Clustering of the Kenyan General Insurance Risk Classes by the Archimedean Copula Theory

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Abstract

Dependence between risks reduces the benefits of diversification. Modern portfolio theory is based on correlation as a measure of dependence while the criterion presented here is based on the copula theory as a measure of the intrinsic relatedness of different risks classes. The dependencies are examined by fitting copulas, estimating the dependence parameters and lastly using distance matrices to cluster the similar risks together. The study derives its data from the general insurance business in Kenya. The motivation of the study was driven by the fact that insurance companies had collapsed in the past, one reason being the type of business classes they engage in. It is therefore important to understand the dependencies between risks for better risk management. Five major clusters stood out each with peculiar characteristics. The first cluster constituted the lines with a high probability of a huge claim amount: Engineering, Liability, Fire industrial and Theft. The second contains lines with moderate claim amounts as compared to the previous cluster but are rather slightly more frequent: Fire domestic, Personal accident, Workman's compensation, Motor commercial and motor private. In the following cluster we have the less popular lines under the umbrella of the miscellaneous class. Marine and Transit which is completely erratic clusters singly while the Aviation line whose main business is exported to foreign countries forming the last cluster. This will assist the companies seeking diversification of their risk portfolio and also entry into re-insurance treaties as a criterion for the determination of forwarding proportions is proposed here.

Keywords: Copula, Measures of dependence, Cluster, Distances, Lines of business

1. Introduction

Once a risk is insured, it is reasonable that the standards for classifying that risk can and should be different from those of marketing and/or underwriting. The variables comprising a classification system should be chosen so that the following guidelines or conditions in addition, of course, to any legal requirements regarding fair discrimination, are generally adhered to in keeping with Walter (1981) who writes on risk classification standards. The whole modern portfolio theory is based on correlation (see Burgi, Dacorogna, and Iles (2008)) as a measure of dependence but the criterion used here is based on the copula theory which is handy when correlation assumptions are violated.

Frees and Valdez (1997) introduces actuaries to the concept of "copulas", as a tool for understanding relationships among multivariate outcomes. The work explores some of the practical applications, including estimation of joint life mortality and multiple decrement models and showed how to fit copulas and hence described their usefulness by pricing a reinsurance contract and estimating expenses for pre-specified losses. Motivated by the fact that most of the Economic Capital assessment models encounter difficulties when trying to incorporate the dependence of claim costs between different Lines of Business (LOBs), Faivre (2003) suggested the use of copula theory as a solution to this problem. Other useful literature include: Embrechts, et al. (2002), Frees and Wang (2004), Pettere and Kollo (2006), Wu, Valdez and Sherris (2006), Gatzert, Schmeiser and Schuckmann, (2008) and Filler, Odening, Okhrin and Xu (2009). Faivre (2003) use Copulas to model the overall distribution of claim costs of four-Lines of Business company. The work utilized different copulas to show that the dependence structure has a substantial impact on the Economic Capital of that firm. Mung'atu, Mwalili and Weke (2011) extended to cover lines of business with sub-classes by employing the Hierarchical Archimedean Copulas. This work proposes the use of the copula theory to model the dependence between business classes and later clustering them into larger homogeneous clusters. The paper is presented in the various sections: introduction; methodology which constitutes: the copula function, distances, clustering, methodology, empirical results; conclusion and recommendations.

1.1. Measures of dependence

Measures of dependence summarize a complicated dependence structure in a single value, in the bivariate case. The three important concepts in measuring dependence include: the linear correlation, rank correlation and the coefficients of tail dependence. The last two provide perhaps the best alternatives to the linear correlation coefficient as a measure of dependence for non-elliptical distributions. Copulas capture the properties of the joint distributions since they are invariant, that is, they remain unchanged under strictly increasing transformations of the random variables.

2. Methodology

2.1. The copula function

The term copula was first used in the work of Sklar (1959) and is derived from the Latin word copulare, meaning to connect or to join and has recently found an extensive acceptance in risk management, financial and insurance applications. The main purpose of copulas is to describe the interrelation of several random variables. A copula is a function that joins or couples a multivariate distribution function to univariate marginal distribution functions and so a copula is a multivariate distribution function. The operational definition of a copula is a multivariate distribution function defined on the unit cube $\left[0,1\right]^n$, with uniformly distributed marginals (see Nelsen, 2006).

Definition 1: A two-dimensional copula function (or a 2-copula) is defined as a binary function $C:[0,1]^2 \rightarrow [0,1]$, which satisfies the following three properties:

$$\begin{array}{l} 1.\ C(u,0) = C(0,u) = 0 \text{ for any } u \in [0,1]. \\ 2.\ C(u,1) = C(1,u) = u \text{ for any } u \in [0,1]. \\ 3.\ \text{For all } 0 \le u_1 \le u_2 \le 1 \text{ and } 0 \le v_1 \le v_2 \le 1 \\ C([u_1,v_1]\times[u_2,v_2]) = C(u_2,v_2) - C(u_{1,}v_2) - C(u_2,v_1) + C(u_1,v_1) \ge 0. \end{array} \qquad (1)$$

From the properties 1-3, when the arguments *u* and *v* are univariate distribution functions F_1 and F_2, the copula function $C(F_1; F_2)$ is a legitimate bivariate distribution function with marginals F_1 and F_2. Conversely, any bivariate distribution function *H(x; y)* with continuous marginals F_1 and F_2 admits a unique representation as a copula function:

$$C(u,v) = H\left(F_1^{-1}(u), F_2^{-1}(v)\right) \qquad (2)$$

In general, an *n*-dimensional Copula is any function $C\colon \left[0,1\right]^n \rightarrow \left[0,1\right]$ such that:

1. C is grounded and *n*-increasing
2. C has margins $C_k, k = 1,2,...,n,$ which satisfy $C_k(u) = u$ for all u in $\left[0,1\right]$.

It is also important to note that for any *n*-copula, $n \ge 3$, each *k*-dimensional margin of *C* is a *k*-copula.

2.1.1. *Archimedean copulas*

Definition 2: Let $\varphi:[0,1]\rightarrow[0,\infty]$ be a continuous, strictly decreasing and convex function such that $\varphi(1)=0$ and $\varphi(0)=\infty$. The function φ has an inverse $\varphi^{-1}:[0,\infty]\rightarrow[0,1]$ with the same properties like φ, except that $\varphi^{-1}(0)=1$ and $\varphi^{-1}(\infty)=0$.

Definition 3: The function $C:[0,1]^n\rightarrow[0,1]$ defined by

$$C(u_1,\ldots,u_n)=\varphi^{-1}(\varphi(u_1)+\ldots+\varphi(u_n)) \tag{3}$$

is called *n*-dimensional Archimedean copula if and only if φ^{-1} is completely monotonic on $[0,\infty)$, that is

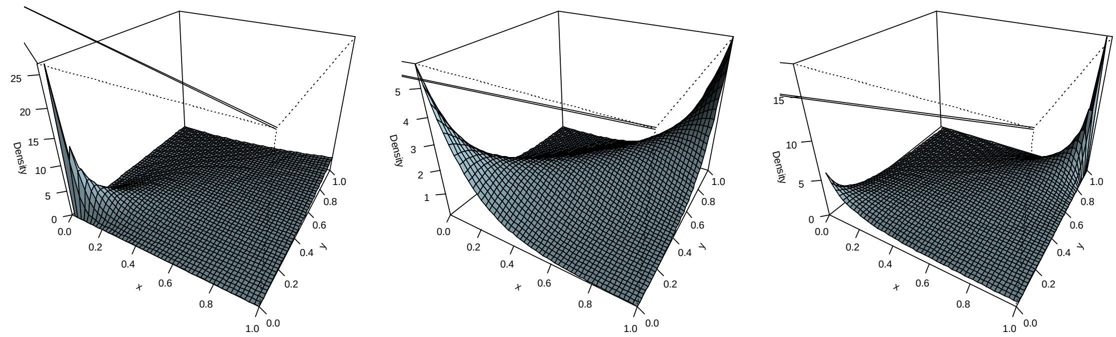
$$(-1)^k\frac{\partial^k}{\partial u^k}\varphi^{-1}(u)\geq 0 \quad \text{for } k\in\aleph \tag{4}$$

The function φ is called the generator of the copula. We assume that the generator φ has only one parameter, denoted as θ. The three often used Archimedean copulas include: Clayton, Gumbel, and Frank.

The Clayton copula: This is an asymmetric Archimedean copula, exhibiting greater dependence in the negative tail than in the positive. This copula is given by $C_\theta(u,v)=\max\left(\left[u^{-\theta}+v^{-\theta}-1\right]^{-1/\theta},0\right)$ and its generator is $\varphi_\theta(x)=\frac{1}{\theta}\left(x^{-\theta}-1\right)$ where $\theta\in[-1,\infty)\backslash\{0\}$. The relationship between Kendall's tau τ and the Clayton copula parameter θ is given by $\theta=2\tau/1-\tau$.

The Gumbel copula: also referred to as the Gumbel-Hougard copula, is an asymmetric Archimedean copula, exhibiting greater dependence in the positive tail than in the negative. This copula is given by $C_\theta(u,v)=\exp\left\{-\left[(-\ln u)^\theta+(-\ln v)^\theta\right]^{1/\theta}\right\}$ and its generator is $\varphi_\theta(x)=(-\ln x)^\theta$ where $\theta\in[1,\infty)$. The relationship between Kendall's tau τ and the Gumbel copula parameter θ is given by $\theta=1/1-\tau$.

The Frank copula: The Frank copula is a symmetric Archimedean copula given by $C_\theta(u,v)=-\frac{1}{\theta}\ln\left\{1+\frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{e^{-\theta}-1}\right\}$ and its generator is $\varphi_\theta(x)=-\ln\left\{\frac{\exp(-\theta x)-1}{\exp(-\theta)-1}\right\}$ where $\theta\in(-\infty,\infty)\backslash\{0\}$. The relationship between Kendall's tau τ and the Frank copula parameter θ is given by $\frac{[D_1(\theta)-1]}{\theta}=\frac{1-\tau}{4}$ where $D_1(\theta)=\frac{1}{\theta}\int_0^\theta\frac{t}{e^t-1}dt$ is a Debye function of the first kind. Figure 1 shows the tail dependencies for the three Archimedean copulas. The Clayton is strong on the lower tail dependence; Frank has no tail dependence while the Gumbel is good for modelling the upper tail dependence.



(a) (b) (c)

Figure 1: Perspective plots for the (a) Clayton, (b) Frank and (c) Gumbel copulas

2.2. Distances

Distance is a numerical description of how far apart objects are. When distances are calculated between various objects, this culminates into a distance matrix.

Definition 4: A metric on a set *X* is a function, called the distance function or simply distance, $d : X \times X \to \Re$ (where $\Re$ is the set of real numbers). For all *x, y, z* in *X*, this function is required to satisfy the following conditions:

1. $d(x, y) \geq 0$ (non-negativity)
2. $d(x, y) = 0$ if and only if $x = y$ (identity of indiscernible).

Condition 1 and 2 together produce positive definiteness

3. $d(x, y) = d(y, x)$ (symmetry)
4. $d(x, z) \leq d(x, y) + d(y, z)$ (subadditivity/triangle inequality).

2.2.1. Euclidean distances

The well-known distance is the Euclidean distance which is defined as $d(\mathbf{x},\mathbf{y}) = \|\mathbf{x},\mathbf{y}\| = \sqrt{(x-y)^T(x-y)} = \sqrt{\sum_i (x_i - y_i)^2}$ (with $\|\mathbf{x}\|$ being the norm of *x*, and x_i and y_i being the *i*-th element of *x* and *y*). The Euclidean distances were used as the criterion to cluster the general insurance risk classes with respect to the spearman's rho, Kendall's tau and the Tail dependence.

2.2.2. Manhattan distances

This is also known as City Block Distance, boxcar distance, absolute value distance, rectilinear distance, Minkowski's L1 distance, or taxi cab metric and it is given by $d = \sum_i^n |\mathbf{x}_i - \mathbf{y}_i|$ where *n* is the number of variables, and x_i and y_i are the values of the *i*-th variable, at points *X* and *Y* respectively.

2.3. Clustering

Clustering is a technique to group objects based on distance or similarity. It is therefore the assignment, grouping or segmenting of a set of observations, individuals, cases, or data rows into subsets, called clusters, so that observations in the same cluster are similar in some sense. The cardinal objective of clustering is to measure the degree of similarity (or dissimilarity) between the individual objects being clustered. In this work we utilize the agglomerative approach under the Hierarchical clustering. The algorithm of agglomerative approach to compute hierarchical clustering is as follows:

(i) Convert object features to distance matrix, in our case we have the matrix of the rank correlation coefficients and the tail dependence.
(ii) Set each object as a cluster, thus for the twelve general insurance lines of business, we will have twelve clusters in the beginning.
(iii) Iterate until the number of cluster is one, that is, by merging the two closest clusters and continuously updating the distance matrix.

2.3.1. Cophenetic correlation coefficient

After the formation of the clusters, the question now is how good is the clustering? There is an index called Cross Correlation Coefficient or Cophenetic Correlation Coefficient that shows the goodness of fit of our clustering similar to the Correlation Coefficient of regression. To compute the Cophenetic Correlation Coefficient of hierarchical clustering, we need a distance matrix and a Cophenetic matrix. To obtain Cophenetic matrix, we need to fill the distance matrix with the minimum merging distance that we obtain in the previous cluster objects. Cophenetic Correlation Coefficient is simply correlation coefficient between distance matrix and Cophenetic matrix.

2.4. The Clustering Algorithm

This is section presents the proposed algorithm for grouping business classes into various lines. For the different business classes follow the algorithm below to cluster them into their respective lines (or departments):

(i) Fit the Copula function (see section 2.1) for each pair of business classes;
(ii) Estimate the dependence parameter, θ;
(iii) Calculate the measures of dependence, the rank correlation (Spearman's rho or the Kendall's tau) and the tail dependence, using the relationships in sub-section 2.1.1;
(iv) Compare closeness of these measures to each other by calculating appropriate distances culminating to a distance matrix (see section 2.2); and
(v) Cluster the business classes into the various homogeneous lines or departments using the minimum distance approach (see section 2.3).

This resulted in the highly related classes being in one cluster with the less dependent classes being in different clusters. The different clusters of business form a diversified portfolio together (with each cluster having homogeneous lines within). The investment should not pick only those business classes/lines within one cluster as this will reduce the diversification benefits.

3. Empirical Results

Data that were collected from thirty-five insurance companies who are members of the Insurance Regulatory Authority (IRA) of Kenya and participating in some class of general insurance for the period 2006 to 2009 were analysed in this section.

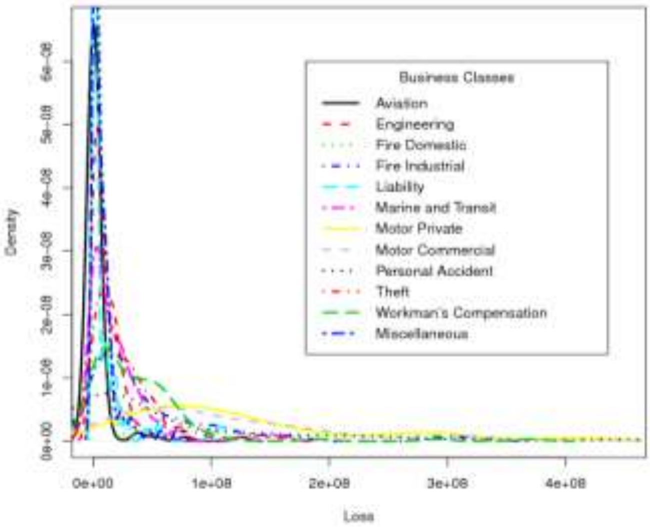


Figure 2: Densities for the twelve classes of business in general insurance, Kenya

A general business insurer, in Kenya, can be registered to transact any or all the twelve classes of general insurance business namely: aviation, engineering, fire-domestic, fire-industrial, liability, marine, motor-private, motor-commercial, personal accident, theft, workman's compensation and miscellaneous. Their densities are shown in Figure 2 which are positively skewed as one would expect with any theoretical loss distribution.

We now compare the clustering based on the Euclidean distances and the Manhattan distances. We suggest the use of the cophenetic correlation coefficient in choosing between the best distances to use. The Euclidean distance performs better than the Manhattan distances, as shown in Table 1 as it consistently produced stronger correlations than for the Manhattan distances, and so we will base our clustering on the Euclidean distances.

Table 1: Comparison between the performance of the Euclidean and the Manhattan distance

	Distance			
	Manhattan		**Euclidean**	
Measure	Cophenetic correlation	Significance	Cophenetic correlation	Significance
Kendall's tau	0.6580	<0.01	0.9029	<0.01
Spearman's Rho	0.7041	<0.01	0.7850	<0.01
Tail index	0.6511	<0.01	0.9067	<0.01

Table 2: The dependence parameter, θ, estimated for each pair of general insurance classes

	Av	Eng	F D	F I	Liab	M & T	M P	M C	P A	Theft	W C
Eng	5.814										
F D	6.025	6.696									
F I	6.173	6.906	6.859								
Liab	5.770	6.508	6.535	7.097							
M & T	5.574	6.171	6.293	6.654	6.580						
M P	6.961	7.212	8.219	8.033	7.643	7.085					
M C	8.068	8.280	8.764	8.945	8.438	8.112	10.102				
P A	6.153	6.815	7.124	7.079	6.832	6.668	8.210	8.814			
Theft	6.451	6.957	7.053	7.322	6.911	6.712	7.936	8.536	6.913		
W C	6.243	6.976	7.265	7.338	7.170	6.727	8.296	9.260	7.408	7.187	
Misc	5.496	6.398	6.566	6.410	6.664	5.917	7.474	7.864	6.669	6.568	6.681

KEY: Av - Aviation, Eng - Engineering, F D - Fire Domestic, F I - Fire Industrial, Liab - Liability, M & T - Marine and Transit, M P - Motor Private, M C - Motor Commercial, P A - Personal Accident, W C - Workman ' s Compensation, Misc - Miscellaneous.

Table 3: Tail index calculated from the fitted dependence parameter, θ, for each pair of general insurance classes

	Av	Eng	F D	F I	Liab	M & T	M P	M C	P A	Theft	W C
Eng	0.873										
F D	0.878	0.891									
F I	0.881	0.894	0.894								
Liab	0.872	0.888	0.888	0.897							
M & T	0.868	0.881	0.884	0.890	0.889						
M P	0.895	0.899	0.912	0.910	0.905	0.897					
M C	0.910	0.913	0.918	0.919	0.914	0.911	0.929				
P A	0.881	0.893	0.898	0.897	0.893	0.890	0.912	0.918			
Theft	0.887	0.895	0.897	0.901	0.895	0.891	0.909	0.915	0.895		
W C	0.883	0.896	0.900	0.901	0.899	0.891	0.913	0.922	0.902	0.899	
Misc	0.866	0.886	0.873	0.886	0.890	0.876	0.903	0.908	0.890	0.889	0.891

Table 4: Euclidean distances from Tail index for each pair of general insurance classes

	Av	Eng	F D	F I	Liab	M & T	Misc	M C	M P	P A	Theft
Eng	0.183										
F D	0.178	0.156									
F I	0.178	0.151	0.151								
Liab	0.187	0.159	0.160	0.146							
M & T	0.189	0.169	0.167	0.159	0.159						
Misc	0.192	0.164	0.181	0.166	0.157	0.176					
M C	0.176	0.151	0.141	0.132	0.145	0.160	0.167				
M P	0.175	0.153	0.134	0.133	0.144	0.161	0.157	0.105			
P A	0.178	0.153	0.146	0.146	0.152	0.159	0.160	0.134	0.130		
Theft	0.171	0.150	0.148	0.141	0.150	0.158	0.163	0.137	0.135	0.149	
W C	0.179	0.151	0.144	0.141	0.145	0.159	0.161	0.125	0.127	0.139	0.144

Table 5: Kendall's tau calculated from the fitted dependence parameter, θ, for each pair of general insurance classes

	Av	Eng	F D	F I	Liab	M & T	M P	M C	P A	Theft	W C
Eng	0.828										
F D	0.834	0.851									
F I	0.838	0.855	0.854								
Liab	0.827	0.846	0.847	0.859							
M & T	0.821	0.838	0.841	0.850	0.848						
M P	0.856	0.861	0.878	0.876	0.869	0.859					
M C	0.876	0.879	0.886	0.888	0.881	0.877	0.901				
P A	0.837	0.853	0.860	0.859	0.854	0.850	0.878	0.887			
Theft	0.845	0.856	0.858	0.863	0.855	0.851	0.874	0.883	0.855		
W C	0.840	0.857	0.862	0.864	0.861	0.851	0.879	0.892	0.865	0.861	
Misc	0.818	0.844	0.848	0.844	0.850	0.831	0.866	0.873	0.850	0.848	0.850

Table 6: Euclidean distances from the Kendall's tau for each pair of general insurance classes

	Av	Eng	F D	F I	Liab	M & T	Misc	M C	M P	P A	Theft
Eng	0.249										
F D	0.244	0.212									
F I	0.241	0.207	0.207								
Liab	0.253	0.218	0.217	0.200							
M & T	0.256	0.230	0.228	0.217	0.217						
Misc	0.260	0.222	0.218	0.225	0.214	0.239					
M C	0.237	0.205	0.189	0.181	0.198	0.217	0.221				
M P	0.237	0.209	0.181	0.183	0.197	0.219	0.209	0.146			
P A	0.242	0.210	0.199	0.200	0.208	0.217	0.216	0.183	0.179		
Theft	0.232	0.205	0.202	0.194	0.206	0.216	0.220	0.187	0.185	0.205	
W C	0.242	0.206	0.196	0.193	0.200	0.217	0.218	0.171	0.175	0.191	0.197

Table 7: Spearman's rho calculated from the fitted dependence parameter, θ, for each pair of general insurance classes

	Av	Eng	F D	F I	Liab	M & T	M P	M C	P A	Theft	W C
Eng	0.958										
F D	0.960	0.968									
F I	0.963	0.970	0.969								
Liab	0.957	0.966	0.966	0.971							
M & T	0.954	0.962	0.964	0.968	0.967						
M P	0.970	0.972	0.978	0.978	0.975	0.971					
M C	0.978	0.979	0.981	0.982	0.980	0.978	0.986				
P A	0.962	0.969	0.971	0.971	0.969	0.968	0.978	0.981			
Theft	0.965	0.970	0.971	0.973	0.970	0.968	0.977	0.980	0.970		
W C	0.963	0.970	0.973	0.973	0.972	0.968	0.979	0.983	0.974	0.972	
Misc	0.953	0.965	0.967	0.965	0.968	0.959	0.974	0.977	0.968	0.967	0.968

Table 8: Euclidean distances from the spearman's rho for each pair of general insurance

	Av	Eng	F D	F I	Liab	M & T	Misc	M C	M P	P A	Theft
Eng	0.064										
F D	0.063	0.046									
F I	0.062	0.044	0.044								
Liab	0.067	0.048	0.048	0.042							
M & T	0.067	0.054	0.054	0.049	0.048						
Misc	0.069	0.051	0.049	0.053	0.047	0.058					
M C	0.072	0.053	0.046	0.042	0.049	0.060	0.060				
M P	0.066	0.049	0.038	0.037	0.044	0.055	0.051	0.025			
P A	0.063	0.045	0.041	0.041	0.044	0.049	0.049	0.042	0.036		
Theft	0.059	0.044	0.042	0.038	0.044	0.049	0.051	0.043	0.038	0.043	
W C	0.064	0.045	0.040	0.038	0.042	0.050	0.050	0.037	0.033	0.037	0.040

The estimated dependence parameter, θ, in Table 2 was used to calculate Table 3 whose distances were calculated and recorded in Table 4. The larger the dependence parameter, θ, the stronger the dependence between pairs of business lines and this leads to a high dependence measure value. Table 4 produce the clusters in Figure 3 which relates to the tail index. A similar approach is done on the Kendall's tau and the Spearman's rho with Tables 6 and 8 leading to Figures 4 and 5 respectively. The three dependence measures are consistent since they produce the same clustering structure as evident from Figures 3, 4 and 5.

This work proposes the use of the upper tail dependence derived from the dependence parameter in determining the retention limits for a re-insurance arrangement. Though the dependence is not the only factor to consider for such re-insurance treaties (Straub 1997) the forwarding proportions should be somewhere proportional to 1/(1 − Tail index). This will ensure that for highly dependent risks in the upper tail will forward higher proportion to the re-insurer and vice versa. The behaviour of this proposed quantity is found in Figure

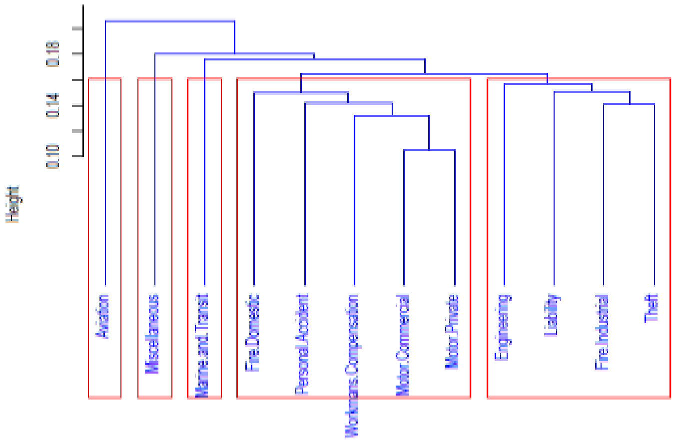


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Figure 3: The general insurance classes clustered by the Euclidean distances for the Tail index

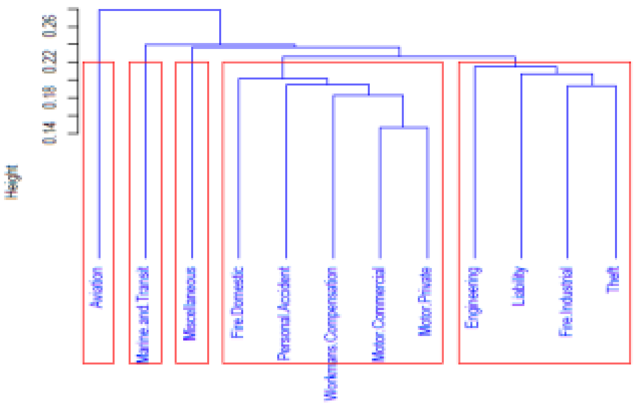

Figure 4: The general insurance classes clustered by the Euclidean distances for the Kendall's tau

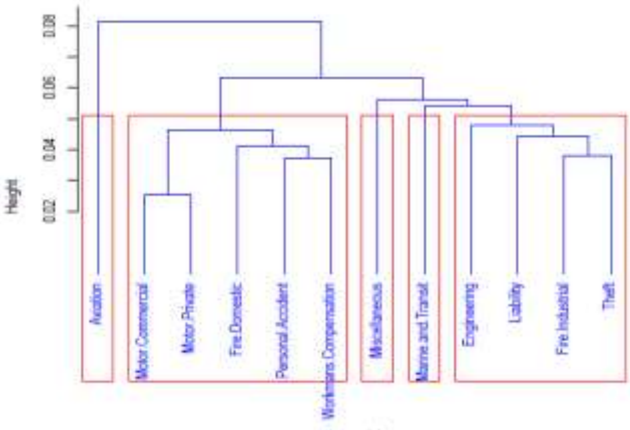

Figure 5: The general insurance classes clustered by the Euclidean distances for the Spearman's rho

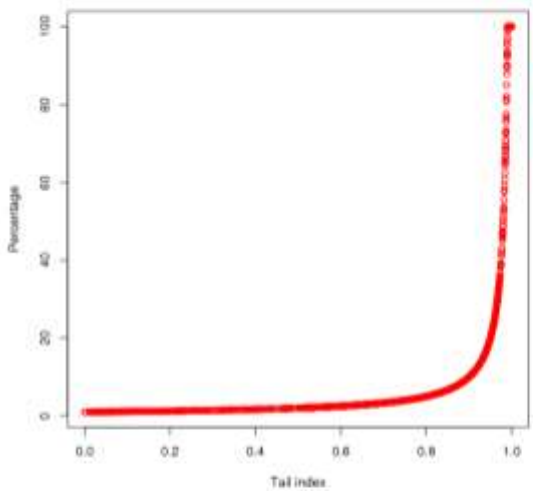

Figure 6: The proposed re-insurance proportions in relation to the tail index

The criteria based on the three dependence measures arrived at five major clusters each with peculiar characteristics. The first cluster involves the business classes/lines with a high probability of a huge claim amount lines: Engineering, Liability, Fire industrial and Theft. The second contain lines with moderate claim amounts as compared to the previous cluster but rather slightly more frequent: Fire domestic, Personal accident, Workman's compensation, Motor commercial and motor private. In the following cluster we have the less popular lines under the umbrella of the miscellaneous class. Marine and Transit which is completely erratic clusters alone while the Aviation line whose main business is exported to foreign countries forming the last cluster. When setting up or restructuring a company it is advisable to select the lines of business from different clusters. This results in a diversified portfolio hence the company enjoys diversification benefits.

4. Conclusion and Recommendations

4.1. Conclusion

The choice of distances for clustering is very crucial as they can vary depending on the problem at hand. Comparison of the results obtained by different cluster analysis methods result different dendrograms and that the cluster analysis should be used. This problem can be surmounted by comparing the cophenetic distances to the initial distances computed as suggested in this manuscript. Just as we suggested, an insurance company can employ the use of tail dependence index to approximate the proportion of retentions in the case of reinsurance arrangements. Finally, it can be observed that with the dendrograms one can choose the number of efficient divisions in the company by just moving up or down the dendrogram tree.

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4.2. Recommendations

We recommend to all investors establishing general insurance business to first consider the dependence structure so as to arrive at a diversified portfolio in order to benefit from diversification benefits. The business classes that form their own individual clusters like the Aviation, Miscellaneous, Marine and Transit should be given special attention when a company engages in them as they present peculiar characteristics within themselves. We do also recommend that the insurance regulator uses the methods outlined in this study in order to understand the dependencies between insurance classes for advisory purposes. This is due to the fact that there may be no single insurance company that operates all the insurance classes for it to have sufficient data. Finally, the proposed algorithm is long and tedious but this can be made easier by having dedicated computer software.

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