Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.4, No.12, 2014

Hadamard Product Of Meromorphic p-valent Functions with Negative Coefficients

Moa'ath N. Oqielat*

Department of Mathematics, Faculty of Science
Al-Balqa Applied University, P.O.Box: Al-Salt 19117, Jordan
* E-mail: moaath.oqielat@bau.edu.jo

Abstract

In this research we introduce the new class $P_p^*(\alpha, \beta, \xi)$ of meromorphic p-valent functions with negative coefficient. Sharp results concerning coefficient inequalities, growth and distortion, radii of starlikeness and convexity and the extreme points for the class $P_p^*(\alpha, \beta, \xi)$ are determined. Furthermore it is shown that the class $P_p^*(\alpha, \beta, \xi)$ is closed under convex linear combinations.

keywords: Analytic functions, Meromorphic functions, P-valent functions and Starlike and convex functions

1 Introduction

Let S_p (p a fixed integer greater that 0) denote the class of functions of the form $f(z) = z^p + \sum_{n=1}^{\infty} a_{p+n} z^{p+n}$ that are holomorphic and p-valent in a punctured disc $E = \{0 < |z| < 1\}$. Further let T_P denote the subclass of S_P consisting of function that can be expressed in the form

$$f(z) = z^{-p} - \sum_{n=1}^{\infty} |a_{p+n}| z^{p+n}. \tag{1}$$

A function $f \in T_P$ is in $P_p^*(\alpha, \beta, \xi)$ if and only if

$$\left| \frac{(f')^{1-p} - p}{2\xi\Big((f')^{1-p} - \alpha\Big) - \Big((f')^{1-p} - p\Big)} \right| < \beta,$$

where $|z| < 1$, $0 \le \alpha < \frac{p}{2\xi}$, $\quad 0 < \beta \le 1 \quad \frac{1}{2} < \xi \le 1$.
Such type of study was carried out by Aouf (1) for $P_p^*(\alpha, \beta)$. We note that $P_1^*(\alpha) = P_1^*(0, \alpha, 1)$ is precisely the class of function in E studied by Caplinger (2). The class $P_1^*(\alpha, 1, \beta) = P_1^*(\alpha, \beta)$ is the class of holomorphic function discussed by Juneja-Mogra (4). Gupta-Jain (3) studied the family of holomorphic univalent functions that have the form $f(z) = z - \sum_{n=2}^{\infty} |a_n| z^n$ and satisfy the condition

$$\left| \frac{f'(z) - 1}{f'(z) + (1 - 2\alpha)} \right| < \beta, \qquad (0 \le \alpha < 1, 0 < \beta \le 1).$$

Kulkarni (5) has studied above mentioned properties for the functions having Taylor series expansion of the type $f(z) = z + \sum_{n=2}^{\infty} a_n z^n$.

For a functions $f \in T_P$ given by (1.1) and $g \in T_P$ given by

$$f(z) = z^{-p} - \sum_{n=1}^{\infty} |b_{p+n}| z^{p+n}. \tag{2}$$

We defined the Hadamard product (convolution) of f and g by

$$h(z) = (f * g)(z) = z^{-p} - \sum_{n=1}^{\infty} |a_{p+n}||b_{p+n}| z^{p+n} = (g * f)(z). \tag{3}$$

Many important properties and characteristics of various interesting subclasses of the class T_P of meromorphic p-valent functions, were studied by Srivastava et.al.(7), Aouf et.al.(8), Mogra(9), Kulkarni et.al.(10), Moa'ath et.al.(11), Saibah and Maslina(12), Ghanim(13), Kamali(14) and Makinde(15).

A function given by (1.3) is said to be a member of the class $P_p^*(\alpha, \beta, \xi)$ *if and only if,*

$$\left| \frac{(f * g)'^{1-p} - p}{2\xi\Big((f * g)'^{1-p} - \alpha\Big) - \Big((f * g)'^{1-p} - p\Big)} \right| \le \beta, \tag{4}$$

where $0 \le \alpha < \frac{p}{2\xi}$, $\quad 0 < \beta \le 1$, $\quad \frac{1}{2} < \xi \le 1$, $\quad$ for all $\quad z \in D = |z| < 1$.

In this paper, sharp results concerning coefficients, distrotion theorem and the radius of convexity for the class $P_p^*(\alpha, \beta, \xi)$ are determined using Hadamard product. Finally we prove that the class $P_p^*(\alpha, \beta, \xi)$ is closed under the arithmetic mean and convex linear combinations.

2 Coefficient Inequalities

In this section, we provide a sufficient condition for a function h, analytic in D to be in $P_p^*(\alpha, \beta, \xi)$.

Theorem 2.1 *A function* $h(z)$ defined by (1.3) *is in the class* $P_p^*(\alpha, \beta, \xi)$, *if and only if,*

$$\sum_{n=1}^{\infty} (p+n)[1 + \beta(2\xi - 1)]|a_{p+n}||b_{p+n}| \le 2\beta\xi(p - \alpha), \tag{5}$$

where $0 \le \alpha < \frac{p}{2\xi}$, $\quad 0 < \beta \le 1 \quad \frac{1}{2} < \xi \le 1$, $\quad$ for all $\quad z \in E$.

Proof. $\Rightarrow$Assume that $|z| = 1$, and $h(z) \in P_p^*(\alpha, \beta, \xi)$, then

$$\begin{aligned} & \left| \frac{(f * g)'^{1-p} - p}{2\xi\Big((f * g)'^{1-p} - \alpha\Big) - \Big((f * g)'^{1-p} - p\Big)} \right| \\ = & \left| \frac{-\sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}|z^n}{2\xi(p-\alpha) - \sum_{n=1}^{\infty}(2\xi - 1)(p+n)|a_{p+n}||b_{p+n}|z^n} \right| \\ \le & \frac{\sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}|}{2\xi(p-\alpha) - \sum_{n=1}^{\infty}(2\xi - 1)(p+n)|a_{p+n}||b_{p+n}|} \le \beta \end{aligned}$$

that is

$$\sum_{n=1}^{\infty} (p+n)[1 + \beta(2\xi - 1)]|a_{p+n}||b_{p+n}| \le 2\beta\xi(p - \alpha)$$

$\Leftarrow$ Conversely, we assume that

$$\sum_{n=1}^{\infty}(p+n)[1+\beta(2\xi-1)]|a_{p+n}||b_{p+n}| \leq 2\beta\xi(p-\alpha).$$

To show $h \in P_p^*(\alpha,\beta,\xi)$, we want to show that (1.4) satisfied.

$$\left|\frac{(f*g)'^{1-p}-p}{2\xi\Big((f*g)'^{1-p}-\alpha\Big)-\Big((f*g)'^{1-p}-p\Big)}\right|$$

$$=\left|\frac{-\sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}|z^n}{2\xi(p-\alpha)-\sum_{n=1}^{\infty}(2\xi-1)(p+n)|a_{p+n}||b_{p+n}|z^n}\right| \leq \beta.$$

Since $|Re(z)| \leq |z|$ for all z we have

$$Re\left[\frac{\sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}|z^n}{2\xi(p-\alpha)-\sum_{n=1}^{\infty}(2\xi-1)(p+n)|a_{p+n}||b_{p+n}|z^n}\right] \leq \beta$$

select the value of z on the real axis so that $(f*g)'(z)z^{1-p}$ is real. By simplifying the denominator in the above expression we get

$$\sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}|z^n \leq 2\beta\xi(p-\alpha)-\sum_{n=1}^{\infty}\beta(2\xi-1)(p+n)|a_{p+n}||b_{p+n}|z^n.$$

Letting $z \to 1$ through real values, we obtain

$$\sum_{n=1}^{\infty}(p+n)[1+\beta(2\xi-1)]|a_{p+n}||b_{p+n}| \leq 2\beta\xi(p-\alpha),$$

then

$$\left|\frac{(f*g)'^{1-p}-p}{2\xi\Big((f*g)'^{1-p}-\alpha\Big)-\Big((f*g)'^{1-p}-p\Big)}\right| \leq \beta.$$

so that $h \in P_p^*(\alpha,\beta,\xi)$.

the result is sharp for a function h of the form

$$h_{p+n}(z)=(f*g)(z)=z^{-p}-\frac{2\beta\xi(p-\alpha)}{(p+n)\Big[1+\beta(2\xi-1)\Big]}z^{p+n} \qquad (n \geq 1) \qquad (6)$$

corollary 2.1 Let the function $h(z)$be defined by (1.3). If $(h)(z) \in P_p^*(\alpha, \beta, \xi)$, then

$$|a_{p+n}||b_{p+n}| \leq \frac{2\beta\xi(p-\alpha)}{(p+n)[1+\beta(2\xi-1)]} \qquad (n \geq 1). \tag{7}$$

the result is sharp for the function h_{p+n} given by (2.2).

3 Distortion Theorem

A distortion property for functions h in the class $P_p^*(\alpha, \beta, \xi)$, is given as follows:

Theorem 3.1. *If the function $(h)(z)$ defined by (1.3) is in the class $P_p^*(\alpha, \beta, \xi)$, then for $0 < |z| = r < 1$, we have*

$$r^p - \frac{2\beta\xi(p-\alpha)}{(p+1)[1+\beta(2\xi-1)]} r^{p+1} \leq |(h)(z)| \leq r^p + \frac{2\beta\xi(p-\alpha)}{(p+1)[1+\beta(2\xi-1)]} r^{p+1} \tag{8}$$

with equality for

$$h_{p+1}(z) = z^p - \frac{2\beta\xi(p-\alpha)}{(p+1)[1+\beta(2\xi-1)]} z^{p+1} \qquad (z = ir, r) \tag{9}$$

and

$$pr^{p-1} - \frac{2\beta\xi(p-\alpha)}{[1+\beta(2\xi-1)]} r^p \leq \left|(h)^{'}(z)\right| \leq pr^{p-1} + \frac{2\beta\xi(p-\alpha)}{[1+\beta(2\xi-1)]} r^p \tag{10}$$

with equality for,

$$h_{p+1}(z) = z^p - \frac{2\beta\xi(p-\alpha)}{(p+1)[1+\beta(2\xi-1)]} z^{p+1} \qquad (z = \pm ir, \pm r)$$

Proof. Since $h \in P_p^*(\alpha, \beta, \xi)$, Theorem 2.1 yields the inequality

$$\sum_{n=1}^{\infty} |a_{p+n}||b_{p+n}| \leq \frac{2\beta\xi(p-\alpha)}{(p+n)[1+\beta(2\xi-1)]}. \tag{11}$$

Thus, for $|z| = r < 1$, and making use of (2.1) we have

$$\begin{aligned}
\left|(h)(z)\right| &= \left|z^p - \sum_{n=1}^{\infty} |a_{p+n}||b_{p+n}| z^{p+n}\right| \\
&\leq r^p + \frac{2\beta\xi(p-\alpha)}{(p+n)[1+\beta(2\xi-1)]} r^{p+1} \quad \text{(substitute in (3.4) when } n=1) \\
&\leq r^p + \frac{2\beta\xi(p-\alpha)}{(p+1)[1+\beta(2\xi-1)]} r^{p+1}.
\end{aligned}$$

And

$$\begin{aligned}\left|(h)(z)\right| &= \left|z^p - \sum_{n=1}^{\infty}|a_{p+n}||b_{p+n}|z^{p+n}\right|, \\ &\geq r^p + \frac{2\beta\xi(p-\alpha)}{(p+n)[1+\beta(2\xi-1)]}r^{p+1} \quad \text{(substitute in (3.4) when } n=1) \\ &\geq r^p - \frac{2\beta\xi(p-\alpha)}{(p+1)[1+\beta(2\xi-1)]}r^{p+1}.\end{aligned}$$

Also from Theorem 2.1, it follows that

$$\sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}|z^{p+n} \leq \frac{2\beta\xi(p-\alpha)}{[1+\beta(2\xi-1)]}. \tag{12}$$

thus

$$\begin{aligned}\left|(h)'(z)\right| &= \left|pz^{p-1} - \sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}|z^{p+n-1}\right| \\ &\leq pr^{p-1} + \frac{2\beta\xi(p-\alpha)}{[1+\beta(2\xi-1)]}r^p.\end{aligned}$$

and

$$\begin{aligned}\left|(h)(z)\right| &= \left|pz^{p-1} - \sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}|z^{p+n-1}\right| \\ &\geq pr^{p-1} - \frac{2\beta\xi(p-\alpha)}{[1+\beta(2\xi-1)]}r^p.\end{aligned}$$

Hence completes the proof of Theorem 3.1.

Theorem 3.2. *Let $(h)(z) \in P_p^*(\alpha,\beta,\xi)$. Then the disc $|z| < 1$ is mapped on to a domain that contains the disc*

$$|w| < \frac{(p+1)+\beta[(2\xi-1)+2\xi\alpha]}{(p+1)[1+\beta(2\xi-1)]}.$$

proof. The result follows upon letting $r \to 1$in (3.3). that is

$$\begin{aligned}|w| &< 1 - \frac{2\beta\xi(p-\alpha)}{(p+1)[1+\beta(2\xi-1)]} \\ &< \frac{(p+1)+(p+1)\beta(2\xi-1)-2\beta\xi(p-\alpha)}{(p+1)[1+\beta(2\xi-1)]}, \\ &< \frac{(p+1)+\beta[(2\xi-1)+2\xi\alpha]}{(p+1)[1+\beta(2\xi-1)]}.\end{aligned}$$

4 Radii of Starlikeness and Convexity

The radii of starlikeness and convexity for the class $P_p^*(\alpha,\beta,\xi)$, is given by the following theorem:

Theorem 4.1. *If the function $(h)(z)$ defined by (1.3) is in the class $P_p^*(\alpha,\beta,\xi)$, then $(h)(z)$ is starlike in the disk $|z| < r(p,\alpha,\beta,\xi)$, where $r(p,\alpha,\beta,\xi)$,is the largest value for which*

$$r = r(p,\alpha,\beta,\xi) = \inf_{n\in N}\left(\frac{p[1+\beta(2\xi-1)]}{2\beta\xi(p-\alpha)}\right)^{\frac{1}{n}} \qquad (n = 1,2,3,...)$$

The result is sharp for functions $h_{p+n}(z)$ given by (2.2).

Proof. It suffices to show that

$$\left|\frac{zh'(z)}{h(z)} - p\right| \le p,$$

for $|z| < 1$, we have

$$\begin{aligned}\left|\frac{zh'(z)}{h(z)} - p\right| &\le \left|\frac{-\sum_{n=1}^{\infty} n|a_{p+n}||b_{p+n}|z^{p+n}}{z^p - \sum_{n=1}^{\infty}|a_{p+n}||b_{p+n}|z^{p+n}}\right| \\ &\le \frac{\sum_{n=1}^{\infty} n|a_{p+n}||b_{p+n}||z|^{p+n}}{|z|^p - \sum_{n=1}^{\infty}|a_{p+n}||b_{p+n}||z|^{p+n}} \le p. \qquad (13)\end{aligned}$$

The inquality (13) above holds true if

$$\sum_{n=1}^{\infty} n|a_{p+n}||b_{p+n}||z|^{p+n} \le p|z|^p - p\sum_{n=1}^{\infty}|a_{p+n}||b_{p+n}||z|^{p+n}$$

and it follows that

$$\sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}||z|^n \le \sum_{n=1}^{\infty}\frac{2\beta\xi(p-\alpha)}{[1+\beta(2\xi-1)]}|z|^n \le p,$$

thus $h(z)$ is starlike if,

$$\frac{2\beta\xi(p-\alpha)}{[1+\beta(2\xi-1)]}|z|^n \le p, \qquad n = 1,2,3,...$$

then we have,

$$r(p,\alpha,\beta,\xi) = \inf_{n\in N}\left(\frac{p[1+\beta(2\xi-1)]}{2\beta\xi(p-\alpha)}\right)^{\frac{1}{n}} \qquad (n = 1,2,3,...)$$

as required.

Theorem 4.2. *If the function $(h)(z)$ defined by (1.3) is in the class $P_p^*(\alpha,\beta,\xi)$, then $(h)(z)$ is convex in the disk $|z| < r(p,\alpha,\beta,\xi)$, where $r(p,\alpha,\beta,\xi)$,is the largest value for which*

$$r(p,\alpha,\beta,\xi) = \inf_{n\in N}\left(\frac{p^2[1+\beta(2\xi-1)]}{(p+n)2\beta\xi(p-\alpha)}\right)^{\frac{1}{n}} \qquad (n = 1,2,3,...)$$

The result is sharp for functions $h_{p+n}(z)$ given by (2.2).

Proof. It suffices to show that

$$\left|\left(1+\frac{zh''(z)}{h'(z)}\right)-p\right| \le p.$$

for $|z|<1$, we have

$$\begin{aligned}
\left|\left(1+\frac{zh''(z)}{h'(z)}\right)-p\right| &= \left|\frac{zh''(z)+(1-p)h'(z)}{h'(z)}\right| \\
&= \left|\frac{\sum_{n=1}^{\infty} n(p+n)|a_{p+n}||b_{p+n}|z^n}{p-\sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}|z^n}\right| \\
&\le \frac{\sum_{n=1}^{\infty} n(p+n)|a_{p+n}||b_{p+n}||z|^n}{p-\sum_{n=1}^{\infty}(p+n)|a_{p+n}||b_{p+n}||z|^n} \le p. \qquad (14)
\end{aligned}$$

The inquality (14) above holds true if

$$\sum_{n=1}^{\infty}\left(\frac{p+n}{p}\right)^2 |a_{p+n}||b_{p+n}||z|^n \le 1.,$$

and it follows that

$$\sum_{n=1}^{\infty}\left(\frac{p+n}{p}\right)^2 \frac{2\beta\xi(p-\alpha)}{[1+\beta(2\xi-1)]}|z|^n \le 1.$$

then $(h)(z)$ is convex if,

$$\left(\frac{p+n}{p}\right)^2 |z|^n \le \frac{(p+n)[1+\beta(2\xi-1)]}{2\beta\xi(p-\alpha)},$$

then we have,

$$r(p,\alpha,\beta,\xi) = \inf_{n\in N}\left(\frac{p^2[1+\beta(2\xi-1)]}{(p+n)2\beta\xi(p-\alpha)}\right)^{\frac{1}{n}} \qquad (n=1,2,3,...)$$

as required.

5 Convex Linear Combination

Our next result involves a linear combination of function h of the type (1.3).

Theorem 5.1. Let

$$h_p(z) = z^p, \qquad (15)$$

and

$$h_{p+n}(z) = z^p - \frac{2\beta\xi(p-\alpha)}{(p+n)[1+\beta(2\xi-1)]} z^{p+n}, \qquad (n \geq 1) \tag{16}$$

then $h \in P_p^*(\alpha, \beta, \xi)$ if and only if it can be expressed in the form

$$h(z) = \sum_{n=1}^{\infty} \lambda_{p+n} h_{p+n}(z), \tag{17}$$

$$\text{where} \qquad \lambda_{p+n} \geq 0 \qquad \text{and} \qquad \sum_{n=1}^{\infty} \lambda_{p+n} = 1.$$

Proof.$\Leftarrow$ From (6.1),(6.12) and (6.13), it is easily seen that

$$\begin{aligned} h(z) &= \sum_{n=0}^{\infty} \lambda_{p+n} h_{p+n}(z), \\ &= z^p - \sum_{n=1}^{\infty} \frac{2\beta\xi(p-\alpha)\lambda_{p+n}}{(p+n)[1+\beta(2\xi-1)]} z^{p+n} \end{aligned}$$

then it follows that

$$\sum_{n=1}^{\infty} \frac{(p+n)[1+\beta(2\xi-1)]}{2\beta\xi(p-\alpha)} \frac{2\beta\xi(p-\alpha)\lambda_{p+n}}{(p+n)[1+\beta(2\xi-1)]} = \sum_{n=1}^{\infty} \lambda_{p+n} = 1 - \lambda_p \leq 1$$

it follows from Theorem 2.1 that the function $h \in P_p^*(\alpha, \beta, \xi)$.

$\Leftarrow$ Conversely, let us suppose that $h \in P_p^*(\alpha, \beta, \xi)$. Then

$$\left|a_{p+n}\right|\left|b_{p+n}\right| \leq \frac{2\beta\xi(p-\alpha)}{(p+n)[1+\beta(2\xi-1)]} \qquad (n \geq 0).$$

Setting

$$\lambda_{p+n} = \frac{(p+n)[1+\beta(2\xi-1)]}{2\beta\xi(p-\alpha)} \left|a_{p+n}\right|\left|b_{p+n}\right| \qquad (n \geq 0),$$

It follows that

$$h(z) = \sum_{n=0}^{\infty} \lambda_{p+n} h_{p+n}(z)$$

this complete the proof of theorem.

Corollary 5.1 The extreme points of $P_p^*(\alpha, \beta, \xi)$ are the function

$$h_p(z) = z^p, \tag{18}$$

and

$$h_{p+n}(z) = z^p - \frac{2\beta\xi(p-\alpha)}{(p+n)[1+\beta(2\xi-1)]} z^{p+n}. \qquad n \geq 1 \tag{19}$$

Theorem 5.2. *The class $P_p^*(\alpha,\beta,\xi)$ is closed under convex linear combinations.*
Proof. Suppose that the functions h_1 and h_2 defined by,

$$h_i(z) = z^p - \sum_{n=1}^{\infty} |a_{p+n,i}||b_{p+n,i}|z^{p+n} \qquad (i = 1, 2; z \in E) \tag{20}$$

are in the class $P_p^*(\alpha,\beta,\xi)$.

Setting $h(z) = \mu h_1(z) + (1-\mu)h_2(z)$ we want to show that $h \in P_p^*(\alpha,\beta,\xi)$. For $(0 \le \mu \le 1)$, we can write

$$h(z) = z^p - \sum_{n=1}^{\infty} \Big\{\mu|a_{p+n,1}||b_{p+n,1}| + (1-\mu)|a_{p+n,2}||b_{p+n,2}|\Big\} z^{p+n}, \qquad (z \in D)$$

In view of theorem 2.1, we have

$$\begin{aligned} \sum_{n=1}^{\infty} & (p+n)[(1+\beta(2\xi-1)]\Big\{\mu|a_{p+n,1}||b_{p+n,1}| + (1-\mu)|a_{p+n,2}||b_{p+n,2}|\Big\} z^{p+n}, \\ = & \ \mu\sum_{n=1}^{\infty}(p+n)[(1+\beta(2\xi-1)]|a_{p+n,1}||b_{p+n,1}| + (1-\mu)\sum_{n=1}^{\infty}|a_{p+n,2}||b_{p+n,2}|, \\ \le & \ \mu\Big\{2\beta\xi(p-\alpha)\Big\} + (1-\mu)\Big\{2\beta\xi(p-\alpha)\Big\} = 2\beta\xi(p-\alpha), \end{aligned}$$

which show that $h \in P_p^*(\alpha,\beta,\xi)$. Hence the theorem.

References

[1] M.K.Aouf, Certain classes of p-valent functions with negative coefficients II, Indian J.Pure appl. Math. 19(8), 1988, 761-767.

[2] T.R.Caplinger, On certain classes of analytic functions, Ph.D. dissertation, University of Mississipi, 1972.

[3] V.P.Gupta, P.K.Jain, Certain classes of univalent functions with negative coefficients, Bull.Austr.Math.Soc 15, 1976, 467-473.

[4] O.P.Juneja, M.L.Mogra, Radii of convexity for certain classes of univalent analytic functions, Pacific Jour.Math. 78, 1978, 359-368.

[5] S.R.Kulkarni, Some problems connected with univalent functions, Ph.D. Thesis, Shivaji University, Kolhapur 1981, unpublished.

[6] P.L.Duren, Univalent functions. In Grundlehren der Mathematischen Wissenschaften, 259, Springer-Verlag, New York, (1983).

[7] H. M. Srivastava, H. M. Hossen and M. K. Aouf., A unified presentation of some classes meromorphically multivalent functions, Comp. Math. with App. 38(1999), 63-700.

[8] M. K. Aouf, On a class meromorphic multivalent functions with positive coefficients, Math. Japan. 35, (1990), 603-608.

[9] M. L. Mogra, Meromorphic multivalent functions with positive coefficients I, Math. Japan. 35, (1990), 1-11.

[10] S. R. Kulkarni, U. H. Naik and H. M. Srivastava, A certain class of meromorphically p-valent, quasi-convex functions, Pan Amer. Math. J. 8 (1998), 57-64.

[11] Moa'ath and Darus, Certain class of meromorphic p-valent functions with positive coefficients, Tamkang Jornal of Mathematics. 37, (2006), 251-260.

[12] Saibah and Maslina, On subclasses of p-valent functions with fixed second negative coefficients, Journal of Analysis and Applications. 3(1), (2006), 19-30.

[13] F. Ghanim and M. Darus On a certain subclass of meromorphic univalent functions wih fixed second positive coefficients, surveys in mathematics and applications. 5, (2010), 5 49-60.

[14] M. Kamali, K. Suchithra, B.A. Stephen and A. Gangadharan, On certain subclass of meromorphic p −valent functions with negative coefficients. General Mathematics. (2011), 19(1), 109-122.

[15] D.O Makinde *On a Certin Family of Meromorphic p−valent Functions with Negative Coefficients*, Mathematical Theory and Modeling. 2, (2012).