,Tb') + d(Sa,Tb')\}$$

$$= kd(Sa,Tb')$$

$\Rightarrow x = y$

Now we prove that $Sx = Tx$

$$d(Sx,Tx) = d(Sx,Ty) = d\big(A(x,y),B(y,x)\big) \le \frac{k}{3}\{d(Sx,Ty) + d(A(x,y),Ty) + d\big(Sx,B(y,x)\big)\}$$

$$= \frac{k}{3}\{d(Sx,Ty) + d(Sx,Ty) + d(Sx,Ty)\}$$

$$= kd(Sx,Tx)$$

$\Rightarrow Sx = Tx$

Also by condition (ii) we have,

$$x = B(x,x)$$

Thus $A(x,x) = T(x) = B(x,x) = S(x) = x.$

Theorem: 3.6 Let (X,d) be a Complex valued metric space. Let $A,B: X \times X \to X$ and $S,T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $d\big(A(x,y),B(u,v)\big) \le \alpha d(Sx,Tu) + \beta d\big(Tu, A(x,y)\big) + \gamma d(B(u,v),Sx)$
for all $x,y,u,v \in X$ and $(\alpha + \beta + \gamma) \le 1$

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x.$

Proof: Since the pairs (A,S) and (B,T) are owc so there are points a,b,a',b' in X such that

$A(a,b) = Sa,\ A(b,a) = Sb$ and $B(a',b') = Ta',\ B(b',a') = Tb'$

We claim that $Sa = Ta'$. If not, by inequality (i) we get

$d(A(a,b), B(a',b')) \leq \alpha d(Sa,Ta') + \beta d(Ta', A(a,b)) + \gamma d(B(a',b'), Sa)$

or $d(Sa,Ta') \leq \alpha d(Sa,Ta') + \beta d(Ta',Sa) + \gamma d(Ta',Sa)$

$$= (\alpha + \beta + \gamma) d(Sa,Ta')$$

$\Rightarrow Sa = Ta'$

Therefore $A(a,b) = Ta' = Sa = B(a',b')$

Similarly $A(b,a) = Tb' = Sb = B(b',a')$

Thus the pairs (A,S) and (B,T)have common coincidence points.

Let $A(a,b) = Ta' = Sa = B(a',b') = x$

and $A(b,a) = Tb' = Sb = B(b',a') = y$

Since (A,S) and (B,T) are owc

So $Sx = SA(a,b) = A(Sa,Sb) = A(x,y)$

and $Sy = SA(b,a) = A(Sb,Sa) = A(y,x)$

Also $Tx = TB(a',b') = B(Ta',Tb') = B(x,y)$

and $Ty = TB(b',a') = B(Tb',Ta') = B(y,x)$

Next we show that $x = y$, for this

putting $x = a, y = b, u = b', v = a'$ in (i),

$$d(x,y) = d(A(a,b), B(b',a')) \leq \alpha d(Sa,Tb') + \beta d(Tb', A(a,b)) + \gamma d(B(b',a'), Sa)$$

$$= \alpha d(Sa,Tb') + \beta d(Tb',Sa) + \gamma d(Tb',Sa)$$

$$= (\alpha + \beta + \gamma) d(Sa,Tb')$$

$\Longrightarrow x = y$

Now we prove that $Sx = Tx$

$$d(Sx,Tx) = d(Sx,Ty) = d(A(x,y), B(y,x)) \leq \alpha d(Sx,Ty) + \beta d(Ty, A(x,y)) + \gamma d(B(y,x), Sx)$$

$$= \alpha d(Sx,Ty) + \beta d(Ty,Sx) + \gamma d(Ty,Sx)$$

$$= (\alpha + \beta + \gamma)\ d(Sx,Ty)$$

$$\Longrightarrow Sx = Tx$$

Also by condition (ii) we have,

$$x = B(x,x)$$

Thus $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Theorem: 3.7. Let (X,d) be a Complex valued metric space. Let $A,B: X \times X \to X$ and $S,T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $d(A(x,y),B(u,v)) \leq \alpha \max[d(Sx,Ty),d(Sx,A(x,y))] + \beta \max[d(B(u,v),Tv),d(A(x,y),Tu)] + \gamma d(B(u,v),Sx)$

for all $x,y,u,v \in X$ and $(\alpha+\beta+\gamma) \leq 1$

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Proof: The proof is similar as theorem 3.6.

Theorem: 3.8 Let (X,d) be a Complex valued metric space. Let $A,B: X \times X \to X$ and $S,T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $d(A(x,y),B(u,v)) \leq$
$k\psi\{d(Sx,Tu),d(Sx,A(x,y)),d(B(u,v),Tu),d(A(x,y),Tu),d(Sx,B(u,v))\}$

where $\psi: [0,1]^5 \to [0,1]$, $\psi(t,1,1,t,t) \leq t$ and $0 < k < 1$, for all $x,y,u,v \in X$

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x$.

Proof: Since the pairs (A,S) and (B,T) are owc so there are points a,b,a',b' in X such that

$A(a,b) = Sa$, $A(b,a) = Sb$ and $B(a',b') = Ta'$, $B(b',a') = Tb'$

We claim that $Sa = Ta'$. If not, by inequality (i) we get

$d(A(a,b),B(a',b')) \leq k\psi\{d(Sa,Ta'),d(Sa,A(a,b)),d(B(a',b'),Ta'),d(A(a,b),Ta'),d(Sa,B(a',b'))\}$

or $d(Sa,Ta') \leq k\psi\{d(Sa,Ta'),d(Sa,Sa),d(Ta',Ta'),d(Sa,Ta'),d(Sa,Ta')\}$

$\leq kd(Sa,Ta')$

$\Rightarrow Sa = Ta'$

Therefore $A(a,b) = Ta' = Sa = B(a',b')$

Similarly $A(b,a) = Tb' = Sb = B(b',a')$

Thus the pairs (A,S) and (B,T)have common coincidence points.

Let $A(a,b) = Ta' = Sa = B(a',b') = x$

and $A(b,a) = Tb' = Sb = B(b',a') = y$

Since (A,S) and (B,T) are owc

So $Sx = SA(a,b) = A(Sa,Sb) = A(x,y)$

and $Sy = SA(b,a) = A(Sb,Sa) = A(y,x)$

Also $Tx = TB(a',b') = B(Ta',Tb') = B(x,y)$

and $Ty = TB(b',a') = B(Tb',Ta') = B(y,x)$

Next we show that $x = y$, for this

Putting $x = a$, $y = b$, $= b'$, $v = a'$ in (i),

$d(x,y) = d\big(A(a,b), B(b',a')\big) \leq$
$k\psi\{d(Sa,Tb'), d\big(Sa, A(a,b)\big), d(B(b',a'),Tb'), d(A(a,b),Tb'), d\big(Sa, B(b',a')\big)\}$

$$= k\psi\{d(Sa,Tb'), d(Sa,Sa), d(Tb',Tb'), d(Sa,Tb'), d(Sa,Tb')\}$$

$$\leq kd(Sa,Tb')$$

$\Longrightarrow x = y$

Now we prove that $Sx = Tx$

$$d(Sx,Tx) = d(Sx,Ty) = d\big(A(x,y), B(y,x)\big)$$
$$\leq k\psi\{d(Sx,Ty), d\big(Sx, A(x,y)\big), d(B(y,x),Ty), d(A(x,y),Ty), d\big(Sx, B(y,x)\big)\}$$
$$= k\psi\{d(Sx,Ty), d(Sx,Sx), d(Ty,Ty), d(Sx,Ty), d(Sx,Ty)\}$$

$$\leq kd(Sx,Tx)$$

$\Longrightarrow Sx = Tx$

Also by condition (ii) we have,

$$x = B(x,x)$$

Thus $A(x,x) = T(x) = B(x,x) = S(x) = x.$

Theorem: 3.9. Let (X,d) be a Complex valued metric space. Let $A,B: X \times X \to X$ and $S,T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $d\big(A(x,y), B(u,v)\big) \leq k\psi\{d(Sx,Tu), d(A(x,y),Tu), d\big(Sx, B(u,v)\big)\}$

where $\psi: [0,1]^3 \to [0,1]$, $\psi(t,t,t) \leq t$ and $0 < k < 1$, for all $x,y,u,v \in X$

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x.$

Proof: The proof is similar as theorem 3.8.

Theorem: 3.10. Let (X,d) be a Complex valued metric space. Let $A,B: X \times X \to X$ and $S,T: X \to X$ be four self-mappings satisfying the following conditions:

(i) $d\big(A(x,y), B(u,v)\big) \leq$
$k\psi max\{d(Sx,Tu), d\big(Sx, A(x,y)\big), d(B(u,v),Tu), d(A(x,y),Tu), d\big(Sx, B(u,v)\big)\}$

where $\psi: [0,1] \to [0,1]$, $\psi(t) \leq t$ and $0 < k < 1$, for all $x,y,u,v \in X$

(ii) $y = B(x,y)$

Moreover if the pairs (A,S) and (B,T) are owc, then there exists a unique point x in X such that $A(x,x) = T(x) = B(x,x) = S(x) = x.$

Proof: The proof is similar as theorem 3.8.

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