Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.4, No.12, 2014

Bayesian Analysis of Log-Generalized Inverse Weibull Distribution using Ranked set sampling

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1. Introduction:

The Weibull distribution is one of the most popular distributions in the lifetime data analyzing because a wide variety of shapes with varying levels of its parameters can be created. During the past decades, extensive work has been done on this distribution in both the frequentist and Bayesian points of view, like, Johnson et al. (1995) and Kundu (2008). Moreover, the Weibull probability density function can be decreasing (or increasing) or unimodal, depending on the shape of distribution parameters. The inverse Weibull distribution (IW) is usually used in reliability and biological studies. The three- parameter generalized inverse Weibull (GIW) distribution, which extends to several distributions, and commonly used in the lifetime-literature, is more flexible than the inverse Weibull distribution. Mudholkar et al. (1994), Jiang et al. (1999) and De Gusmao et al. (2011) introduced and discussed the three-parameter GIW distribution. Helu et al. (2010) discussed Bayes estimation of parameters of weibull distribution using ranked set sampling.

Let T be a random variable having the probability density function of three parameter Generalized Inverse Weibull given by

$$f(t)=\gamma\,\beta\,\alpha^{\beta}t^{-(\beta+1)}\exp\left[-\gamma\left(\frac{\alpha}{\beta}\right)^{t}\right];\ t>0;\alpha,\beta,\gamma>0 \qquad (1.1)$$

On parameterzing, $\beta=1/\sigma$ *and* $\mu=\log(\alpha)$ in (1.1) the random variable $Y=\log(T)$ has probability density function of Log- Generalized Inverse Weibull (LGIW) given as

$$f(y,\gamma,\sigma,\mu)=\frac{\gamma}{\sigma}\exp\left\{-\left(\frac{y-\mu}{\sigma}\right)-\gamma\exp\left[-\left(\frac{y-\mu}{\sigma}\right)\right]\right\}\ ;-\infty<y<\infty\ ;\gamma>0;\sigma>0\ ;-\infty<\mu<\infty$$

(1.2)

The CDF for LGIW can be defined as

$$F(y)=\exp\left\{-\gamma\exp\left[-\left(\frac{y-\mu}{\sigma}\right)\right]\right\} \qquad (1.3)$$

and the corresponding survival and hazard functions are given as

$$S(y)=1-\exp\left\{-\gamma\exp\left[-\left(\frac{y-\mu}{\sigma}\right)\right]\right\}$$

& $$h(y)=\frac{\gamma}{\sigma}\exp\left\{-\left(\frac{y-\mu}{\sigma}\right)-\gamma\exp\left[-\left(\frac{y-\mu}{\sigma}\right)\right]\right\}\left[1-\exp\left\{-\gamma\exp\left[-\left(\frac{y-\mu}{\sigma}\right)\right]\right\}\right]^{-1}$$

2. Maximum Likelihood Estimation for the parameters γ & μ of LGIW assuming σ to be known:

Let $y_1, y_2, ..., y_n$ be identically and independently distributed as LGIW. Then the likelihood function of for (1.2) is given as

$$L(y\mid\gamma,\mu,\sigma)=\left(\frac{\gamma}{\sigma}\right)^n \exp\sum_{i=1}^{n}\left\{-\left(\frac{y_i-\mu}{\sigma}\right)-\gamma\exp\left[-\left(\frac{y_i-\mu}{\sigma}\right)\right]\right\} \tag{2.1}$$

$$l=\ln L=n\ln\gamma-n\ln\sigma-\sum_{i=1}^{n}\left(\frac{y_i-\mu}{\sigma}\right)-\gamma\sum_{i=1}^{n}\exp\left[-\left(\frac{y_i-\mu}{\sigma}\right)\right] \tag{2.2}$$

$$\Rightarrow \qquad \hat{\gamma}=\frac{n}{\sum_{i=1}^{n}\exp\left[-\left(\frac{y_i-\mu}{\sigma}\right)\right]} \tag{2.3}$$

$$\frac{\partial l}{\partial\mu}=\frac{n}{\sigma}-\frac{\gamma}{\sigma}\sum_{i1}^{n}\exp\left[-\left(\frac{y_i-\mu}{\sigma}\right)\right]=0 \tag{2.4}$$

3. Bayesian Estimation:

Let $y_1, y_2, ..., y_n$ be distributed as LGIW. We would like to study the Bayes estimators for γ and μ. Assume γ and μ to be independent with prior density function for γ to be non-informative given as $\phi_1(\gamma)=1/\gamma\,;\gamma>0$ and for μ it be standard normal variate given by $\phi_2(\mu)=\frac{1}{\sqrt{2\pi}}e^{-\frac{\mu^2}{2}}$.

Thus using the Bayes theorem the posterior distribution is as follows

$$\pi(\gamma,\mu\mid y,\sigma)\propto\left(\frac{\gamma}{\sigma}\right)^n \exp\sum_{i=1}^{n}\left\{-\left(\frac{y_i-\mu}{\sigma}\right)-\gamma\exp\left[-\left(\frac{y_i-\mu}{\sigma}\right)\right]\right\}\frac{1}{\gamma}e^{-\frac{\mu^2}{2}}$$

or

$$\pi(\gamma,\mu\mid y,\sigma)=k\gamma^{n-1}\exp\sum_{i=1}^{n}\left\{-\left(\frac{y_i-\mu}{\sigma}\right)\right\}\exp\sum_{i=1}^{n}\left\{-\gamma\exp\left[-\left(\frac{y_i-\mu}{\sigma}\right)\right]\right\}\exp\left(-\frac{\mu^2}{2}\right)$$

(3.1)

where $K^{-1}=\int_{-\infty}^{\infty}\int_{0}^{\infty}\gamma^{n-1}\exp\sum_{i=1}^{n}\left\{-\left(\frac{y_i-\mu}{\sigma}\right)\right\}\exp\sum_{i=1}^{n}\left\{-\gamma\exp\left[-\left(\frac{y_i-\mu}{\sigma}\right)\right]\right\}\exp\left(-\frac{\mu^2}{2}\right)d\gamma\, d\mu$

$$=\int_{-\infty}^{\infty}\exp\sum_{i=1}^{n}\left\{-\left(\frac{y_i-\mu}{\sigma}\right)\right\}\exp\left(-\frac{\mu^2}{2}\right)\frac{\Gamma n}{\left\{\sum_{i=1}^{n}\exp\left[-\left(\frac{y_i-\mu}{\sigma}\right)\right]\right\}^n}d\mu$$

Therefore equation (3.1) becomes

$$\pi(\gamma,\mu \mid y,\sigma) = \frac{\gamma^{n-1} \exp \sum_{i=1}^{n} \left\{ -\left(\frac{y_i - \mu}{\sigma} \right) \right\} \exp \sum_{i=1}^{n} \left\{ -\gamma \exp \left[-\left(\frac{y_i - \mu}{\sigma} \right) \right] \right\} \exp\left(-\frac{\mu^2}{2} \right)}{\Gamma n \; k}$$

(3.2)

Thus marginal posterior pdf for γ is

$$\pi_1(\gamma \mid y) = \int_{-\infty}^{\infty} \pi(\gamma,\mu \mid y,\sigma)\, d\mu$$

$$= \int_{-\infty}^{\infty} \frac{\gamma^{n-1} \exp\left(\sum_{i=1}^{n} \left\{ -\left(\frac{y_i - \mu}{\sigma} \right) - \gamma \exp\left[-\left(\frac{y_i - \mu}{\sigma} \right) \right] \right\} - \frac{\mu^2}{2} \right)}{\Gamma n \; k} d\mu \qquad (3.3)$$

Thus marginal posterior pdf for μ is

$$\pi_2(\mu \mid y) = \int_{0}^{\infty} \pi(\gamma,\mu \mid y,\sigma)\, d\gamma$$

$$= \int_{0}^{\infty} \frac{\gamma^{n-1} \exp\left(\sum_{i=1}^{n} \left\{ -\left(\frac{y_i - \mu}{\sigma} \right) - \gamma \exp\left[-\left(\frac{y_i - \mu}{\sigma} \right) \right] \right\} - \frac{\mu^2}{2} \right)}{\Gamma n \; k} d\gamma \qquad (3.4)$$

$\therefore$ Bayes estimate for γ is

$$\hat{\gamma}_{bayes} = E(\gamma \mid y) = \int_{0}^{\infty} \gamma \, \pi_1(\gamma \mid y)\; d\gamma$$

$$= \int_{0}^{\infty} \frac{\gamma^{n}}{k \Gamma n} \exp\left(\sum_{i=1}^{n} \left\{ -\left(\frac{y_i - \mu}{\sigma} \right) - \gamma \exp\left[-\left(\frac{y_i - \mu}{\sigma} \right) \right] \right\} - \frac{\mu^2}{2} \right) d\gamma \qquad (3.5)$$

And the Bayes estimate for μ is

$$\mu_{bayes} = E(\mu \mid y) = \int_{-\infty}^{\infty} \mu \; \pi_2(\mu \mid y)\; d\mu$$

$$= \int_{-\infty}^{\infty} \frac{\mu \gamma^{n-1}}{k \Gamma n} \exp\left(\sum_{i=1}^{n} \left\{ -\left(\frac{y_i - \mu}{\sigma} \right) - \gamma \exp\left[-\left(\frac{y_i - \mu}{\sigma} \right) \right] \right\} - \frac{\mu^2}{2} \right) d\mu \qquad (3.6)$$

4. Estimation of parameters of LGIW using ranked set sampling (RSS):

The concept of Ranked Set Sampling was first introduced by McIntyre (1952) so as to increase the efficiency of an estimator of the population mean. RSS is used to provide efficient inferential procedures to collected sample data. Patil (1995), Barnett and Moore (1997), and Chen et al. (2003) have found the applications where RSS techniques are useful. Authors like Al-Saleh and Al-Shrafat (2000), Zheng and Al-Saleh (2001) evaluated the performance of RSS using real application. A comparison between the Bayes risks of the Bayes estimators obtained by using RSS and SRS for the exponential distribution was studied by Al-Saleh and Muttlak (1998). Lavine (1999) examined from a Bayesian point of view and explored some optimality questions on different aspects of Bayesian ranked set sampling. Al-Saleh et al (2000) studied the concept of Bayesian methods with RSS for exponential family with conjugate prior and found that the RSS Bayes estimator has smaller Bayes risk than SRS Bayes estimator. Sadek et al (200) obtained the Bayesian estimate of the parameter of the exponential distribution based on ranked set sampling (RSS) under the Linex loss function and used both conjugate and Jeffreys prior distributions. The procedure of RSS is described as follows.

Step1. Select randomly m^2 sample units denoted by Yij, i = 1,…, m and j =1,…, m, from the population.

Step2. Allocate the m^2 selected units randomly into m sets each of size m as.

$$\begin{pmatrix} y_{11} & y_{21} & \cdots & y_{m1} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ y_{1m} & y_{2m} & \cdots & y_{mm} \end{pmatrix}$$

Step3. Without taking any measurements, rank units within each row based on a criterion and are presented as:

$$\begin{pmatrix} y_{(1)1} & y_{(2)1} & \cdots & y_{(m)1} \\ \vdots & \vdots & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ y_{(1)m} & y_{(2)m} & \cdots & y_{(m)m} \end{pmatrix}$$ which will be one cycle.

Step4. Choose a sample by taking the smallest ranked unit from the first row, then the second smallest ranked unit from the second row, and continue in this fashion until the largest ranked unit is selected from the last row. As a result, the ranked set sample associated with this cycle will be $\left(y_{(1)1}, y_{(2)2}, \ldots, y_{(m)m}\right)$. Note that $y_{(i)i}$ is the ith order statistic $y_{(i)}$ of a sample of size m.

Step5. Repeat Steps 1–4 (k) times until the desired sample size n = k × m, is obtained for analysis, and our RSS sample using k cycles each of size m such that n = k × m will be: $\{y_{(i)ic}, i=1,\ldots,m; c=1,\ldots.,k\}$.

For simplification we use the notation, $X_{(ic)} = Y_{(i)ic}$ then for fixed c, $X_{ic}, i=1,\ldots.,m$ are independent with pdf equal to pdf of $Y_{(i)}$ and given by:

$$g(x_{ic}) = \frac{m!}{(i-1)!(m-i)!}[F(x_{ic})]^{i-1} f(x_{ic})[1-F(x_{ic})]^{m-i}$$

Let $\underline{X} = (\underline{X}_1, \underline{X}_2, \ldots, \underline{X}_k)$, then the conditional joint pdf of $\underline{X}$ is given by

$$L(\underline{X} \mid \gamma, \mu) = \prod_{c=1}^{k}\prod_{i=1}^{m} g(x_{ic} \mid \gamma, \mu)$$

To derive the Bayes estimators for the parameters of LGIW using RSS, let y_{ij} be distributed as (1.2) then the conditional pdf of $\underline{X}$ given γ and μ will be:

$$L(\underline{X}\mid\gamma,\mu)=\prod_{c=1}^{k}\prod_{i=1}^{m}\frac{m!}{(i-1)!(m-i)!}[F(x_{ic})]^{i-1}f(x_{ic})[1-F(x_{ic})]^{m-i}$$

$$L=\left(\frac{m!\gamma}{\sigma}\right)^{km}\exp\sum_{c=1}^{k}\sum_{i=1}^{m}\left\{-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right\}\prod_{c=1}^{k}\prod_{i=1}^{m}\frac{\left[1-\exp\left\{-\gamma\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]\right\}\right]^{m-i}\left[\exp\left\{-\gamma\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]\right\}\right]^{i}}{(i-1)!(m-i)!}$$

(4.2)

4.1. Maximum Likelihood Estimation using ranked set sampling (RSS):

Using equation (4.2) obtained in section 4 as a likelihood function, the MLE of the parameters γ and μ are as

$$\ln L=km\ln m!+km\ln\gamma-km\ln\sigma-\sum_{c=1}^{k}\sum_{i=1}^{m}\left(\frac{x_{ic}-\mu}{\sigma}\right)-\gamma\sum_{c=1}^{k}\sum_{i=1}^{m}i\,\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]+$$

$$\sum_{c=1}^{k}\sum_{i=1}^{m}(m-i)\ln\left[1-\exp\left\{-\gamma\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]\right\}\right]-\sum_{c=1}^{k}\sum_{i=1}^{m}(i-1)!(m-i)!$$

(4.1.1)

$$\frac{\partial\ln L}{\partial\gamma}=\frac{km}{\gamma}+\sum_{c=1}^{k}\sum_{i=1}^{m}i\exp\left(\frac{-(x_{ic}-\mu)}{\sigma}\right)+\frac{\sum_{c=1}^{k}\sum_{i=1}^{m}(m-i)\exp\left\{-\left(\frac{x_{ic}-\mu}{\sigma}\right)-\gamma\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]\right\}}{1-\exp\left\{-\gamma\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]\right\}}=0$$

(4.1.2)

$$\frac{\partial\ln L}{\partial\mu}=\frac{km}{\sigma}+\frac{\partial}{\partial\mu}\left[\gamma\sum_{c=1}^{k}\sum_{i=1}^{m}i\,\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]+\sum_{c=1}^{k}\sum_{i=1}^{m}(m-i)\ln\left[1-\exp\left\{-\gamma\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]\right\}\right]\right]=0$$

(4.1.3)

4.2 Bayes Estimation using ranked set sampling (RSS):

Using the same priors as in section 3, the joint posterior pdf γ and μ given $\underline{X}=\underline{x}$ is

$$\phi(\gamma,\mu\mid\underline{x})=\frac{L(\underline{x}\mid\gamma,\mu)\times 1/\gamma\times e^{-\mu^2/2}}{\int_{-\infty}^{\infty}\int_{0}^{\infty}L(\underline{x}\mid\gamma,\mu)\frac{e^{-\mu^2/2}}{\gamma}\,d\gamma\,d\mu}$$

$$=\frac{\left(\frac{m!\gamma}{\sigma}\right)^{km}\frac{e^{-\mu^2/2}}{\gamma}\exp\sum_{c=1}^{k}\sum_{i=1}^{m}\left\{-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right\}\prod_{c=1}^{k}\prod_{i=1}^{m}\frac{\left[1-\exp\left\{-\gamma\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]\right\}\right]^{m-i}\left[\exp\left\{-\gamma\exp\left[-\left(\frac{x_{ic}-\mu}{\sigma}\right)\right]\right\}\right]^{i}}{(i-1)!(m-i)!}}{\int_{-\infty}^{\infty}\int_{0}^{\infty}L(\underline{x}\mid\gamma,\mu)\frac{e^{-\mu^2/2}}{\gamma}\,d\gamma\,d\mu}$$

(4.2.1)

The marginal posterior pdf of γ is obtained as:

$$\phi_1(\gamma \mid \underline{x}) = \int_{-\infty}^{\infty} \phi(\gamma, \mu \mid \underline{x})\, d\mu \tag{4.2.2}$$

Thus marginal posterior pdf for μ is

$$\phi_2(\mu \mid \underline{x}) = \int_{0}^{\infty} \phi(\gamma, \mu \mid \underline{x})\, d\gamma \tag{4.2.3}$$

$\therefore$ Bayes estimate for γ is

$$\hat{\gamma}_{bayes} = E(\gamma \mid \underline{x}) = \int_{0}^{\infty} \gamma\, \phi_1(\gamma \mid \underline{x})\, d\gamma \tag{4.2.4}$$

And the Bayes estimate for μ is

$$\mu_{bayes} = E(\mu \mid \underline{x}) = \int_{-\infty}^{\infty} \mu\, \phi_2(\mu \mid \underline{x})\, d\mu \tag{4.2.5}$$

5. Estimation of parameters of LGIW using modified ranked set sampling (MRSS):

Modified Ranked Set Sampling is used as a special case of RSS. The first three steps of MRSS are same as that of RSS, the sample is chosen by taking the smallest ranked unit from each row of the m rows. For one cycle the MRSS will be $\left(y_{(1)1}, y_{(1)2}, \ldots, y_{(1)m}\right)$, and for c cycles the MRSS sample is $\left(y_{(1)1c}, y_{(1)2c}, \ldots, y_{(1)mc}\right)$. Repeating it k times, till the $n = k \times m$ sample size is obtained. For each cycle consisting of independent and identically distributed variables the MRSS is obtained, as $Y_{(1)ic}$ is distributed as the first order statistics.

Thus required MRSS is $\{y_{(i)ic}, i = 1, \ldots, m; c = 1, \ldots., k\}$ .let $Z_{(ic)} = Y_{(1)ic}$,

$$h(z_{ic}) = \frac{m\gamma}{\sigma} \exp m\left\{-\left(\frac{z_{ic}-\mu}{\sigma}\right) - \gamma \exp\left[-\left(\frac{z_{ic}-\mu}{\sigma}\right)\right]\right\}$$

Let $\underline{Z}_c = (\underline{Z}_{1c}, \underline{Z}_{2c}, \ldots, \underline{Z}_{mc})$, and $\underline{Z} = (\underline{Z}_1, \underline{Z}_2, \ldots, \underline{Z}_k)$ then the conditional joint pdf of $\underline{Z}$ is given by

$$L(\underline{z} \mid \gamma, \mu) = \prod_{c=1}^{k} \prod_{i=1}^{m} \frac{m\gamma}{\sigma} \exp m\left\{-\left(\frac{z_{ic}-\mu}{\sigma}\right) - \gamma \exp\left[-\left(\frac{z_{ic}-\mu}{\sigma}\right)\right]\right\} \tag{5.1}$$

5.1 Maximum Likelihood Estimation modified ranked set sampling (MRSS):

The maximum likelihood estimators of γ and μ can be obtained as

$$L(\underline{z}\mid\gamma,\mu)=\left(\frac{m\gamma}{\sigma}\right)^{km}\exp m\sum_{c=1}^{k}\sum_{i=1}^{m}\left\{-\left(\frac{z_{ic}-\mu}{\sigma}\right)-\gamma\exp\left[-\left(\frac{z_{ic}-\mu}{\sigma}\right)\right]\right\}$$

$$l=km\ln m+km\ln\gamma-km\ln\sigma-m\sum_{c=1}^{k}\sum_{i=1}^{m}\left(\frac{z_{ic}-\mu}{\sigma}\right)-m\gamma\sum_{c=1}^{k}\sum_{i=1}^{m}\left\{\exp\left[-\left(\frac{z_{ic}-\mu}{\sigma}\right)\right]\right\}$$

(5.1.1)

$$\therefore\frac{\partial l}{\partial\gamma}=\frac{km}{\gamma}-m\sum_{c=1}^{k}\sum_{i=1}^{m}\left\{\exp\left[-\left(\frac{z_{ic}-\mu}{\sigma}\right)\right]\right\}=0 \qquad (5.1.2)$$

$$\therefore\frac{\partial l}{\partial\mu}=\frac{km^2}{\sigma}-\frac{m\gamma}{\sigma}\sum_{c=1}^{k}\sum_{i=1}^{m}\left\{\exp\left[-\left(\frac{z_{ic}-\mu}{\sigma}\right)\right]\right\}=0 \qquad (5.1.3)$$

5.2 Bayes Estimation modified ranked set sampling (MRSS):

The posterior density function for the parameters of LGIW using MRSS can be obtained as:

$$\xi(\gamma,\mu\mid\underline{z})=\frac{L(\underline{z}\mid\gamma,\mu)\times 1/\gamma\times e^{-\mu^2/2}}{\int_{-\infty}^{\infty}\int_{0}^{\infty}L(\underline{z}\mid\gamma,\mu)\frac{e^{-\mu^2/2}}{\gamma}\,d\gamma\,d\mu}$$

$$=\frac{\frac{e^{-\mu^2/2}}{\gamma}\left(\frac{m\gamma}{\sigma}\right)^{km}\exp m\sum_{c=1}^{k}\sum_{i=1}^{m}\left\{-\left(\frac{z_{ic}-\mu}{\sigma}\right)-\gamma\exp\left[-\left(\frac{z_{ic}-\mu}{\sigma}\right)\right]\right\}}{\int_{-\infty}^{\infty}\int_{0}^{\infty}\left(\frac{m\gamma}{\sigma}\right)^{km}\exp m\sum_{c=1}^{k}\sum_{i=1}^{m}\left\{-\left(\frac{z_{ic}-\mu}{\sigma}\right)-\gamma\exp\left[-\left(\frac{z_{ic}-\mu}{\sigma}\right)\right]\right\}\frac{e^{-\mu^2/2}}{\gamma}\,d\gamma\,d\mu}$$

(5.2.1)

The marginal posterior pdf of γ is obtained as:

$$\xi_1(\gamma\mid\underline{z})=\int_{-\infty}^{\infty}\xi(\gamma,\mu\mid\underline{z})\,d\mu \qquad (5.2.2)$$

Thus marginal posterior pdf for μ is

$$\xi_2(\mu\mid\underline{x})=\int_{0}^{\infty}\xi(\gamma,\mu\mid\underline{z})\,d\gamma \qquad (5.2.3)$$

$\therefore$ Bayes estimate for γ is

$$\hat{\gamma}_{bayes}=E(\gamma\mid\underline{z})=\int_{0}^{\infty}\gamma\,\xi_1(\gamma\mid\underline{z})\,d\gamma \qquad (5.2.4)$$

And the Bayes estimate for μ is

$$\mu_{bayes} = E(\mu \mid \underline{z}) = \int_{-\infty}^{\infty} \mu \ \xi_2(\mu \mid \underline{z}) \ d\mu \qquad (5.2.5)$$

References:

- D. Kundu (2008): Bayesian Inference and Life Testing Plan for the Weibull Distribution in Presence of Progressive Censoring. Technometrics, vol. 50, no. 2, 144–154.
- F. R. S. de Gusmão, E. M. M. Ortega, G. M. Cordeiro (2011): The Generalized Inverse Weibull Distribution. Stat Papers -Springer-Verlag52:591–619.
- G. A. McIntyre (1952): A Method of Unbiased Selective Sampling, using Ranked Sets. Austral.J. Agricult. Res. 3:385–390.
- G. Zheng and M. Fraiwan Al-Saleh (2001): Modified maximum likelihood estimators based on ranked set sampling. The Annals of the Institute of Statistical Mathematics
- G.P. Patil (1995): Ranked set sampling, Environ. Ecol. Stat., 2, 271-285.
- G.S. Mudholkar, G.D Kollia (1994) Generalized Weibull family: a structural analysis. Commun. Stat. Ser. 23:1149–1171.
- Helu, M. Abu-Salih, and O. Alkam (2010): Bayes Estimation of Weibull Distribution Parameters Using Ranked Set Sampling. Communications in Statistics—Theory and Methods, 39: 2533–2551.
- Jiang R, Zuo MJ, Li HX (1999) Weibull and Weibull inverse Mixture Models allowing Negative Weights. Reliab Eng Syst Saf 66:227–234.
- Lavine, M. (1999): The Bayesian of ranked set sampling. J. Environ. Ecol. Statist. 6:47–57.
- M. F. Al-Saleh and H. Muttlak (1998): A Note on the Estimation of the Parameter of the Exponential Distribution using Bayesian RSS. Pakistan Journal of Statistics, 14(1998), 49- 56.
- M. F. Al-Saleh and K. Al-Shrafat (2000): Estimation of the Average Milk Yield of Sheep using Ranked Set Sampling. Environmentrics, 12, 395-399.
- N. L. Johnson, S. Kotz, and N. Balakrishnan,(1995) Continuous Univariate Distribution, John Wiley & Sons, New York, 2nd edition,.
- V. Barnett and K. Moore (1997): Best linear unbiased estimates in ranked-set sampling with particular reference to imperfect ordering, J. Appl. Stat., 24, 697-710.
- Z. Chen, Z. Bai and B.K. Sinha (2003): Ranked Set Sampling Look, Springer-Verlage, New York.