A Common Fixed Point Theorem for Four Compatible Mappings of Type (P) In Complete Metric Space

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Abstract: *The purpose of this paper is to present a common unique fixed-point theorem for four self mappings in complete metric space using weaker condition such as compatible of type (P) and associated sequence in place of compatible mappings .Our result generalizes the results of Sharma, Badshah and Gupta [5], Lohani and Badshah [3] and Singh and Chouhan [6].*

Keywords: *Complete metric Space, compatible mappings, compatible mappings of type (P), common fixed point.*

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1. Introduction:

Jungck [1,2], introduced more generalized commutativity; known as compatibility, which is weaker than weakly commuting maps and proved a common fixed point theorem for weakly commuting maps. After various author proved a common fixed point theorem for compatible mappings satisfying contractive type conditions and continuity of one of the mapping is required.

In 1994 Pathak, Chang and Cho [4] introduced the concept of compatible mappings of type (P) and proved a common fixed point theorem for compatible mappings of type (P). In 2014 Srinivas and Rao [7] and Srinivas and Raju [8] proved common fixed point theorem on compatible mappings of type (P).

The purpose of this paper is to generalize some common fixed point theorems, which extend the results of Sharma, Badshah and Gupta [5], Lohani and Badshah [3], Singh and Chouhan [6] by using a rational inequality and compatible mappings of type (P) instead of compatible mappings. To illustrate our main theorems, an example is also given.

2. Preliminaries:

Definition 2.1 : Two mapping S and T from a metric space (X, d) into itself ,are called commuting on X, if $d(STx, TSx) = 0$ i.e. $STx = TSx$ for all x in X.

Definition 2.2 : Two mapping S and T from a metric space (X, d) into itself ,are called weakly commuting on X, if $d(STx, TSx) \le d(Sx, Tx)$ for all x in X.

Clearly, commuting mappings are weakly commuting, but converse is not necessarily true, given by following example :

Example2.1 [5]

Let X = [0, 1] with the Euclidean metric d. Define S and $T : X \to X$ by

$$Sx = \frac{x}{3-x} \quad \text{and} \quad Tx = \frac{x}{3} \text{ for all } x \text{ in } X.$$

Then for any x in X ,

$$d\left(STx, TSx\right) = \left|\frac{x}{9-x} - \frac{x}{9-3x}\right|$$
$$= \left|\frac{-2x^2}{(9-x)(9-3x)}\right|$$

$$\le \frac{x^2}{9-3x}$$
$$= \left|\frac{x}{3-x} - \frac{x}{3}\right|$$
$$= d\left(Sx, Tx\right)$$

i.e. $d\left(STx, TSx\right) \le d\left(Sx, Tx\right)$ for all x in X.

Thus S and T are weakly commuting mappings on X, but they are not commuting on X, because

$$STx = \frac{x}{9-x} < \frac{x}{9-3x} = TSx \text{ for any } x \ne 0 \text{ in } X.$$

i.e. $STx < TSx$ for any $x \ne 0$ in X.

Definition 2.3. If Two mapping S and T from a metric space (X, d) into itself, are called compatible mappings on X, if $\lim_{m\to\infty} d\left(STx_m, TSx_m\right) = 0$, when $\{x_m\}$ is a sequence in X such that $\lim_{m\to\infty} Sx_m = \lim_{m\to\infty} Tx_m = x$ for some x in X.

Clearly Two mapping S and T from a metric space (X, d) into itself, are called compatible mappings on X, then $d(STx, TSx) = 0$ when $d(Sx, Tx) = 0$ for some x in X . Note that weakly commuting mappings are compatible, but the converse is not necessarily true.

Example2.2 [5]

Let X = [0, 1] with the Euclidean metric d. Define S and $T : X \to X$ by

$$Sx = x \quad \text{and} \quad Tx = \frac{x}{x+1} \text{ for all } x \text{ in } X.$$

Then for any x in X ,

$$STx = S\left(Tx\right) = S\left(\frac{x}{x+1}\right) = \frac{x}{x+1}$$
$$TSx = T\left(Sx\right) = T\left(x\right) = \frac{x}{x+1}$$
$$d\left(Sx, Tx\right) = \left|x - \frac{x}{x+1}\right| = \left|\frac{x^2}{x+1}\right|$$

Thus we have

$$d\left(STx, TSx\right) = \left|\frac{x}{x+1} - \frac{x}{x+1}\right|$$
$$= 0 \le \frac{x^2}{x+1} \text{ for all } x \text{ in } X.$$
$$= d\left(Sx, Tx\right)$$

i.e. $d\left(STx, TSx\right) \le d\left(Sx, Tx\right)$ for all x in X.

Thus S and T are weakly commuting mappings on X, and then obviously S and T are compatible mappings on X .

Example2.3 [5]

Let $X = R$ with the Euclidean metric d. Define S and $T : X \to X$ by

$Sx = x^2$ and $Tx = 2x^2$ for all x in X.

Then for any x in X ,

$$STx = S\left(Tx\right) = S\left(2x^2\right) = 4x^4$$
$$TSx = T\left(Sx\right) = T\left(x^2\right) = 2x^4 \text{ are compatible mappings on } X \text{, because}$$
$$d\left(Sx, Tx\right) = \left|x^2 - 2x^2\right| = \left|-x^2\right| \to 0 \text{ as } x \to 0$$

Then

$$d(STx, TSx) = |4x^4 - 2x^4| = 2|x^4| \to 0 \ \ as \ \ x \to 0$$

But $d(STx, TSx) \le d(Sx, Tx)$ is not true for all *x* in *X*.

Thus *S* and *T* are not weakly commuting mappings on X.Hence all weakly commuting mappings are compatible, but converse is not true.

Definition 2.4 If Two mapping *S* and *T* from a metric space (*X*, *d*) into itself, are called **compatible mappings of type (A)** on X, if $\lim_{n\to\infty} d(STx_n, TTx_n) = 0$ and $\lim_{n\to\infty} d(TSx_n, SSx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

Definition 2.5 If Two mapping *S* and *T* from a metric space (*X*, *d*) into itself, are called **compatible mappings of type (B)** on X, if

$$\lim_{n\to\infty} d(STx_n, TTx_n) \le \frac{1}{2}\left[\lim_{n\to\infty} d(STx_n, St) + \lim_{n\to\infty} d(St, SSx_n)\right]$$

and
$$\lim_{n\to\infty} d(TSx_n, SSx_n) \le \frac{1}{2}\left[\lim_{n\to\infty} d(TSx_n, Tt) + \lim_{n\to\infty} d(Tt, TTx_n)\right]$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

Definition 2.6 If Two mapping *S* and *T* from a metric space (*X*, *d*) into itself, are called **compatible mappings of type (P)** on X, if $\lim_{n\to\infty} d(SSx_n, TTx_n) = 0$ whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Sx_n = \lim_{n\to\infty} Tx_n = t$ for some $t \in X$.

Example2.4

Let *X* = [0, 1) with the usual metric $d(x, y) = |x - y|$.

Define *S* and $T : X \to X$ by

$$Sx = \begin{cases} 1/3 & \text{when } 0 \le x < \dfrac{1}{2} \\ 1/2 & \text{when } \dfrac{1}{2} \le x \le 1 \end{cases}$$

$Tx = 1 - x$ for all *x* in *X*.

Then clearly the pair (*S*, *T*) is compatible of type (P) .For this take a sequence $x_n = \frac{1}{2} + \frac{1}{n}$, $n \ge 3$.

Then $\lim_{n\to\infty} Sx_n = \frac{1}{2}$, $\lim_{n\to\infty} Tx_n = \frac{1}{2}$ Also $\lim_{n\to\infty} SSx_n = \lim_{n\to\infty} S\left(\frac{1}{2}\right) = \frac{1}{2}$.

and $\lim_{n\to\infty} TTx_n = \lim_{n\to\infty} T\left(\frac{1}{2} - \frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{1}{2}\right) = \frac{1}{2}$. so that $\lim_{n\to\infty}(SSx_n, TTx_n) = 0$.

Hence the pair (*S*, *T*) is compatible of type (P) on X . But the pair (*S*, *T*) is not compatible on X, for this take a sequence $x_n = \frac{1}{2} + \frac{1}{n}$, $n \ge 3$.

Then $\lim_{n\to\infty} Sx_n = \frac{1}{2}$, $\lim_{n\to\infty} Tx_n = \frac{1}{2}$ Also $\lim_{n\to\infty} TSx_n = \lim_{n\to\infty} T\left(\frac{1}{2}\right) = \lim_{n\to\infty}\left(1 - \frac{1}{2}\right) = \frac{1}{2}$.

But $\lim_{n\to\infty} STx_n = \lim_{n\to\infty} S\left(1 - \frac{1}{2} - \frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{1}{2}\right) = \frac{1}{2}$. so that $\lim_{n\to\infty}(TSx_n, STx_n) \ne 0$.

Hence (*S*, *T*) is not compatible on *X*. Note that compatible mappings are compatible of type (P), but the converse is not necessarily true.

Singh and Chouhan [6] proved the following theorem.

Theorem2.1 : , Let *A*, *B*, *S* and *T* be mappings from a complete metric space (*X*, *d*) into itself satisfying the following conditions:

$$A(X) \subseteq T(X) \text{ and } B(X) \subseteq S(X) \tag{3.1}$$

One of P, Q, S and T is continuous,

$$\begin{aligned}[d(Ax,By)]^2 \le\ & k_1\left[d(Ax,Sx)d(By,Ty)+d(By,Sx)d(Ax,Ty)\right] \\ & +k_2\left[d(Ax,Sx)d(Ax,Ty)+d(By,Ty)d(By,Sx)\right]\end{aligned} \tag{3.2}$$

for all $x, y \in X$, where $k_1, k_2 \ge 0$ and $0 \le k_1 + k_2 < 1$.
The pairs (*A*, *S*) and (*B*, *T*) are compatible on *X*, then A, *B*, *S* and *T* have a unique common fixed point in *X*.
Lohani and Badshah [3] proved the following theorem.

Theorem 2.2: Let *P*,*Q*,*S* and *T* be self mappings from a complete metric space (*X*,*d*) into itself satisfying the following conditions

$$S(X) \subseteq Q(X),\ T(X) \subseteq P(X) \tag{3.3}$$

$$d(Sx,Ty) \le \alpha \frac{d(Qy,Ty)[1+d(Px,Sx)]}{[1+d(Px,Qy)]} + \beta d(Px,Qy) \tag{3.4}$$

for all *x*,*y* in *X* where $\alpha, \beta \ge 0$, $\alpha + \beta < 1$.
Suppose that
(i) One of *P*, *Q*,*S* and *T* is continuous
(ii) Pairs (*S*, *P*) and (*T*, *Q*) are compatible on *X*. Then *P*, *Q*, *S* and *T* have a unique common fixed point in *X*.
Sharma, Badshah and Gupta [15] proved the following theorem.

Theorem 2.3 Let *P*, *Q*, *S* and *T* be mappings from a complete metric space (*X*, *d*) into itself satisfying the conditions

$$S(X) \subseteq Q(X),\ T(X) \subseteq P(X) \tag{3.5}$$

$$d(Sx,Ty) \le \left\{\alpha + \beta \frac{d(Sx,Px)}{1+d(Px,Qy)}\right\} d(Ty,Qy) \tag{3.6}$$

for all $x, y \in X$, where $\alpha, \beta \ge 0$ and $\alpha + \beta < 1$.
Suppose that
(i) One of *P*, *Q*, *S* and *T* is continuous,
(ii) Pairs (*S*, *P*) and (*T*, *Q*) are compatible on *X*.
Then P, Q, S and T have a unique common fixed point in *X*.

Now we generalize the **theorem 2.3** using weakly compatible mappings in place of compatible mappings also condition of any one of the mapping is being dropped.
Associated Sequence: Suppose *P*, *Q*, *S* and *T* be mappings from a complete metric space (*X*, *d*) into itself satisfying the conditions:

$$S(X) \subseteq Q(X) \text{ and } T(X) \subseteq P(X) \tag{3.5}$$

$$d(Sx,Ty) \le \left\{\alpha + \beta \frac{d(Sx,Px)}{1+d(Px,Qy)}\right\} d(Ty,Qy) \tag{3.6}$$

for all $x, y \in X$, where $\alpha, \beta \ge 0$ and $\alpha + \beta < 1$.
Then for an arbitrary point x_0 in *X*, by (3.5) we choose a point x_1 in *X* such that $Sx_0 = Qx_1$ and for this point x_1, there exists a point x_2 in *X* such that $Tx_1 = Px_2$ and so on. Proceeding in the similar manner, we can define a sequence $\{y_m\}$ in *X* such that

$y_{2m+1} = Qx_{2m+1} = Sx_{2m}$ for $m \ge 0$ and $y_{2m} = Px_{2m} = Tx_{2m-1}$ for $m \ge 1$ (3.7)

we shall call this sequence as an "Associated sequence of x_0" relative to four self mappings *P*, *Q*, *S* and *T*.

Lemma 2.1 . Let *P*, *Q*, *S* and *T* be mappings from a complete metric space (*X*, *d*) into itself satisfying the conditions (3.5) and (3.6). Then the Associated sequence $\{y_m\}$ relative to four self mappings *P*, *Q*, *S* and *T* defined in (3.7) is a Cauchy sequence in *X*.

Proof. By definition (3.7) we have

$$d(y_{2m+1}, y_{2m}) = d(Sx_{2m}, Tx_{2m-1})$$

$$\le \left\{\alpha+\beta\frac{d(Sx_{2m},Px_{2m})}{1+d(Px_{2m},Qx_{2m-1})}\right\}d(Tx_{2m-1},Qx_{2m-1})$$

$$\le \left\{\alpha+\beta\frac{d(y_{2m+1},y_{2m})}{1+d(y_{2m},y_{2m-1})}\right\}d(y_{2m},y_{2m-1})$$

$$\le \alpha d(y_{2m},y_{2m-1})+\beta d(y_{2m+1},y_{2m})$$

i.e $d(y_{2m+1},y_{2m})\le\dfrac{\alpha}{1-\beta}d(y_{2m},y_{2m-1})$

Hence $d(y_{2m+1},y_{2m})\le h\,d(y_{2m},y_{2m-1})$

Where $h=\dfrac{\alpha}{1-\beta}<1$

Similarly we can show that

$$d(y_{2m+1},y_{2m})\le h^{2m}\,d(y_1,y_0)$$

For k > m , we have

$$d(y_{m+k},y_m)\le\sum_{i=1}^{k}d(y_{n+i},y_{n+i-1})$$

$$\le\sum_{i=1}^{k}h^{n+i-1}d(y_1,y_0)$$

i.e. $d(y_{m+k},y_m)\le\left(\dfrac{h^n}{1-h}\right)d(y_1,y_0)\to 0$ as $n\to\infty$

Since $h<1$, $h^n\to 0$ as $n\to\infty$, so that $d(y_m,y_{m+k})\to 0$. This Show that the sequence $\{y_m\}$ is a Cauchy's sequence in X. and since X is a complete metric space, it converges to a limit, say u in X. The converse of the lemma is not true, that is P, Q, S and T satisfying (3.5) and (3.6) , even if for x_0 in X and the Associated sequence of x_0 converges, the metric space (X, d) need not be complete. The following example establishes this.

Example 2.5

Let $X = [0, 1]$ with usual metric $d(x, y) = |x-y|$. Define Self mappings P, Q, S and T on X by

$$Sx=Tx=\begin{cases}\frac{1}{3} & \text{if } 0\le x<\frac{1}{2}\\ \frac{1}{2} & \text{if } \frac{1}{2}\le x\le 1\end{cases}, \text{ and } Px=Qx=1-x \text{ for all } X.$$

Then $S(X)=T(X)=\left\{\frac{1}{2},\frac{1}{3}\right\}$ while, $P(X)=Q(X)=[0,1]$

Clearly $S(X)\subseteq Q(X)$ and $T(X)\subseteq P(X)$. Also inequality (3.6) can be easily verified with appropriate values of α and β. Also the sequence $Sx_0, Tx_1, Sx_2, Tx_3, \ldots Sx_{2n}, Tx_{2n+1}, \ldots$ converges to $\frac{1}{2}$. But (X, d) is not a complete metric space.

3. Main Result

Theorem 3.1 Let P, Q, S and T be mappings from a metric space (X, d) into itself satisfying the conditions (3.5) and (3.6). Suppose that the pairs (S, P) and (T, Q) are compatible mappings of type (P) on X. Further the associated sequence relative to four self mappings P, Q, S and T such that Sx_0, Tx_1,…, Sx_{2m}, Tx_{2m+1} converges to u in X as $n\to\infty$ .Then P, Q, S and T have a unique common fixed point u in X.

Proof. Let $\{y_m\}$ be the Associated sequence in X defined by (3.7).By **lemma 2.1** , the Associated $\{y_m\}$ is a Cauchy sequence in X and hence it converges to some point u in X. Consequently, the subsequences $\{Sx_{2m}\}$,{

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Px_{2m} }, { Tx_{2m-1} } and { Qx_{2m-1} } of $\{y_m\}$ also converges to u. Suppose S is continuous . Then $S^2x_{2m} \to Su$, $SPx_{2m} \to Su$ as $n \to \infty$.

Since (S, P) is compatible of type (P) , then $\lim_{m\to\infty} d(SSx_{2m}, TTx_{2m}) = 0$. This gives $\lim_{m\to\infty} PPx_{2m} = Su$.

Hence $\lim_{m\to\infty} PPx_{2m} = \lim_{m\to\infty} SSx_{2m} = Su.$

To prove $Su = u$ put $x = Px_{2m}$, $y = x_{2m-1}$ in (3.6), we get

$$d(SPx_{2m}, Tx_{2m-1}) \leq \lim_{m\to\infty}\left\{\alpha + \beta\frac{d(SPx_{2m}, PPx_{2m})}{1 + d(PPx_{2m}, Qx_{2m-1})}\right\} d(Tx_{2m-1}, Qx_{2m-1})$$

Letting $m \to \infty$

$$d(Su, u) \leq \left\{\alpha + \beta\frac{d(Su, Su)}{1 + d(Su, u)}\right\} d(u, u)$$

i.e. $d(Su, u) \leq 0$

So that $u = Su$. since $S(X) \subseteq Q(X)$ there exists $v \in X$ such that $u = Sv = Qv$. we prove that $Tv = Qv$.

To prove $Tv = u$ put $x = x_{2m}$, $y = v$ in (3.6), we get

$$d(Sx_{2m}, Tv) \leq \left\{\alpha + \beta\frac{d(Sx_{2m}, Px_{2m})}{1 + d(Px_{2m}, Qv)}\right\} d(Tv, Qv)$$

Letting $m \to \infty$

$$d(u, Tv) \leq \left\{\alpha + \beta\frac{d(u, u)}{1 + d(u, Qv)}\right\} d(Tv, u)$$

$$(1-\alpha)\, d(u, Tv) \leq 0$$

i.e. $d(u, Tv) \leq 0$

So that $u = Tv$. Hence $u = Tv = Qv$.

Since (T, Q) is compatible of type (P) and $u = Tv = Qv$, we get $d(TTv, QQv) = 0$. This gives $d(Tu, Qu) = 0$. Hence $Tu = Qu$.

To prove $Tu = u$ put $x = u$, $y = x_{2m-1}$ in (3.6), we get

$$d(Su, Tx_{2m-1}) \leq \left\{\alpha + \beta\frac{d(Su, Pu)}{1 + d(Pu, Qx_{2m-1})}\right\} d(Tx_{m-1}, Qx_{2m-1})$$

Letting $m \to \infty$

$$d(u, Tu) \leq \left\{\alpha + \beta\frac{d(Su, Su)}{1 + d(u, Qv)}\right\} d(u, u)$$

i.e. $d(u, Tu) \leq 0$

So that $u = Tu$. Hence $u = Tu = Qu$, therefore u is a common fixed point of T and Q .Since $T(X) \subseteq P(X)$ there exists $v' \in X$ such that $u = Tu = Pv'$. we prove that $Sv' = Pv'$.

To prove $Sv' = u$ put $x = v'$, $y = u$ in (3.6), we get

$$d(Sv', Tu) \leq \left\{\alpha + \beta\frac{d(Sv', Pv')}{1 + d(Pv', Qu)}\right\} d(Tu, Qu)$$

$$d(Sv', u) \leq 0$$

So that $u = Sv'$. Hence $u = Sv' = Pv'$ .Since (S, P) is compatible of type (P) and $u = Sv' = Pv'$, we get $d(PPv', SSv') = 0$. This gives $d(Pu, Su) = 0$. Hence $Pu = Su.$

Hence $u = Pu = Su$ therefore u is a common fixed point of P and S. Thus $u = Tu = Qu = Su = Pu$. Hence u is a common fixed point of P, Q, S and T.

For uniqueness of u, suppose u and z , $u \neq z$, are common fixed points of P, Q, S and T. Then by (3.6), we obtain

$$\begin{aligned} d(u,z) &= d(Su,Tz) \\ &\le \left\{\alpha + \beta \frac{d(Su,Pu)}{1+d(Pu,Qz)}\right\} d(Tz,Qz) \\ &\le \left\{\alpha + \beta \frac{d(u,u)}{1+d(u,z)}\right\} d(z,z) \\ &\le 0 \end{aligned}$$

i.e. $$d(u,z) \le 0$$

which is a contradiction .Hence $u = z$. So u is a unique common fixed point of P, Q, S and T.
This completes the proof.

Remark 3.1 From the example 2.5

Since $SPx = S(1-x) = \begin{cases} \frac{1}{3} & \text{if } 0 \le x < \frac{1}{2} \\ \frac{1}{2} & \text{if } \frac{1}{2} \le x \le 1 \end{cases}$

and $PSx = P\left[\begin{cases} \frac{1}{3} & \text{if } 0 \le x < \frac{1}{2} \\ \frac{1}{2} & \text{if } \frac{1}{2} \le x \le 1 \end{cases}\right] = \begin{cases} \frac{2}{3} & \text{if } 0 \le x < \frac{1}{2} \\ \frac{1}{2} & \text{if } \frac{1}{2} \le x \le 1 \end{cases}$ .Then clearly $PSx \neq SPx$.

Hence the pair (S, P) is not commuting on X. Similarly easily verified that the pair (T, Q) is not commuting on X. Also the pairs (S, T) and (T, Q) are not compatible on X, for this take a sequence $x_n = \frac{1}{2} + \frac{1}{n}$, $n \ge 3$.

Then $\lim_{n\to\infty} Sx_n = \frac{1}{2}$, $\lim_{n\to\infty} Px_n = \lim_{n\to\infty}\left(\frac{1}{2} - \frac{1}{n}\right) = \frac{1}{2}$.

Also $\lim_{n\to\infty} PSx_n = \lim_{n\to\infty} P\left(\frac{1}{2}\right) = \lim_{n\to\infty}\left(1 - \frac{1}{2}\right) = \frac{1}{2}$.

But $\lim_{n\to\infty} SPx_n = \lim_{n\to\infty} S\left(1 - \frac{1}{2} - \frac{1}{n}\right) = \lim_{n\to\infty} S\left(\frac{1}{2} - \frac{1}{n}\right) = \frac{1}{2}$. so that $\lim_{n\to\infty}\left(PSx_n, SPx_n\right) \neq 0$.

Hence (S, P) is not compatible on X. Similarly easily verified that the pair (T, Q) is not compatible on X. It can be easily verified that the pairs (S, T) and (T, Q) are not compatible of type (A), compatible of type (B) .But the pairs (S, T) and (T, Q) are compatible of type (P) .For this take a sequence $x_n = \frac{1}{2} + \frac{1}{n}$, $n \ge 3$.

Then $\lim_{n\to\infty} Sx_n = \frac{1}{2}$, $\lim_{n\to\infty} Tx_n = \frac{1}{2}$ Also $\lim_{n\to\infty} SSx_n = \lim_{n\to\infty} S\left(\frac{1}{2}\right) = \frac{1}{2}$.

and $\lim_{n\to\infty} TTx_n = \lim_{n\to\infty} T\left(\frac{1}{2} - \frac{1}{n}\right) = \lim_{n\to\infty}\left(\frac{1}{2}\right) = \frac{1}{2}$. so that $\lim_{n\to\infty}\left(SSx_n, TTx_n\right) = 0$.

Hence the pair (S, T) is compatible of type (P) on X. Similarly easily verified that the pair (T, Q) is compatible mappings of type (P) on X. Also the rational inequality (3.6) holds for appropriate value of α, β with $\alpha, \beta \ge 0$ and $\alpha + \beta < 1$ and $\alpha < 1$. Clearly $\frac{1}{2}$ is the unique common fixed point of P, Q, S and T.

Conclusion:- The conclusion of this paper that we shown a unique common fixed point theorem with generalization result of Sharma, Badshah and Gupta [5] ,using weaker condition compatible of type (P) , instead of compatible mappings also we illustrate our result by example.

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