

Analysis of Single Server Queueing System With Batch Service Under Multiple Vacations With Loss And Feedback

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Abstract

Consider a single server queueing system with foxed batch service under multiple vacations with loss and feedback in which the arrival rate λ follows a Poisson process and the service time follows an exponential distribution with parameter μ. Assume that the system initially contain k customers when the server enters the system and starts the service in batch. The concept of feedback is incorporated in this model (i.e) after completion of the service, if this batch of customers dissatisfied then this batch may join the queue with probability q and with probability (1-q) leaves the system. This q is called a feedback probability. After completion of the service if he finds more than k customers in the queue then the first k customers will be taken for service and service will be given as a batch of size k and if he finds less than k customers in the queue then he leaves for a multiple vacation of exponential length α. The impatient behaviour of customer is also studied in this model (i.e) the arriving customer may join the queue with probability p when the server is busy or in vacation. This probability p is called loss probability. This model is completely solved by constructing the generating function and Rouche's theorem is applied and we have derived the closed form solutions for probability of number of customers in the queue during the server busy and in vacation. Further we are providing the analytical solution for mean number of customers and variance of the system. Numerical studies have been done for analysis of mean and variance for various values of λ, μ, α, p, q and k and also various particular cases of this model have been discussed.

Keywords : Single Server , Batch Service, Loss and Feedback, Multiple vacations, Steady state distribution.

1. Introduction

Queueing systems where services are offered in batches instead of personalized service of one at a time. A service may be of a fixed size (say) k, such a system is called fixed batch size. Bailey (1954), was the first to consider bulk service. Such models find applications in several situations, such as transportation. Mass transit vehicles are natural batch servers. There are a great number of numerical and approximations methods are available, in this paper we will place more emphasis on the solutions by Probability Generating Function.

Formulation of queues with feedback mechanism was first introduced by Takacs. The concepts loss and feedback are introduced for customers only. After completion of the service, if the customer dissatisfied then he may join the queue with probability q and with probability (1-q) he leaves the system. This is called **feedback** in queueing theory. If the server is busy at the time of the arrival of customer, then due to impatient behaviour of the customer, customer may or may not join the queue. This is called **loss** in queueing theory. We assume that p is the probability that the customer joins the queue and (1-p) is the probability that he leaves the system without getting service (due to impatient).

Feedback queues play a vital role in the areas of Computer networks, Production systems subject to rework, Hospital management, Super markets and Banking business etc. Takacs (1963), introduced the concept of feedback queues. Disney, McNickle and Simmon (1980), D'Avignon and Disney (1976), Krishnakumar (2002),Thangaraj and Vanitha . (2009,2010). Ayyappan et al., (**2010**), Farahmand .K and T. Li(2009) are a few to be mentioned for their contribution.

In this paper we are going to concentrate on a very special batch service queue called the fixed size batch service queue under multiple vacations with loss and feedback. The model under consideration is described in Section 2. In Section 3 we analyze the model by deriving the system steady state equations and probability generating functions. Using these generating functions, steady state probabilities are obtained in Section 4. The operating characteristics are obtained in Section 5. A numerical study is carried out in Section 6 to test the effect of the system performance measure discussed in Section 5. We are providing the analytical solution for mean number of customers and variance of the system. Numerical studies have been done for analysis of mean and variance for various values of λ, μ, α, p, q and k and also various particular cases of this model have been discussed.

2. DESCRIBE OF THE MODEL

Consider a single server queueing system with foxed batch service under multiple vacations with loss and feedback in which the arrival rate λ follows a Poisson process and the service time follows an exponential distribution with parameter μ. Assume that the system initially contain k customers when the server enters the system and starts the service in batch. The concept of feedback is incorporated in this model (i.e) after completion of the service, if this batch of customers dissatisfied then this batch may join the queue with probability q and with probability (1-q) leaves the system. This q is called a feedback probability. After completion of the service if he finds more than k customers in the queue then the first k customers will be taken for service and service will be given as a batch of size k and if he finds less than k customers in the queue then he leaves for a multiple vacation of exponential length α. The impatient behaviour of customer is also studied in this model (i.e) the arriving customer may join the queue with probability p when the server is busy or in vacation. This probability p is called loss probability.
Let < N(t),C(t) > be a random process where N(t) be the random variable which represents the number of customers in queue at time t and C(t) be the random variable which represents the server status (busy/vacation) at time t.

We define

$P_{n,1}(t)$ - probability that the server is in busy if there are n customers in the queue at time t.
$P_{n,2}(t)$ - probability that the server is in vacation if there are n customers in the queue at time t
The Chapman- Kolmogorov equations are

$$P_{0,1}^{'}(t) = -(\lambda p + \mu(1-q))P_{0,1}(t) + \mu(1-q)P_{k,1}(t) + \alpha P_{k,2}(t) \quad (1)$$

$$P_{n,1}^{'}(t) = -(\lambda p + \mu(1-q))P_{n,1}(t) + \lambda p P_{n-1,1}(t) + \mu(1-q)P_{n+k,1}(t) + \alpha P_{n+k,2}(t) \text{ for } n = 1,2,3,... \quad (2)$$

$$P_{0,2}^{'}(t) = -\lambda p P_{0,2}(t) + \mu(1-q)P_{0,1}(t) \quad (3)$$

$$P_{n,2}^{'}(t) = -\lambda p P_{n,2}(t) + \lambda p P_{n-1,2}(t) + \mu(1-q)P_{n,1}(t) \text{ for } n = 1,2,3,...,k-1 \quad (4)$$

$$P_{n,2}^{'}(t) = -(\lambda p + \alpha)P_{n,2}(t) + \lambda p P_{n-1,2}(t) \text{ for } n \geq k \quad (5)$$

3. EVALUATION OF STEADY STATE PROBABILITIES

In this section we are finding the closed form solutions for number of customers in the queue when the server is busy or in vacation using Generating function technique.

When steady state prevails, the equations (1) to (5) becomes

$$(\lambda p+\mu(1-q))P_{0,1}=\mu(1-q)P_{k,1}+\alpha \mathrm{P}_{k,2} \quad (6)$$

$$(\lambda p+\mu(1-q))P_{n,1}=\lambda pP_{n-1,1}+\mu(1-q)P_{n+k,1}+\alpha \mathrm{P}_{n+k,2} \text{ for } n=1,2,3,.... \quad (7)$$

$$\lambda pP_{0,2}=\mu(1-q)P_{0,1} \quad (8)$$

$$\lambda pP_{n,2}=\lambda pP_{n-1,2}+\mu(1-q)P_{n,1} \text{ for } n=1,2,3,...,k-1 \quad (9)$$

$$(\lambda p+\alpha)P_{n,2}=\lambda pP_{n-1,2} \text{ for } n\geq k \quad (10)$$

Generating functions for the number of customers in the queue when the server is busy or in the vacation are defined as

$$G(z)=\sum_{n=0}^{\infty}P_{n,1}z^{n} \text{ and } H(z)=\sum_{n=0}^{\infty}P_{n,2}z^{n}$$

Multiply the equation (6) with 1 and (7) with z^n on both sides and summing over n = 0 to ∞, we get

$$G(z)[\lambda pz^{k+1}-(\lambda p+\mu(1-q))z^{k}+\mu]+\alpha H(z)=\mu(1-q)\sum_{n=0}^{k-1}P_{n,1}z^{n}+\alpha\sum_{n=0}^{k-1}P_{n,2}z^{n} \quad (11)$$

Adding equation (8), (9) and (10) after multiply with 1, z^n (n = 1, 2, 3, . . ., k-1) and z^n (n = k, k+1, k+2, . . .) respectively, we get

$$H(z)[\alpha+\lambda p(1-z)]=\mu(1-q)\sum_{n=0}^{k-1}P_{n,1}z^{n}+\alpha\sum_{n=0}^{k-1}P_{n,2}z^{n} \quad (12)$$

From the equations (11) and (12), we get

$$G(z)=\frac{H(z)\lambda p(1-z)}{\lambda pz^{k+1}-(\lambda p+\mu(1-q))z^{k}+\mu(1-q)} \quad (13)$$

From equation (12), we get

$$H(z)=\frac{\mu(1-q)\sum_{n=0}^{k-1}P_{n,1}z^{n}+\alpha\sum_{n=0}^{k-1}P_{n,2}z^{n}}{\alpha+\lambda p(1-z)} \quad (14)$$

Equation (14) represents the probability generating function for number of customers in the queue when the server is in vacation.

From the equations (13) and (14), we get

$$G(z)=\frac{(\lambda p(1-z))\mu(1-q)\sum_{n=0}^{k-1}P_{n,1}z^{n}+\alpha\sum_{n=0}^{k-1}P_{n,2}z^{n}}{[\lambda pz^{k+1}-(\lambda p+\mu(1-q))z^{k}+\mu][\alpha+\lambda p(1-z)]} \quad (15)$$

Equation (15) represents the probability generating for number of customers in the queue when the server is busy.

Put z = 1 in equation (13), we get

$$G(1)=\frac{\lambda p}{k\mu(1-q)-\lambda p}H(1) \quad (16)$$

The normalized condition is $G(1)+H(1)=1$ (17)

Using the equation (1) in (17), we get

$$H(1)=\frac{k\mu(1-q)-\lambda p}{k\mu(1-q)} \quad (18)$$

From the equations (16) and (18) , we get

Steady state probability that the server is busy = $G(1)=\dfrac{\lambda p}{k\mu(1-q)}$

Steady state probability that the server in vacation = $H(1)=\dfrac{k\mu(1-q)-\lambda p}{k\mu(1-q)}$

The generating function G(z) has the property that it must converge inside the unit circle. We notice that the denominator of G(z), $\lambda pz^{k+1}-(\lambda p+\mu(1-q))z^{k}+\mu(1-q)$ has k+1 zero's. Applying Rouche's theorem, we notice that k zeros of this expression lies inside the circle $|z|<1$ and must coincide with k zeros of numerator of G(z) and one zero lies outside the circle $|z|<1$. Let z_0 be a zero which lies outside the circle $|z|<1$.

As G(z) converges, k zeros of numerator and denominator will be cancelled, we get

$$G(z)=\frac{A}{(\alpha+\lambda p(1-z))\lambda(z-z_{0})} \quad (19)$$

Put z = 1 in the equation (19), we get

$$G(1)=\frac{A}{\lambda(1-z_{0})\alpha} \quad (20)$$

Using (16) and (18) in (20), we obtain

$$A = \frac{\lambda^2 p}{k\mu(1-q)}(1-z_0)\alpha \tag{21}$$

From the equation (19) and (21), we get

$$G(z) = \frac{\lambda p}{(k\mu(1-q)-\lambda p)} \frac{(1-z_0)\alpha}{(z-z_0)(\alpha+\lambda p-\lambda pz)} \tag{22}$$

By applying partial fractions, we get

$$G(z) = \frac{\alpha}{(k\mu(1-q)-\lambda p)} \frac{(1-z_0)\lambda p}{(\alpha+\lambda p-\lambda pz_0)}\left[\sum_{n=0}^{\infty}\left(\frac{\lambda p}{\alpha+\lambda p}\right)^{n+1} z^n - \sum_{n=0}^{\infty}\frac{z^n}{z_0^{n+1}}\right] \tag{23}$$

Comparing the coefficient of z^n on both sides of the equation (23), we get

$$P_{n,1} = \frac{\alpha}{(k\mu(1-q)-\lambda p)} \frac{(1-r)s}{(s-r)}(s^{n+1} - r^{n+1}) \text{ for n = 0,1,2,...} \tag{24}$$

where $r = \frac{1}{z_0}$ and $s = \frac{\lambda p}{\lambda p+\alpha}$

Using equation (24) in (8) and (9) , apply recursive for n = 1,2,3,. . . ,k-1, we get

$$P_{n,2} = \frac{\mu(1-q)}{\lambda p}\sum_{t=0}^{n} P_{t,1} \text{ for n = 0,1,2,3,. . . ,k-1} \tag{25}$$

$$P_{n,2} = \left(\frac{\lambda p}{\lambda p+\alpha}\right)^{n-k+1} P_{k-1,2} \quad \text{for} \quad n \geq k \tag{26}$$

Equations (24), (25) and (26) represent the steady state probabilities for number of customers in the queue when the server is busy / in vacation.

4. STABILITY CONDITION

The necessary and sufficient condition for the system to the stable is $\frac{\lambda p}{k\mu(1-q)} < 1$.

5. PARTICULAR CASE

If we take p =1 and q = 0, the results coincides with the results of the model single server batch service under multiple vacation.

6. SYSTEM PERFORMANCE MEASURES

In this section, we will list some important performance measures along with their formulas. These measures are used to bring out the qualitative behaviour of the queueing model under study. Numerical study has been dealt in very large scale to study the following measures

a. Probability that there are n customers in the queue when the server is busy

$$P_{n,1} = \frac{\alpha}{(k\mu(1-q)-\lambda p)}\frac{(1-r)s}{(s-r)}(s^{n+1} - r^{n+1}) \text{ for n = 0,1,2,...}$$

Where $r_1 = \frac{1}{z_0}$ and $s = \frac{\lambda p}{\lambda p+\alpha}$

b. Probability that there are n customers in the queue when the server in vacation

$$P_{n,2} = \frac{\mu(1-q)}{\lambda p}\sum_{t=0}^{n} P_{t,1} \text{ for n = 0,1,2,...,k-1}$$

$$P_{n,2} = \left(\frac{\lambda p}{\lambda p+\alpha}\right)^{n-k+1} P_{k-1,2} \text{ for } n \geq k$$

c. Mean number of customers in the queue

$$L_q = \sum_{n=0}^{\infty} n(P_{n,1} + P_{n,2})$$

d.Variance of the number of customers in the queue

$$V(x) = \left(\sum_{n=0}^{\infty} n^2 P_{n,1} + \sum_{n=0}^{\infty} n^2 P_{n,2}\right) - (L_q)^2$$

.

7. NUMERICAL STUDIES

The values of parameters λ, μ, α, p, q and k are chosen so that they satisfy the stability condition discussed in section 4. The system performance measures of this model have been done and expressed in the form of tables for various values λ, μ, α and k.

Tables 3, 6 and 9 show the impact of arrival rate λ and k over Mean number of customers in the queue.

Tables 10 show the impact of p and k over Mean number of customers in the queue.

Further we infer the following

- Mean number of customers in the queue increases as arrival rate λ increases.
- P_1 and P_2 are probabilities of server in busy and in vacation

Tables 1, 4and 7 show the steady probabilities distribution when server is busy for various values of λ and different batch size k.

Tables 2, 5 and 8 show the steady state probabilities distribution when the server is in vacation for various values of λ and different batch size k.

Table 1: Steady state Probabilities distribution for various values of λ and μ = 10, α = 5, p = 0.8, q = 0.2 when the server is busy and batch size is K = 2

Λ	p_{01}	p_{11}	p_{21}	p_{31}	p_{41}	p_{51}	p_{61}	p_{71}	p_{81}	p_{91}	p_{101}
1	0.0392	0.0090	0.0016	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.5	0.0525	0.0171	0.0042	0.0009	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0628	0.0260	0.0081	0.0023	0.0006	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
2.5	0.0708	0.0349	0.0130	0.0043	0.0014	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.0769	0.0435	0.0186	0.0071	0.0026	0.0009	0.0003	0.0001	0.0000	0.0000	0.0000
3.5	0.0814	0.0516	0.0246	0.0105	0.0042	0.0017	0.0006	0.0002	0.0001	0.0000	0.0000
4	0.0846	0.0589	0.0309	0.0145	0.0064	0.0027	0.0011	0.0005	0.0002	0.0001	0.0000
4.5	0.0868	0.0655	0.0373	0.0189	0.0090	0.0042	0.0019	0.0008	0.0004	0.0002	0.0001
5	0.0881	0.0714	0.0435	0.0237	0.0121	0.0060	0.0029	0.0013	0.0006	0.0003	0.0001
5.5	0.0886	0.0764	0.0495	0.0286	0.0155	0.0081	0.0041	0.0021	0.0010	0.0005	0.0002
6	0.0885	0.0807	0.0553	0.0337	0.0193	0.0106	0.0057	0.0030	0.0016	0.0008	0.0004
6.5	0.0878	0.0842	0.0606	0.0388	0.0234	0.0135	0.0076	0.0042	0.0023	0.0012	0.0007
7	0.0867	0.0870	0.0655	0.0439	0.0276	0.0167	0.0098	0.0056	0.0032	0.0018	0.0010
7.5	0.0852	0.0891	0.0699	0.0488	0.0319	0.0201	0.0123	0.0074	0.0044	0.0025	0.0015
8	0.0834	0.0906	0.0738	0.0535	0.0363	0.0237	0.0151	0.0094	0.0057	0.0035	0.0021
8.5	0.0813	0.0914	0.0772	0.0579	0.0407	0.0275	0.0181	0.0116	0.0074	0.0046	0.0029
9	0.0789	0.0917	0.0799	0.0620	0.0450	0.0314	0.0213	0.0142	0.0093	0.0060	0.0038
9.5	0.0763	0.0914	0.0822	0.0657	0.0492	0.0354	0.0247	0.0169	0.0114	0.0076	0.0050

Table 2: Steady state Probabilities distribution for various values of λ and μ = 10, α = 5, p = 0.8, q = 0.2 when the server is in vacation and batch size is K = 2

Λ	p_{02}	p_{12}	p_{22}	p_{32}	p_{42}	p_{52}	p_{62}	p_{72}	p_{82}	p_{92}	p_{102}
1	0.3915	0.4814	0.0664	0.0092	0.0013	0.0002	0.0000	0.0000	0.0000	0.0000	0.0000
1.5	0.3498	0.4639	0.0898	0.0174	0.0034	0.0007	0.0001	0.0000	0.0000	0.0000	0.0000
2	0.3141	0.4439	0.1076	0.0261	0.0063	0.0015	0.0004	0.0001	0.0000	0.0000	0.0000
2.5	0.2832	0.4227	0.1208	0.0345	0.0099	0.0028	0.0008	0.0002	0.0001	0.0000	0.0000
3	0.2562	0.4012	0.1301	0.0422	0.0137	0.0044	0.0014	0.0005	0.0002	0.0000	0.0000
3.5	0.2325	0.3798	0.1363	0.0489	0.0176	0.0063	0.0023	0.0008	0.0003	0.0001	0.0000
4	0.2115	0.3588	0.1400	0.0546	0.0213	0.0083	0.0032	0.0013	0.0005	0.0002	0.0001
4.5	0.1928	0.3385	0.1417	0.0593	0.0248	0.0104	0.0044	0.0018	0.0008	0.0003	0.0001
5	0.1761	0.3188	0.1417	0.0630	0.0280	0.0124	0.0055	0.0025	0.0011	0.0005	0.0002
5.5	0.1611	0.3000	0.1404	0.0657	0.0308	0.0144	0.0067	0.0032	0.0015	0.0007	0.0003
6	0.1475	0.2819	0.1381	0.0676	0.0331	0.0162	0.0079	0.0039	0.0019	0.0009	0.0005
6.5	0.1351	0.2646	0.1349	0.0688	0.0351	0.0179	0.0091	0.0046	0.0024	0.0012	0.0006
7	0.1239	0.2482	0.1311	0.0693	0.0366	0.0193	0.0102	0.0054	0.0029	0.0015	0.0008
7.5	0.1136	0.2324	0.1268	0.0692	0.0377	0.0206	0.0112	0.0061	0.0033	0.0018	0.0010
8	0.1042	0.2174	0.1221	0.0685	0.0385	0.0216	0.0121	0.0068	0.0038	0.0021	0.0012
8.5	0.0956	0.2031	0.1171	0.0675	0.0389	0.0224	0.0129	0.0074	0.0043	0.0025	0.0014
9	0.0876	0.1895	0.1118	0.0660	0.0390	0.0230	0.0136	0.0080	0.0047	0.0028	0.0016
9.5	0.0803	0.1765	0.1064	0.0642	0.0387	0.0234	0.0141	0.0085	0.0051	0.0031	0.0019

Table 3: Average number of customers in the queue and Variance for various values of λ, p = 0.8, q = 0.2 and μ = 10, α = 5 and batch size is K = 2

Λ	μ	α	P_1	P_2	Mean	Variance
1	10	5	0.0500	0.9500	0.6608	0.4366
1.5	10	5	0.0750	0.9250	0.7427	0.5511
2	10	5	0.1000	0.9000	0.8260	0.6809
2.5	10	5	0.1250	0.8750	0.9112	0.8269
3	10	5	0.1500	0.8500	0.9986	0.9905
3.5	10	5	0.1750	0.8250	1.0885	1.1729
4	10	5	0.2000	0.8000	1.1814	1.3758
4.5	10	5	0.2250	0.7750	1.2775	1.6010
5	10	5	0.2500	0.7500	1.3774	1.8507
5.5	10	5	0.2750	0.7250	1.4813	2.1275
6	10	5	0.3000	0.7000	1.5900	2.4344
6.5	10	5	0.3250	0.6750	1.7038	2.7753
7	10	5	0.3500	0.6500	1.8236	3.1545
7.5	10	5	0.3750	0.6250	1.9500	3.5775
8	10	5	0.4000	0.6000	2.0839	4.0509
8.5	10	5	0.4250	0.5750	2.2264	4.5830
9	10	5	0.4500	0.5500	2.3785	5.1838
9.5	10	5	0.4750	0.5250	2.5419	5.8661

Table 4: Steady state Probabilities distribution for various values of λ and μ = 10, α = 5, p = 0.8, q = 0.2 when the server is busy and batch size is K = 4

Λ	p_{01}	p_{11}	p_{21}	p_{31}	p_{41}	p_{51}	p_{61}	p_{71}	p_{81}	p_{91}	p_{101}
1	0.0196	0.0045	0.0008	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.5	0.0263	0.0085	0.0021	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0316	0.0129	0.0040	0.0011	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2.5	0.0357	0.0174	0.0064	0.0021	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000
3	0.0390	0.0216	0.0091	0.0034	0.0012	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000
3.5	0.0415	0.0257	0.0120	0.0050	0.0020	0.0008	0.0003	0.0001	0.0000	0.0000	0.0000
4	0.0435	0.0294	0.0151	0.0069	0.0030	0.0013	0.0005	0.0002	0.0001	0.0000	0.0000
4.5	0.0450	0.0329	0.0182	0.0090	0.0042	0.0019	0.0008	0.0004	0.0002	0.0001	0.0000
5	0.0461	0.0360	0.0212	0.0112	0.0056	0.0027	0.0013	0.0006	0.0003	0.0001	0.0001
5.5	0.0469	0.0388	0.0242	0.0135	0.0071	0.0036	0.0018	0.0009	0.0004	0.0002	0.0001
6	0.0475	0.0413	0.0271	0.0159	0.0088	0.0047	0.0024	0.0012	0.0006	0.0003	0.0002
6.5	0.0478	0.0435	0.0298	0.0183	0.0105	0.0059	0.0032	0.0017	0.0009	0.0005	0.0002
7	0.0479	0.0454	0.0324	0.0207	0.0124	0.0072	0.0041	0.0022	0.0012	0.0007	0.0004
7.5	0.0479	0.0471	0.0349	0.0230	0.0143	0.0086	0.0050	0.0029	0.0016	0.0009	0.0005
8	0.0478	0.0486	0.0372	0.0254	0.0163	0.0101	0.0061	0.0036	0.0021	0.0012	0.0007
8.5	0.0475	0.0498	0.0393	0.0277	0.0183	0.0117	0.0072	0.0044	0.0027	0.0016	0.0009
9	0.0472	0.0509	0.0413	0.0299	0.0203	0.0133	0.0085	0.0053	0.0033	0.0020	0.0012
9.5	0.0468	0.0518	0.0431	0.0320	0.0223	0.0150	0.0098	0.0063	0.0040	0.0025	0.0016

Table 5: Steady state Probabilities distribution for various values of λ and μ = 10, α = 5, p = 0.8, q = 0.2 when the server is in vacation and batch size is K = 4

Λ	p_{02}	p_{12}	p_{22}	p_{32}	p_{42}	p_{52}	p_{62}	p_{72}	p_{82}	p_{92}	p_{102}
1	0.1959	0.2408	0.2486	0.2498	0.0345	0.0048	0.0007	0.0001	0.0000	0.0000	0.0000
1.5	0.1753	0.2321	0.2461	0.2492	0.0482	0.0093	0.0018	0.0003	0.0001	0.0000	0.0000
2	0.1578	0.2224	0.2424	0.2480	0.0601	0.0146	0.0035	0.0009	0.0002	0.0001	0.0000
2.5	0.1428	0.2122	0.2378	0.2462	0.0703	0.0201	0.0057	0.0016	0.0005	0.0001	0.0000
3	0.1299	0.2020	0.2323	0.2438	0.0791	0.0256	0.0083	0.0027	0.0009	0.0003	0.0001
3.5	0.1186	0.1920	0.2263	0.2408	0.0864	0.0310	0.0111	0.0040	0.0014	0.0005	0.0002
4	0.1087	0.1823	0.2200	0.2372	0.0926	0.0361	0.0141	0.0055	0.0021	0.0008	0.0003
4.5	0.0999	0.1730	0.2133	0.2333	0.0976	0.0409	0.0171	0.0072	0.0030	0.0013	0.0005
5	0.0922	0.1642	0.2066	0.2289	0.1017	0.0452	0.0201	0.0089	0.0040	0.0018	0.0008
5.5	0.0853	0.1558	0.1998	0.2243	0.1050	0.0491	0.0230	0.0108	0.0050	0.0024	0.0011
6	0.0791	0.1479	0.1930	0.2194	0.1075	0.0526	0.0258	0.0126	0.0062	0.0030	0.0015
6.5	0.0735	0.1404	0.1863	0.2144	0.1093	0.0557	0.0284	0.0145	0.0074	0.0038	0.0019
7	0.0685	0.1334	0.1797	0.2092	0.1105	0.0584	0.0308	0.0163	0.0086	0.0045	0.0024
7.5	0.0639	0.1267	0.1732	0.2039	0.1112	0.0607	0.0331	0.0181	0.0098	0.0054	0.0029
8	0.0597	0.1204	0.1669	0.1986	0.1115	0.0626	0.0351	0.0197	0.0111	0.0062	0.0035
8.5	0.0559	0.1145	0.1608	0.1933	0.1114	0.0642	0.0370	0.0213	0.0123	0.0071	0.0041
9	0.0524	0.1090	0.1548	0.1880	0.1110	0.0655	0.0386	0.0228	0.0135	0.0079	0.0047
9.5	0.0492	0.1037	0.1491	0.1827	0.1102	0.0665	0.0401	0.0242	0.0146	0.0088	0.0053

Table 6: Average number of customers in the queue and Variance for various values of λ, p = 0.8, q = 0.2 and μ = 10, α = 5 and batch size is K = 4

Λ	μ	Α	P_1	P_2	Mean	Variance
1	10	5	0.0250	0.9750	1.6600	1.4356
1.5	10	5	0.0375	0.9625	1.7400	1.5477
2	10	5	0.0500	0.9500	1.8202	1.6726
2.5	10	5	0.0625	0.9375	1.9004	1.8106
3	10	5	0.0750	0.9250	1.9809	1.9618
3.5	10	5	0.0875	0.9125	2.0616	2.1263
4	10	5	0.1000	0.9000	2.1427	2.3045
4.5	10	5	0.1125	0.8875	2.2243	2.4966
5	10	5	0.1250	0.8750	2.3065	2.7030
5.5	10	5	0.1375	0.8625	2.3893	2.9239
6	10	5	0.1500	0.8500	2.4728	3.1599
6.5	10	5	0.1625	0.8375	2.5571	3.4112
7	10	5	0.1750	0.8250	2.6423	3.6785
7.5	10	5	0.1875	0.8125	2.7286	3.9623
8	10	5	0.2000	0.8000	2.8159	4.2631
8.5	10	5	0.2125	0.7875	2.9044	4.5816
9	10	5	0.2250	0.7750	2.9942	4.9184
9.5	10	5	0.2375	0.7625	3.0854	5.2744

Table 7: Steady state Probabilities distribution for various values of λ and μ = 10,

α = 5, p = 0.8, q = 0.2 when the server is busy and batch size is K = 6

Λ	p_{01}	p_{11}	p_{21}	p_{31}	p_{41}	p_{51}	p_{61}	p_{71}	p_{81}	p_{91}	p_{101}
1	0.0131	0.0030	0.0005	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
1.5	0.0175	0.0057	0.0014	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
2	0.0210	0.0086	0.0027	0.0007	0.0002	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
2.5	0.0238	0.0116	0.0043	0.0014	0.0004	0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
3	0.0260	0.0144	0.0061	0.0023	0.0008	0.0003	0.0001	0.0000	0.0000	0.0000	0.0000
3.5	0.0277	0.0171	0.0080	0.0034	0.0013	0.0005	0.0002	0.0001	0.0000	0.0000	0.0000
4	0.0290	0.0196	0.0100	0.0046	0.0020	0.0008	0.0003	0.0001	0.0001	0.0000	0.0000
4.5	0.0301	0.0219	0.0121	0.0060	0.0028	0.0012	0.0005	0.0002	0.0001	0.0000	0.0000
5	0.0308	0.0240	0.0141	0.0074	0.0037	0.0018	0.0008	0.0004	0.0002	0.0001	0.0000
5.5	0.0314	0.0259	0.0161	0.0089	0.0047	0.0024	0.0012	0.0006	0.0003	0.0001	0.0001
6	0.0319	0.0276	0.0180	0.0105	0.0058	0.0031	0.0016	0.0008	0.0004	0.0002	0.0001
6.5	0.0321	0.0291	0.0198	0.0121	0.0069	0.0038	0.0021	0.0011	0.0006	0.0003	0.0002
7	0.0323	0.0304	0.0216	0.0137	0.0082	0.0047	0.0026	0.0015	0.0008	0.0004	0.0002
7.5	0.0324	0.0316	0.0232	0.0152	0.0094	0.0056	0.0033	0.0019	0.0011	0.0006	0.0003
8	0.0324	0.0326	0.0248	0.0168	0.0107	0.0066	0.0040	0.0023	0.0014	0.0008	0.0005
8.5	0.0323	0.0335	0.0262	0.0183	0.0120	0.0076	0.0047	0.0028	0.0017	0.0010	0.0006
9	0.0322	0.0343	0.0276	0.0198	0.0133	0.0087	0.0055	0.0034	0.0021	0.0013	0.0008
9.5	0.0320	0.0350	0.0288	0.0212	0.0146	0.0097	0.0063	0.0040	0.0025	0.0016	0.0010

Table 8: Steady state Probabilities distribution for various values of λ and μ = 10,

α = 5, p = 0.8, q = 0.2 when the server is in vacation and batch size is K = 6

Λ	p_{02}	p_{12}	p_{22}	p_{32}	p_{42}	p_{52}	p_{62}	p_{72}	p_{82}	p_{92}	p_{102}
1	0.1306	0.1605	0.1657	0.1665	0.1666	0.1667	0.0230	0.0032	0.0004	0.0001	0.0000
1.5	0.1169	0.1547	0.1641	0.1661	0.1666	0.1666	0.0323	0.0062	0.0012	0.0002	0.0000
2	0.1052	0.1483	0.1616	0.1653	0.1663	0.1666	0.0404	0.0098	0.0024	0.0006	0.0001
2.5	0.0952	0.1415	0.1585	0.1641	0.1659	0.1664	0.0476	0.0136	0.0039	0.0011	0.0003
3	0.0866	0.1347	0.1549	0.1625	0.1653	0.1662	0.0539	0.0175	0.0057	0.0018	0.0006
3.5	0.0791	0.1281	0.1509	0.1605	0.1643	0.1658	0.0595	0.0214	0.0077	0.0028	0.0010
4	0.0726	0.1216	0.1467	0.1582	0.1632	0.1652	0.0645	0.0252	0.0098	0.0038	0.0015
4.5	0.0668	0.1155	0.1424	0.1556	0.1617	0.1645	0.0689	0.0288	0.0121	0.0051	0.0021
5	0.0617	0.1097	0.1379	0.1527	0.1601	0.1636	0.0727	0.0323	0.0144	0.0064	0.0028
5.5	0.0572	0.1042	0.1335	0.1497	0.1582	0.1625	0.0761	0.0356	0.0167	0.0078	0.0037
6	0.0531	0.0990	0.1290	0.1465	0.1562	0.1613	0.0790	0.0387	0.0190	0.0093	0.0045
6.5	0.0494	0.0942	0.1247	0.1433	0.1540	0.1599	0.0815	0.0416	0.0212	0.0108	0.0055
7	0.0462	0.0896	0.1204	0.1399	0.1516	0.1583	0.0836	0.0442	0.0233	0.0123	0.0065
7.5	0.0432	0.0853	0.1163	0.1366	0.1491	0.1566	0.0854	0.0466	0.0254	0.0139	0.0076
8	0.0405	0.0813	0.1122	0.1332	0.1466	0.1548	0.0869	0.0488	0.0274	0.0154	0.0086
8.5	0.0380	0.0775	0.1083	0.1298	0.1439	0.1529	0.0881	0.0508	0.0293	0.0169	0.0097
9	0.0357	0.0739	0.1045	0.1265	0.1413	0.1509	0.0890	0.0525	0.0310	0.0183	0.0108
9.5	0.0337	0.0705	0.1009	0.1231	0.1385	0.1488	0.0897	0.0541	0.0326	0.0197	0.0119

Table 9: Average number of customers in the queue and Variance for various values of λ, p = 0.8, q =0 .2 and μ = 10, α = 5 and batch size is K = 6

Λ	μ	A	P_1	P_2	Mean	Variance
1	10	5	0.0167	0.9833	2.6600	3.1023
1.5	10	5	0.0250	0.9750	2.7400	3.2143
2	10	5	0.0333	0.9667	2.8200	3.3391
2.5	10	5	0.0417	0.9583	2.9000	3.4767
3	10	5	0.0500	0.9500	2.9800	3.6271
3.5	10	5	0.0583	0.9417	3.0601	3.7904
4	10	5	0.0667	0.9333	3.1402	3.9667
4.5	10	5	0.0750	0.9250	3.2204	4.1558
5	10	5	0.0833	0.9167	3.3007	4.3580
5.5	10	5	0.0917	0.9083	3.3811	4.5734
6	10	5	0.1000	0.9000	3.4617	4.8020
6.5	10	5	0.1083	0.8917	3.5425	5.0440
7	10	5	0.1167	0.8833	3.6235	5.2995
7.5	10	5	0.1250	0.8750	3.7048	5.5686
8	10	5	0.1333	0.8667	3.7864	5.8517
8.5	10	5	0.1417	0.8583	3.8683	6.1488
9	10	5	0.1500	0.8500	3.9507	6.4603
9.5	10	5	0.1583	0.8417	4.0334	6.7862

Table 10: Average number of customers in the queue and Variance for various values of p, λ = 5, q = 0.2 and μ = 10, α = 5 and batch size is K = 2

p	μ	λ	P1	P2	Mean	Variance
0.1	10	5	0.0313	0.9688	0.6002	0.3602
0.2	10	5	0.0625	0.9375	0.7016	0.4920
0.3	10	5	0.0938	0.9063	0.8050	0.6469
0.4	10	5	0.1250	0.8750	0.9112	0.8269
0.5	10	5	0.1563	0.8438	1.0208	1.0343
0.6	10	5	0.1875	0.8125	1.1346	1.2717
0.7	10	5	0.2188	0.7813	1.2532	1.5425
0.8	10	5	0.2500	0.7500	1.3774	1.8507
0.9	10	5	0.2813	0.7188	1.5080	2.2012
1	10	5	0.3125	0.6875	1.6462	2.6004

8. Conclusion:

The Numerical studies show the changes in the system due to impact of batch size, vacation rate , arrival rate . The mean number of customers in the system increases as batch size and arrival rate increase . The mean number of customers in the system decreases as vacation rate increases. Various special cases have been discussed, which are particular cases of this research work. This research work can be extended further by introducing various concepts like bulk arrival , interruption etc.

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