Estimation of Parameters of Linear Econometric Model and the Power of Test in the Presence of Heteroscedasticity Using Monte-Carlo Approach

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Abstract

This paper is concerned with the estimation of parameters of linear econometric model and the power of test in the presence of heteroscedasticity using Monte-Carlo approach. The Monte Carlo approach was used for the study in which random samples of sizes 20, 50 and 100, each replicated 50 times were generated. Since the linear econometric model was considered, a fixed X variable for the different sample sizes was generated to follow a uniform distribution while 50 replicates of the stochastic error term for different sample sizes followed a normal distribution. Two functional form of heteroscedasticity $h(x) = X \quad and \quad h(x) = X^{1/2}$ were introduced into the econometric model with the aim of studying the behaviour of the parameters to be estimated. 50 replicates of the dependent variable for each sample size was generated from the model $Y = \alpha + \beta x_i + u_i\left(h(x)\right)$ where the parameters, $\alpha \quad and \quad \beta$ were assumes to be 0.5 and 2.0 respectively. The Ordinary Least Squares (OLS) and the Generalized Least Squares (GLS) estimators were studied to identify which is more efficient in the presence of the two functional forms of heteroscedasticity considered. Both estimators were unbiased and consistent but none was convincingly more efficient than the other. The power of test was used to examine which test of heteroscedasticity (i.e., Glejser, Breusch-Pagan and White) is most efficient in the detection of any of the two forms of heteroscedasticity using different sample sizes. Glejser test detects heteroscedasticity more efficiently even in small sample sizes while White test is not as efficient when sample size is small compared to when the sample size is large.

Keywords: Heteroscedasticity, Monte Carlo, Power of Test, Ordinary Least Squares Estimator, Generalized Least Squares Estimator, Breusch-Pagan test, Glejser test, White test, Bias, Variance, Root Mean Square Error

1. Introduction

Linear statistical models for regression, analysis of variance and experimental design are widely used today in business administration, economics, engineering and the social, health and biological sciences. Successful application of these models requires a sound understanding of both the underlying theory and the practical problems encountered in using the models in real life situations. Econometric models are statistical models used in econometrics to specify the statistical relationship that is believed to hold between the various economic quantities that pertain to a particular economic phenomenon under study. An econometric model can be derived from a deterministic economic model by allowing for uncertainty or from an economic model which itself is stochastic. However, it is also possible to use econometric models that are not tied to any specific economic theory. In econometrics, it is presupposed that the quantities being analyzed can be treated as random variables. An econometric model then is a set of joint probability distribution to which the true joint probability distribution of the variables under study is supposed to belong. In the case in which the elements of this set can be indexed by a finite number of real value parameters, the model is called a parametric model; otherwise it is a non-parametric or semi-parametric model. A large part of econometrics is the study by methods for selecting models, estimating them and carrying out inference on them. The most common econometric models are structural, in that they convey causal and counterfactual information and are used for policy evaluation. For example, an equation modeling consumption spending based on income could be used to see what consumption would be contingent on any of various hypothetical levels of income, only one of which (depending on the choice of a fiscal policy) will end up actually occurring. An important econometric model to be considered is the Linear Regression. Since econometric models deal with economic theories of real life situations and can be treated as random variables, it will be very important to consider the variances of the estimates obtained whether it will have constant variances (Homoscedasticity) or different variances (Heteroscedasticity). Since it is not always possible to have constant variances in a real life situation, this necessitate the in depth study of the concept of heteroscedasticity and the behaviour of estimates of parameter in the presence of heteroscedasticity.

2 Material and Methods

In this section attention is focused on the specification of the models used in the study, the design of the Monte-Carlo experiment is clearly spelt out. Details of the procedure for evaluating the performance of the heteroscedasticity tests considered are stated. Also, the procedures used in generating the data and the associated error terms are discussed.

2.1 The Monte – Carlo Approach

Under the Monte Carlo approach, the experimenter specifies a model and assumes specific numeric values to its parameters. He also specifies the distribution of the error term of the model. He then makes random selection from the distribution to obtain values for the error term.

Depending on the model, the experimenter selects values for the independent variable(s) Xs and given the chosen values of the error term, he solves the equation of the model and obtains values for the dependent variable Y. for each randomly drawn value of the error term, a new generated value of the dependent variable is obtained.

Using this procedure, the experimenter forms sample of generated observations of the dependent variable, which together with the X values, the generated error terms and the assumed specification are used to estimate the true parameter values or carry out the appropriate test of hypothesis. Such experiments are repeated many times and the result of the tests are summarized and used in drawing general conclusions about each test.

Each author who has introduced a new test usually evaluated its performance or relative performance vis-à-vis other test under various scenarios (varying sample size, replication numbers, stochastic terms and other model characteristics) using the Monte Carlo approach.

Goldfield – Quandt (1965), in developing their test, varied the number of observations, using two sample size (30 and 60). They also varied the number of omitted observations as well as their independent variable X by assuming different mean μ_x and standard deviations σ_x. For each combination of μ_x and σ_x, one sample of X was generated and for each of such sample, 100 samples of 30 (or 60) disturbance term values were generated from N(0,1) and corresponding samples of y values were calculated. The relative frequency (in 100 trials) of cases in which the correct statistical decision is reached is recorded as the estimate of the power of the test.

Glejser (1969), using three sample sizes (20, 30, 60) and a fixed standard deviation of their independent variable σ_x, varied the form of heterocedasticity. Each of the three sample sizes was replicated 100 times and their dependent variable (y) was generated using the functional form:

$$y = \beta_0 + \beta_1 X_1 + u_i \qquad (2.1.1)$$

Las-Forsberg et-al (1999) used two sample sizes (30 and 120) and generated their disturbance terms u_i from $N\left(0, X^{\delta}\right)$ where $0.0 < \delta \leq 2.8$ with 0.4 grid and they assumed

$$y = u_i \qquad (2.1.2)$$

for their endogenous variable; using equally spaced regressors.

Udoko (1990) used three sample sizes $n(20, 40, 60)$ and varying, the form of heteroscedasticity generated his disturbance terms from U (0,1) using linear congruential method, i.e.

$$u_n = \left(A u_{n-1} + C\right)\left(Mod \quad M\right) \qquad (2.1.3)$$

$n = 1, 2, \ldots, M$ where A is the multiplier $0 \leq A \leq M$ and

C is the increment $0 \leq A \leq M$, the uniform terms were later transformed into standard normal errors.

In this work, we use three sample sizes ($20, 50, 100$), equally replicated (r=50 replicates) and two forms of heteroscedasticity. At each combination of sample size, replication and form of heteroscedasticity, we undertake a comparative study of the performance of the three tests for heteroscedasticity using the power of the tests as calculated from the result of the Monte-Carlo experiment.

2.2 The Design of the Monte-Carlo Experiment

2.2.1 Basic Model

In this study, we use the model

$$y_i = \alpha + \beta x_i + u_i \qquad (2.2.1)$$

where the independent variable X is invariant and identical in repeated samples, the values of α and β are arbitrarily chosen because all test statistics used are expected to produce the same result no matter which value we choose and u_i, the disturbance terms are chosen from normal distribution. In other words, X is said to be non-stochastic and Y is stochastic because U is stochastic

The use of an intrinsically linear model is to reduce the complexity of the estimation of parameters. Parameters of intrinsically non-linear models can only be estimated using algorithms, which yield approximation values of the parameter after numerous iterations. Also all the known tests for heteroscedasticity are based on intrinsically linear models.

2.2.2 Choice of Parameters and the Independent Variable X

In the past, computer programmes were tailed to specific data generating process for the Monte-Carlo experiment, that is all the necessary constraints and impositions on the data to be generated were spelt out in the computer programme by researchers. Consequently, the algorithms underlying the data generation process have had to be documented and reported in detail. This complicated the verification of data used in such works

Today, softwares are available in their large numbers that can attend to all these constraints and restrictions and will produce what was obtained in the past with relatively higher speed and precision.

For the first round of our experiment, three equally spaced values of regressor sets are chosen to indicate three different samples of sizes n= 20, 50, and 100. The regressor sets are

$$X_I = 1, 2, \ldots, 20$$
$$X_{II} = 1, 2, \ldots, 50$$
$$X_{III} = 1, 2, \ldots, 100$$

The sample sizes chosen are multiples of 'three' to allow for unambiguous one-third of observations to be omitted in the case of Goldfeld-Quandt test which is samples often encountered in applied works.

Given the equation:

$$y_i = \alpha + \beta x_i + u_i \qquad (2.2.1)$$

y could not be determined except values are set for α and β, and u_i , we therefore arbitrarily set 0.5 and 2.0 for α and β respectively.

The Microsoft Excel package was used to generate random deviates which were later standardized as u_i. We then resort to the equation

$$y_i = \alpha + \beta x_i + u_i$$

to determine the values of y . However, the following three transformations are made depending on the form of heteroscedasticity to be introduced using

$$y^* = \alpha + \beta x_i + u_i (h(x)) \qquad (2.2.2)$$

where $h(x_i)$ is the form of heteroscedasticity.

The two forms of $h(x)$ used in this study are:

1. X
2. $X^{1/2}$

The main criterion on the choice of the form of x is to consider transformations of dependent variables most frequently used in applied regression analysis.

2.2.3 The Generation of Sample Data

The disturbance terms to be used are generated as follows:

1 Generate 20 random deviates using Microsoft Excel package

2 For a replication of size 50, repeat step1, 50 times, to obtain 50 different random samples, each of size 20.

3 Standardize, each of the 50 replications of random deviates to obtain 50 groups of different standard normal deviates of size 20 each having mean 0 and variance 1.

4 Values of the standardized deviates u_i obtained in step 3 are used to calculate

$$y^* = \alpha + \beta x_i + U_i (h(x_i)) \qquad (2.2.2)$$

For other sample sizes and replications, steps 1 to 3 are repeated to meet their specifications. Step 4 is used according to the desired form of heteroscedasticity $h(x)$.

Consequently, in the sample design for generating the data sets each of the three sample sizes 20, 50 and 100 are replicated 50 times. This is repeated for each of the three specifications of the independent variable X. the generated data set for the specification $h(x)$, when n = 20 for equally spaced regressor set is displayed in table 2.2.1

As shown by (2.2.2), the design of this Monte-Carlo experiment is such that regressor – induced heteroscedasticity is used in generating the regressant. Three transformations of the equally spaced and unequally spaced regressors are assumed. Three tests for homoscedasticity of the error term are then applied to the data generated with the objective of studying their power to detect the introduced heteroscedasticity.

2.3 Results and Discussion

The bias, variances and the Root Mean Square Error for the two estimators, OLS and GLS, in the presence of the two different functional forms of heteroscedasticity will be considered to see what happens as the sample size increases.

Table 1: Bias for Estimators of β

BIAS FOR ESTIMATORS OF β				
HSC	ESTIMATORS	SMPL 20		
		INTERCEPT	SLOPE	SBIAS
X	OLS	0.009998	-0.019226	-0.009228
	GLS	0.009798	-0.019226	-0.009428
SQRT X	OLS	0.016372	-0.027068	-0.010696
	GLS	0.016370	-0.027068	-0.010698
HSC	ESTIMATORS	SMPL 50		
		INTERCEPT	SLOPE	SBIAS
X	OLS	0.011086	-0.028418	-0.017332
	GLS	0.011086	-0.028418	-0.017332
SQRT X	OLS	0.015104	-0.033408	-0.018304
	GLS	0.015104	-0.033408	-0.018304
HSC	ESTIMATORS	SMPL 100		
		INTERCEPT	SLOPE	SBIAS
X	OLS	-0.020190	0.045106	0.024916
	GLS	-0.016191	0.044112	0.027921
SQRT X	OLS	-0.019232	0.048193	0.028961
	GLS	-0.019352	0.048188	0.028836

The bias of the OLS and GLS estimators of β using the three different sample sizes reveals that both estimators are unbiased in the presence of the two functional forms of heteroscedasticity considered. This is clearly seen in the result from the table above as the sum of the bias are very small. Also, from the table 1 above, the sum of the bias tends to reduce as the sample size increases from 20 to 50 but then increased as the sample size increased from 50 to 100. This is observed in the two different functional forms of heteroscedasticity.

Table 2: Variance for Estimators of β

VARIANCE FOR ESTIMATORS OF β				
HSC	ESTIMATORS	SMPL 20		
		INTERCEPT	SLOPE	SVAR
X	OLS	0.019198	0.131505	0.150703
	GLS	0.019141	0.131505	0.150646
SQRT X	OLS	0.039320	0.194280	0.233601
	GLS	0.039321	0.194280	0.233601
HSC	ESTIMATORS	SMPL 50		
		INTERCEPT	SLOPE	SVAR
X	OLS	0.010062	0.085597	0.095660
	GLS	0.010062	0.085597	0.095660
SQRT X	OLS	0.018594	0.121316	0.139910
	GLS	0.018594	0.121316	0.139910
HSC	ESTIMATORS	SMPL 100		
		INTERCEPT	SLOPE	SVAR
X	OLS	0.003854	0.029581	0.033435
	GLS	0.003647	0.028642	0.032289
SQRT X	OLS	0.006724	0.042718	0.049442
	GLS	0.006723	0.042715	0.049438

The variances of the OLS and GLS estimators of β using the three different sample sizes reveals that the sum of variances of both estimators are approximately equal in the presence of the two functional forms of heteroscedasticity considered. Although, the OLS estimator was observed to have a smaller variance at most times. Also, from the table 2 above, the sum of the variances tends to reduce as the sample size increases from 20 to 50 and from 50 to 100. This is observed in the two different functional forms of heteroscedasticity.

Table 3: Root Mean Squares Error for Estimators of β

RMSE FOR ESTIMATORS OF β				
HSC	**ESTIMATORS**	**SMPL 20**		
		INTERCEPT	**SLOPE**	**SRMSE**
X	**OLS**	0.138918	0.363145	0.502063
	GLS	0.138699	0.363145	0.501844
SQRT X	**OLS**	0.198968	0.441603	0.640571
	GLS	0.198970	0.441603	0.640573
HSC	**ESTIMATORS**	**SMPL 50**		
		INTERCEPT	**SLOPE**	**SRMSE**
X	**OLS**	0.100922	0.293947	0.394869
	GLS	0.100922	0.293947	0.394869
SQRT X	**OLS**	0.137195	0.349903	0.487098
	GLS	0.137195	0.349903	0.487098
HSC	**ESTIMATORS**	**SMPL 100**		
		INTERCEPT	**SLOPE**	**SRMSE**
X	**OLS**	0.065282	0.177808	0.243090
	GLS	0.062522	0.174894	0.237416
SQRT X	**OLS**	0.084223	0.212227	0.296450
	GLS	0.084245	0.212219	0.296464

The RMSE of the OLS and GLS estimators of β using the three different sample sizes reveals that the sum of RMSE of both estimators are approximately equal in the presence of the two functional forms of heteroscedasticity considered. Here, the GLS estimator was observed to have a smaller RMSE value at some times (at sample size 20 with heteroscedasticity $h(x) = X$ and at sample size 100 with same functional form of heteroscedasticity). The OLS estimator had the smaller RMSE at sample size 20 and 100 with the functional form of heteroscedasticty $h(x) = X^{1/2}$. Also, from the table 3 above, the sum of the RMSE tends to reduce as the sample size increases from 20 to 50 and from 50 to 100 and this was observed in the two different functional forms of heteroscedasticity.

2.4 Power of Test

Here, the efficiency of three tests in the detection of the functional forms of heteroscedasticity introduced into the model will be considered. The EVIEWS 7.0 was used to obtain these results. For the different sample sizes considered and in the presence of the two functional forms of heteroscedasticity introduced into the model, the test for the detection of heteroscedasticity using the three tests considered are given below. A total of 900 heteroscedasticity test were run, (300 Glejser tests, 300 Breusch-Pagan tests and 300 White tests), for the two forms of heteroscedasticity. A few samples for each of the three tests are presented in the appendix.

Table 4: Summary Table showing the frequency of detection of the three tests in the presence of the two functional forms of heteroscedasticity.

HSC	TESTS	SAMPLES SIZES			OBSERVED TOTAL	EXPECTED TOTAL	PERCENTAGE
		n = 20	n = 50	n = 100			
X	BPG	34	50	50	**134**	**150**	**89%**
	GLEJSER	45	50	50	**145**	**150**	**97%**
	WHITE	17	49	50	**116**	**150**	**77%**
SQRT X	BPG	16	41	50	**107**	**150**	**71%**
	GLEJSER	25	43	50	**118**	**150**	**79%**
	WHITE	9	36	48	**93**	**150**	**62%**

The table above reveals the number of times and the corresponding percentages that each of the three tests considered (Breusch Pagan Test, Glejser and White test) was able to detect the presence of the two forms of heteroscedasticity in the linear econometric model. White test had the least number of times (i.e., 34 out of 50) of correctly detecting the presence of heteroscedasticity for sample size 20 for the two different forms of heteroscedasticity considered while Glejser had the highest value (i.e. 50 out of 50). This pattern is still maintained as the sample size increases from 20 to 50 and from 50 to 100. Also, the number of times that the test detected the presence of heteroscedasticity improved as the sample size increases. From the table, the percentages of number of correct detection of heteroscedasticity for BPG, Glejser and White are 89%, 97% and 77% respectively for functional form of heteroscedasticity $h(x) = X$ and 71%, 79% and 62% respectively for functional form of heteroscedasticity $h(x) = X^{1/2}$. The Glejser test is observed to have the highest frequency in heteroscedasticity detection.

2.5 Conclusion

The two estimators OLS and GLS are both unbiased and could be used since none of the estimator can be convincingly said to be better than the other especially when the form of functional heteroscedasticity is known. Both estimators are asymptotically good in the sense that as the sample sizes increases, the estimates of the two estimators tend towards the true parameter. The Glejser test is the best test among the three tests considered since it consistently detects the presence of the two functional forms of heteroscedasticity in the various sample sizes considered. The White test cannot be reliable in the detection of heteroscedasticity if the sample size is small.

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Appendix

Fig. 1: Heteroscedasticity Test: Glejser

F-statistic	13.21323	Prob. F(1,18)	0.0019
Obs*R-squared	8.466427	Prob. Chi-Square(1)	0.0036
Scaled explained SS	8.225716	Prob. Chi-Square(1)	0.0041

Test Equation:
Dependent Variable: ARESID
Method: Least Squares
Date: 07/06/11 Time: 19:27
Sample: 1 20
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.038061	0.107756	0.353212	0.7280
X1	0.609778	0.167752	3.635000	0.0019

R-squared	0.423321	Mean dependent var	0.376667
Adjusted R-squared	0.391284	S.D. dependent var	0.310484
S.E. of regression	0.242240	Akaike info criterion	0.096866
Sum squared resid	1.056247	Schwarz criterion	0.196440
Log likelihood	1.031336	Hannan-Quinn criter.	0.116304
F-statistic	13.21323	Durbin-Watson stat	2.380648
Prob(F-statistic)	0.001894		

Fig. 2: Heteroscedasticity Test: Glejser

F-statistic	17.23959	Prob. F(1,48)	0.0001
Obs*R-squared	13.21252	Prob. Chi-Square(1)	0.0003
Scaled explained SS	18.96216	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: ARESID
Method: Least Squares
Date: 07/12/11 Time: 06:08
Sample: 1 50
Included observations: 50

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.031961	0.082721	0.386373	0.7009
X1	0.610081	0.146935	4.152058	0.0001

R-squared	0.264250	Mean dependent var	0.323602
Adjusted R-squared	0.248922	S.D. dependent var	0.356493
S.E. of regression	0.308953	Akaike info criterion	0.527925
Sum squared resid	4.581704	Schwarz criterion	0.604406
Log likelihood	-11.19812	Hannan-Quinn criter.	0.557049
F-statistic	17.23959	Durbin-Watson stat	1.981991
Prob(F-statistic)	0.000134		

Fig. 3: Heteroscedasticity Test: Glejser

F-statistic	51.87468	Prob. F(1,98)	0.0000
Obs*R-squared	34.61204	Prob. Chi-Square(1)	0.0000
Scaled explained SS	50.88712	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: ARESID
Method: Least Squares
Date: 07/13/11 Time: 04:05
Sample: 1 100
Included observations: 100

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.044604	0.071431	-0.624435	0.5338
X1	0.925412	0.128486	7.202408	0.0000

R-squared	0.346120	Mean dependent var	0.400522

Adjusted R-squared	0.339448	S.D. dependent var	0.440689
S.E. of regression	0.358167	Akaike info criterion	0.804164
Sum squared resid	12.57181	Schwarz criterion	0.856268
Log likelihood	-38.20821	Hannan-Quinn criter.	0.825251
F-statistic	51.87468	Durbin-Watson stat	2.152390
Prob(F-statistic)	0.000000		

Fig. 4: Heteroscedasticity Test: White

F-statistic	3.571095	Prob. F(2,17)	0.0507
Obs*R-squared	5.916771	Prob. Chi-Square(2)	0.0519
Scaled explained SS	5.074610	Prob. Chi-Square(2)	0.0791

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/06/11 Time: 19:28
Sample: 1 20
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.089837	0.215042	-0.417764	0.6813
X1	0.605692	1.021273	0.593076	0.5609
X1^2	-0.031610	0.943265	-0.033511	0.9737

R-squared	0.295839	Mean dependent var	0.233458
Adjusted R-squared	0.212996	S.D. dependent var	0.348561
S.E. of regression	0.309220	Akaike info criterion	0.627952
Sum squared resid	1.625487	Schwarz criterion	0.777312
Log likelihood	-3.279521	Hannan-Quinn criter.	0.657109
F-statistic	3.571095	Durbin-Watson stat	2.695626
Prob(F-statistic)	0.050724		

Fig. 5: Heteroscedasticity Test: White

F-statistic	6.686476	Prob. F(2,47)	0.0028
Obs*R-squared	11.07528	Prob. Chi-Square(2)	0.0039
Scaled explained SS	36.67321	Prob. Chi-Square(2)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/12/11 Time: 06:08
Sample: 1 50
Included observations: 50

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.244334	0.253867	0.962449	0.3408
X1	-1.491342	1.195926	-1.247018	0.2186

X1^2	2.201805	1.111806	1.980386	0.0535

R-squared	0.221506	Mean dependent var	0.229263
Adjusted R-squared	0.188378	S.D. dependent var	0.620815
S.E. of regression	0.559293	Akaike info criterion	1.733838
Sum squared resid	14.70200	Schwarz criterion	1.848559
Log likelihood	-40.34594	Hannan-Quinn criter.	1.777524
F-statistic	6.686476	Durbin-Watson stat	2.155578
Prob(F-statistic)	0.002783		

Fig. 6: Heteroscedasticity Test: White

F-statistic	14.46955	Prob. F(2,97)	0.0000
Obs*R-squared	22.97864	Prob. Chi-Square(2)	0.0000
Scaled explained SS	41.63895	Prob. Chi-Square(2)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/13/11 Time: 04:06
Sample: 1 100
Included observations: 100

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.068880	0.191117	-0.360406	0.7193
X1	0.344631	0.906473	0.380189	0.7046
X1^2	0.827623	0.892303	0.927513	0.3560

R-squared	0.229786	Mean dependent var	0.352683
Adjusted R-squared	0.213906	S.D. dependent var	0.688562
S.E. of regression	0.610493	Akaike info criterion	1.880441
Sum squared resid	36.15205	Schwarz criterion	1.958596
Log likelihood	-91.02203	Hannan-Quinn criter.	1.912071
F-statistic	14.46955	Durbin-Watson stat	2.291218
Prob(F-statistic)	0.000003		

Fig. 7: Heteroscedasticity Test: Breusch-Pagan-Godfrey

F-statistic	7.560631	Prob. F(1,18)	0.0132
Obs*R-squared	5.915841	Prob. Chi-Square(1)	0.0150
Scaled explained SS	5.073812	Prob. Chi-Square(1)	0.0243

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/06/11 Time: 19:26
Sample: 1 20
Included observations: 20

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.084297	0.133679	-0.630596	0.5362
X1	0.572229	0.208109	2.749660	0.0132

R-squared	0.295792	Mean dependent var	0.233458
Adjusted R-squared	0.256669	S.D. dependent var	0.348561
S.E. of regression	0.300518	Akaike info criterion	0.528018
Sum squared resid	1.625594	Schwarz criterion	0.627591
Log likelihood	-3.280182	Hannan-Quinn criter.	0.547456
F-statistic	7.560631	Durbin-Watson stat	2.692565
Prob(F-statistic)	0.013182		

Fig. 8: Heteroscedasticity Test: Breusch-Pagan-Godfrey

F-statistic	8.908718	Prob. F(1,48)	0.0045
Obs*R-squared	7.827200	Prob. Chi-Square(1)	0.0051
Scaled explained SS	25.91794	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/12/11 Time: 06:08
Sample: 1 50
Included observations: 50

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.161641	0.154238	-1.047996	0.2999
X1	0.817730	0.273970	2.984748	0.0045

R-squared	0.156544	Mean dependent var	0.229263
Adjusted R-squared	0.138972	S.D. dependent var	0.620815
S.E. of regression	0.576064	Akaike info criterion	1.773984

Sum squared resid	15.92881	Schwarz criterion	1.850465
Log likelihood	-42.34959	Hannan-Quinn criter.	1.803108
F-statistic	8.908718	Durbin-Watson stat	2.082459
Prob(F-statistic)	0.004455		

Fig. 9: Heteroscedasticity Test: Breusch-Pagan-Godfrey

F-statistic	28.11890	Prob. F(1,98)	0.0000
Obs*R-squared	22.29555	Prob. Chi-Square(1)	0.0000
Scaled explained SS	40.40113	Prob. Chi-Square(1)	0.0000

Test Equation:
Dependent Variable: RESID^2
Method: Least Squares
Date: 07/13/11 Time: 04:05
Sample: 1 100
Included observations: 100

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-0.205516	0.121666	-1.689179	0.0944
X1	1.160490	0.218848	5.302726	0.0000

R-squared	0.222955	Mean dependent var	0.352683
Adjusted R-squared	0.215026	S.D. dependent var	0.688562
S.E. of regression	0.610058	Akaike info criterion	1.869270
Sum squared resid	36.47268	Schwarz criterion	1.921374
Log likelihood	-91.46352	Hannan-Quinn criter.	1.890358
F-statistic	28.11890	Durbin-Watson stat	2.309894
Prob(F-statistic)	0.000001		