Uncertainty of Position of a Photon and Concomitant and Consummating Manifestation of Wave Effects

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Abstract

A system of uncertainty of the position of the particle (photon, electron etc.,) is investigated. Very observation affects the quantum mechanical reality, and the nature like a coy bride hides herself. Measurement always disturbs the true nature of 'nature' We discuss stability,solutional behaviour, and asymptotic behaviors of the system

Keywords: Uncertainty of the position of the photon, Wave pattern, Duality Theory, Double Slit experiment, Superposition, Dark Photon, Annihilating particles.

INTRODUCTION:

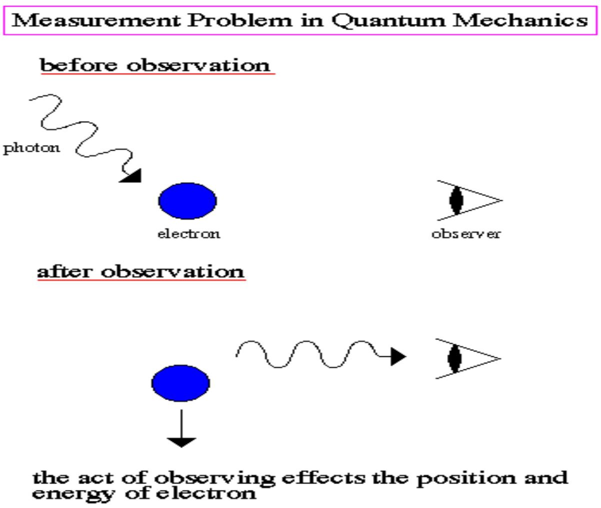
The wave-like properties of light were demonstrated by the famous experiment first performed by Thomas Young in the early nineteenth century. In original experiment, a point source of light illuminates two narrow adjacent slits in a screen, and the image of the light that passes through the slits is observed on a second screen. **The two slit experiment is the key to understanding the microscopic world. Waves can interfere, for light, this will make a series of light and dark band** .The dark and light regions are called interference fringes, the constructive and destructive interference of light waves. So the question is will matter also produce interference patterns. The answer is yes, tested by firing a stream of electrons. **Matter particles, such as electrons, also produce interference patterns due to their wave-like nature so with a high flux of either photons or electrons, the characteristic interference pattern is visible .These characteristic interference pattern is visible Experiment, and cannot isolate the observer or their effects** However, notice that electrons do act as particles, **as do photons**. For example, they make a single strike on a cathode ray tube screen. So if we **lower the number of electrons in the beam** to, say, one per second. Does the interference pattern disappear? **If we lower the intensity of light, or the flux of electrons (the electric current), we should be able to see each photon strike the screen each photon makes a dot on the screen, but where is the interference pattern?** The answer is no, we do see the individual electrons (and photons) strike the screen, and **with time the interference pattern builds up**. Notice that with such a slow rate, each photon (or electron) is not interacting with other photons **to produce** the interference pattern. In fact, the photons are **interacting** with themselves, within their own wave packets for **the interference pattern is still there, it simply takes some time for enough photons, or electrons, to strike the screen to build** (produce) interference **up a recognizable pattern or electrons, through one at a time. So what are the individual particles interfering with? Apparently, themselves.**

Interference, or a wave phenomenon, is still occurring even if we only let the photons or electrons to collide.

What if we do this so slow that only one electron or one photon passes through the slits at a time, then what is interfering with what? I.e. there are not two waves to **destructively and constructively interfere**. It appears, in some strange way, that each photon or electron is interfering with itself. That its wave nature is interfering with its own wave (!).The formation of the interference pattern **requires** the existence of two slits, but how can a single photon passing through one slit **`know'** about the existence of the other slit? We are stuck going back to thinking of each photon as a wave that hits both slits. Or we have to think of the photon as splitting and going through each slit separately (but how does the photon know a pair of slits is coming?). **The only solution is to give up the idea of a photon or an electron having location. The location of a subatomic particle is not defined until it is observed (such as striking a screen).**

ROLE OF THE OBSERVER

The quantum world can be not be perceived directly, but rather through **the use** of instruments. There is a problem with the fact that the act of measuring **disturbs** the energy and position of subatomic particles. This is called the measurement problem. **In order for a particle to interfere with itself, it must pass through both slits. Since the quantum world cannot be observed directly, we are forced to use instruments as extensions of our senses however, quantum entities are so small that even contact with one photon changes their position and momentum - measurement problem hint that the observer is an important piece of any quantum forces us to give up the common sense notion of location**



Thus, we begin to see a strong **coupling** of the properties of a quantum object and the act of measuring those properties. The question of the reality of quantum properties remains unsolved. All quantum mechanical principles must reduce to Newtonian principles at the macroscopic level (there is continuity between quantum and Newtonian mechanics).How does the role of the **observer effect** the wave and particle nature of the quantum world? One test is to return to the two slit experiment and try to determine count which slit the photon goes through. If the photon is a particle, then it has to go through one or the other slit. Doing this experiment results in wiping out the interference pattern. The wave nature of the light is eliminated, only the particle nature remains and particles cannot make interference patterns. Clearly the two slit experiments, for the first time in physics, indicates that there is a much **deeper relationship between the observer and the phenomenon**, at least at the subatomic level. This is an extreme break from the idea of an objective reality or one where the laws of Nature have a special, Platonic existence. **The two**

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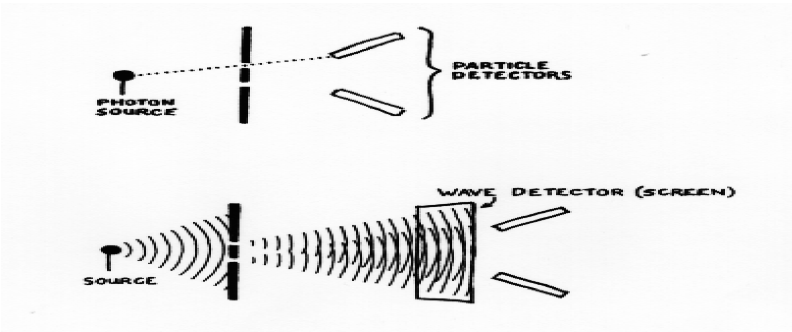
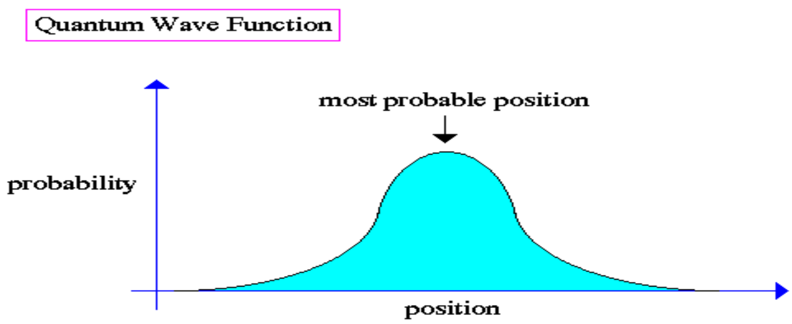
slit experiment is a good test of the role of the observer in the quantum theory. Many experimental designs that attempts to determine which slit a photon has passed through (test for its particle nature) destroys the interference pattern (its wavelike nature).This is a breakdown of objective reality. Each quantum entity has dual potential properties, which become- an actual characteristic if and when it is observed, an actual characteristic if and when it is under observance.



If the physicist looks for a particle (uses particle detectors), then a particle is found. If the physicist looks for a wave (uses a wave detector), then a wave pattern is found. A quantum entity has a dual *potential* nature, but its *actual* (observed) nature is one or the other.

QUANTUM WAVE FUNCTION

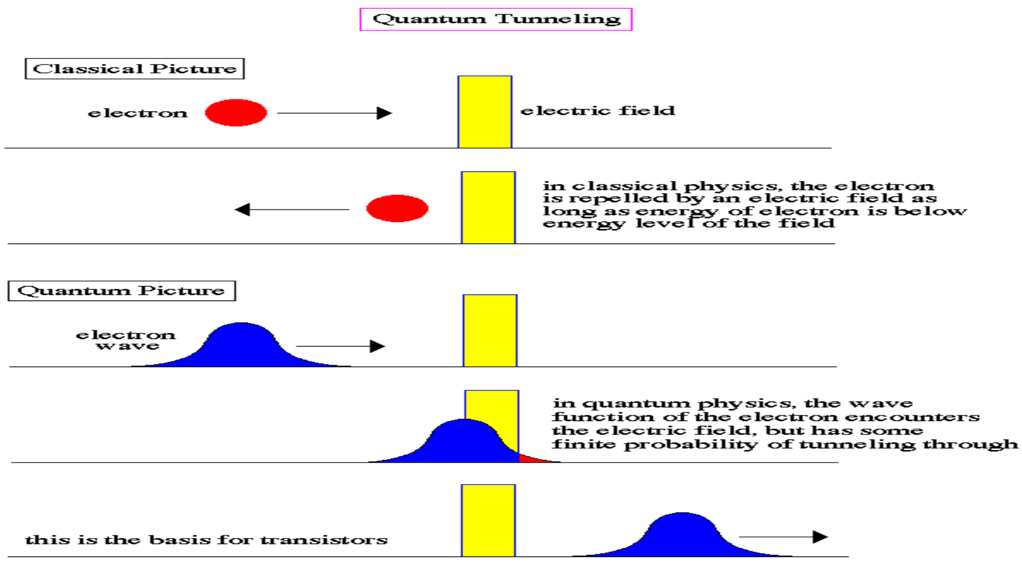
The wave nature of the microscopic world makes the concept of `**position'** difficult for subatomic particles. Even a wave packet has some `fuzziness' associated with it. An electron in orbit has no position to speak of, other than it is somewhere in its orbit. To deal with this problem, quantum physics developed the tool of the quantum wave function as a mathematical description of the superposition **associated w**ith a quantum **a wave packet interpretation for particles means there is an intrinsic fuzziness assigned to them. The wave function is the mathematical tool to describe quantum entities** entity at any particular moment



The key point to the wave function is that the position of a particle is only expressed as likelihood or probability *until* a measurement is made. For example, striking an electron with a photon results in a position measurement and we say that the wave function has `**collapsed**' (i.e. the wave nature of the electron converted to a particle one.

SUPERPOSITION:

The fact that quantum systems, such as electrons and protons, have indeterminate aspects means they exist as possibilities rather than actualities. This gives them the property of being things that might be or might happen, rather than things that are. This is in sharp contrast to Newtonian physics where things are or are not; there is no uncertainty except those imposed by poor data or limitations of the data gathering equipment. Further experimentation showed that reality at the quantum (microscopic) level consists of twins of reality, **actual and potential**. The actual is what we get when we see or measure a quantum entity, the potential is the state in which the object existed before it was measured. The **result is** that a quantum entity (a photon, electron, neutron, etc) exists in multiple possibilities of realities known as superposition. The superposition of possible positions for an electron can be demonstrated by the observed phenomenon called quantum tunneling. **Quantum physics is a science of possibilities rather than exactness of Newtonian physics quantum objects and quantities becomes actual when observed; key proof of quantum super positions is the phenomenon of quantum tunneling; the position of the electron, the wave function, is truly spread out, not uncertain**



Observation causes the wave function to collapse. Quantum existence is tied to the environment, opposite to the independence of macroscopic objects. The collapse of the wave function by observation is a transition from the many to the one, from possibility to actuality. (Wave function collapses because of observation) The identity and existence of a quantum entities are bound up with its overall environment (this is called conceptualism). Like homonyms, words that depend on the context in which they are used, quantum reality shifts its nature according to its surroundings. In the macroscopic world ruled by classical physics, things are what they are. In the microscopic world ruled by quantum physics, there is an existential dialogue among the particle, its surroundings and the person studying

UNCERTAINTY OF POSITION OF A PHOTON CONCOMITANT AND CONSUMMATING MANIFESTATION OF WAVE EFFECTS

ASSUMPTIONS:

Uncertainty of position of a photon concomitant and consummating manifestation of wave effects are classified into three categories;

1) Category 1 representative of the **uncertainty of position of a photon concomitant and consummating**

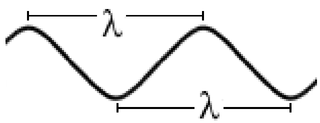
manifestation of wave effects in the first interval vis-à-vis category1

2) Category 2 (second interval) comprising of **uncertainty of position of a photon concomitant and consummating manifestation of wave effects** corresponding to category 2

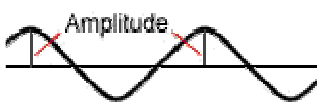
3) Category 3 constituting **uncertainty of position of a photon concomitant and consummating manifestation of wave effects** which belong to higher age than that of category 1 and category 2

Mechanical waves **require** a material medium to travel (air, water, ropes). These waves are divided into three different types. Transverse waves **cause** the medium to move perpendicular to the direction of the wave. Longitudinal waves **cause** the medium to move parallel to the direction of the wave. Surface waves are both transverse waves and longitudinal waves mixed in one medium. Electromagnetic waves do not require a medium to travel (light, radio) **Matter waves are produced by electrons/photons and particles Different stars have different wave pattern**. By moving a rope once, a single vibration is produced. This is a pulse. The shortest time that a point takes to return to the initial position (one vibration) is called period, T. In this example, every vibration is marked with a short pause.

The number of vibrations per second is called frequency and is measured in hertz (Hz). **$f = 1 / T$ where f is the frequency**.The shortest distance between peaks, the highest points, and troughs, the lowest points, is the wavelength, λ.



By knowing the frequency of a wave and its wavelength, we can find its velocity. Here is the equation for the velocity of a wave: $v = \lambda f$ However, the velocity of a wave is only **affected** by the properties of the medium. It is not possible to increase the speed of a wave by **increasing** its wavelength. By doing this, the number of vibrations per **second decreases** and therefore the velocity **remains the same**.. The amplitude of a wave is the distance from a crest to where the wave is at equilibrium. The amplitude is used to measure the energy transferred by the wave. The bigger the distance, the greater the energy transferred; the amplitude of a wave is the distance from a crest to where the wave is at **equilibrium.** The amplitude is **used** to measure the energy transferred by the wave; **the bigger the distance, the greater the energy transferred**



In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye on the **manifestation of wavelength**, wave pattern. Vis-à-vis the uncertainty of photon position. Note that this wavelength differs from one particle to another and hence the interference structure notwithstanding the generalization and universalistic pattern of wavelength would be in the fitness of things. "Over and above" nomenclature could be used to encompass a wider range of **uncertainty of position of a photon concomitant and consummating manifestation of wave effects**. Similarly, a "less than" scale for category 1 can be used. The speed of growth of **uncertainty of position of a photon concomitant and consummating manifestation of wave effects** under category 1 is proportional to the total quantum of on **uncertainty of position of a photon concomitant and consummating manifestation of wave effects** under category 2. In essence the accentuation coefficient in the model is representative of the constant of proportionality between category 1 and category 2 this assumptions is made to foreclose the necessity of addition of one more variable, that would render the systemic equations unsolvable

NOTATION :

G_{32} : Quantum of uncertainty of position of a photon concomitant and consummating manifestation of wave effects in category1

G_{33} : Quantum of uncertainty of position of a photon concomitant and consummating manifestation of wave effects in category 2

G_{34} : Quantum of uncertainty of position of a photon concomitant and consummating manifestation of wave effects in category 3

$(a_{32})^{(6)}, (a_{33})^{(6)}, (a_{34})^{(6)}$: Accentuation coefficients

$(a'_{32})^{(6)}, (a'_{33})^{(6)}, (a'_{34})^{(6)}$: Dissipation coefficients

FORMULATION OF THE SYSTEM :

In the light of the assumptions stated in the foregoing, we infer the following:-

a) The growth speed in category 1 is the sum of a accentuation term $(a_{32})^{(6)}G_{33}$ and a dissipation term $-(a'_{32})^{(6)}G_{32}$, the amount of dissipation taken to be proportional to the total quantum in category 2

b) The growth speed in category 2 is the sum of two parts $(a_{33})^{(6)}G_{32}$ and $-(a'_{33})^{(6)}G_{33}$ the inflow from the category1 dependent on the total amount standing in that category.

c) The growth speed in category 3 is equivalent to $(a_{34})^{(6)}G_{33}$ and $-(a'_{34})^{(6)}G_{34}$ dissipation ascribed only to depletion phenomenon.

Model makes allowance for the new quantum of photon collision and also usage of detectors in the augmented double slit experiment with the ramification of arousal and carousal of **uncertainty of position of a photon concomitant and consummating manifestation of wave effects. We shall note that this is something like the 'augmented reality' or 'dissipated reality', which are being referred to in information science.**

GOVERNING EQUATIONS:

The differential equations governing the above system can be written in the following form

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - (a'_{32})^{(6)}G_{32} \quad 1$$

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - (a'_{33})^{(6)}G_{33} \quad 2$$

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} \quad (a'_{34})^{(6)}G_{34} \quad 3$$

$(a_i)^{(6)} > 0 \quad , \quad i = 32,33,34$ 4

$(a_i')^{(5)} > 0 \quad , \quad i = 32,33,34$ 5

$(a_{33})^{(6)} < (a_{32}')^{(6)}$ 6

$(a_{34})^{(6)} < (a_{33}')^{(6)}$ 7

We can rewrite equation 1, 2 and 3 in the following form

$$\frac{dG_{32}}{(a_{32})^{(6)}G_{33} - (a_{32}')^{(6)}G_{32}} = dt$$ 8

$$\frac{dG_{33}}{(a_{33})^{(6)}G_{32} - (a_{33}')^{(6)}G_{33}} = dt$$

Or we write a single equation as

$$\frac{dG_{32}}{(a_{32})^{(6)}G_{33} - (a_{32}')^{(6)}G_{32}} = \frac{dG_{33}}{(a_{33})^{(6)}G_{32} - (a_{33}')^{(6)}G_{33}} = \frac{dG_{34}}{(a_{34})^{(6)}G_{33} - (a_{34}')^{(6)}G_{34}} = dt$$ 9

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.

For constant multiples α ,β ,γ all positive we can write equation (10) as 10

$$\frac{\alpha dG_{32}}{\alpha\left((a_{32})^{(6)}G_{33} - (a_{32}')^{(6)}G_{32}\right)} = \frac{\beta dG_{33}}{\beta\left((a_{33})^{(6)}G_{32} - (a_{33}')^{(6)}G_{33}\right)} = \frac{\gamma dG_{34}}{\gamma\left((a_{34})^{(6)}G_{33} - (a_{34}')^{(6)}G_{34}\right)} = dt$$

The general solution of the system can be written in the form

$\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e_i^{\lambda_i t}$ Where $i = 32,33,34$ and C_{32}, C_{33}, C_{34} are arbitrary constant coefficients. 11

STABILITY ANALYSIS :

Supposing $G_i(0) = G_i^0(0) > 0$, and denoting by λ_i the characteristic roots of the system, it easily results that

1. If $(a_{32}')^{(6)}(a_{33}')^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} > 0$ all the components of the solution, ie all the three parts of the system tend to zero, and the solution is stable with respect to the initial data.

2. If $(a_{32}')^{(6)}(a_{33}')^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$ and $\left(\lambda_{33} + (a_{32}')^{(6)}\right)G_{32}^0 - (a_{32})^{(6)}G_{33}^0 \neq 0, (\lambda_{33} < 0)$, the first two components of the solution tend to infinity as $t \to \infty$, and $G_{34} \to 0$, ie. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 tends to zero.

3. If $(a_{32}')^{(6)}(a_{33}')^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$ and $\left(\lambda_{33} + (a_{32}')^{(5)}\right)G_{32}^0 - (a_{32})^{(6)}G_{33}^0 = 0$ Then all the three parts tend to zero, but the solution is not stable i.e. at a small variation of the initial values of G_i, the corresponding solution tends to infinity.

From the above stability analysis we infer the following:

1. The adjustment process is stable in the sense that the system converges to equilibrium.
2. The approach to equilibrium is a steady one , and there exists progressively diminishing oscillations around the equilibrium point
3. Conditions 1 and 2 are independent of the size and direction of initial disturbance
4. The actual shape of the time path of **uncertainty of position of a photon concomitant and consummating manifestation of wave effects** (it will be self explicit that the detectors are used in the experiment and there is interaction of the instrumentation with nature, radically producing wave effect in its ramification ,manifestation and phenomenological observation) is determined by efficiency parameter , the strength of the response of the portfolio in question, and the initial disturbance for the photon perturbation in the experiment.

MANIFESTATION OF WAVE EFFECTS CONCOMITANT AND CONSUBTANTIATORY TO UNCERTAINTY IN POSITION OF PHOTON IN THE AUGMENTED DOUBLE SLIT EXPERIMENT WITH ATTACHED DETECTORS.

It is to be noted that the wave effects shall differ with the usage of detectors and number of photons and the photons' collision allowed which are all subjective considerations of the person who is performing the experiment. It is also to be stated that that person or the instrument which is measuring the wave pattern is also governed by the same laws as that of the photon or photon numbers that lead collision and the corresponding wavelength pattern manifestation.

ASSUMPTIONS:

Manifestation of wave effects concomitant and consubtantiatory to uncertainty in position of photon in the augmented double slit experiment with attached detectors are stratified in to three categories analogous to the stratification above. Category 1 is representative of the **manifestation of wave effects concomitant and consubtantiatory to uncertainty in position of photon in the augmented double slit experiment with attached detectors**. So also is the case with category 2 and category3

The dissipation coefficient in the growth model is attributable to two factors with the progress of time .(here we refer the reader to birth and death of photon, and the intermittent and interregnum collisions and consequences thereof such as the pernicious to the very existence of the photon in the closed system or emission of the photon from the system to the outside the environment, or the examples in galaxy where such incidents take place in legion-all these aspectionalities and attributions and assignments have been discussed in detail..) Manifestation of effects of waveform become eligible for transfer to the next category. Notwithstanding Category 3 does not have such a provision for further transference which constitutes the end of the journey of the photon under investigation In reality it is to be noted that such collisions would take place in nonlinear and highly per mutative and commutative manner **leading to** manifestation of wave front pattern For details about such in formations of stars ,and specifically in the double slit experiment

NOTATION :

T_{32} : Category 1 of **manifestation of wave effects concomitant and consubtantiatory to uncertainty in position of photon in the augmented double slit experiment with attached detectors**

T_{33} :Category 2 of **manifestation of wave effects concomitant and consubtantiatory to uncertainty in position of photon in the augmented double slit experiment with attached detectors**

T_{34} : Category 3 of **manifestation of wave effects concomitant and consubtantiatory to uncertainty in position of**

photon in the augmented double slit experiment with attached detectors

$(b_{32})^{(6)}, (b_{33})^{(6)}, (b_{34})^{(6)}$: Accentuation coefficients

$(b'_{32})^{(6)}, (b'_{33})^{(6)}, (b'_{34})^{(6)}$: Dissipation coefficients

FORMULATION OF THE SYSTEM :

Under the above assumptions, we derive the following

The growth speed in category 1 is the sum of two parts:

A term $+(b_{32})^{(6)}T_{33}$ proportional to the **manifestation of wave effects concomitant and consubtantiatory to uncertainty in position of photon in the augmented double slit experiment with attached detectors** the category 2.
And as usual on term $-(b'_{32})^{(6)}T_{32}$ representing the quantum of balance dissipated from category 1. Such transference depends on the age of the photon we have used in the experiment under consideration. For birth, death, dark photon, which annihilates the Photon kindly see the author's Photon Energy and Dark Bands The growth speed in category 2 is the sum of two parts. Imagine numerable double slit experiments are conducted at various time intervals. Also note the fact that the photon gets aged for it is the one under considerations, For details about the birth, death of photon please see the main paper

1. A term $+(b_{33})^{(6)}T_{32}$ constitutive of the amount of inflow from the category 1
2. A term $-(b'_{33})^{(6)}T_{33}$ the dissipation factor.

The growth speed under category 3 is attributable to inflow from category 2 and any dissipation due to dark photon, anti electron etc., especially because of annihilation.

GOVERNING EQUATIONS:

Following are the differential equations that govern the growth in the portfolio

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - (b'_{32})^{(6)}T_{32} \quad 12$$

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - (b'_{33})^{(6)}T_{33} \quad 13$$

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - (b'_{34})^{(6)}T_{34} \quad 14$$

$$(b_i)^{(6)} > 0 \quad , \quad i = 32,33,34 \quad 15$$

$$(b'_i)^{(6)} > 0 \quad , \quad i = 32,33,34 \quad 16$$

$$(b_{33})^{(6)} < (b'_{32})^{(6)} \quad 17$$

$$(b_{34})^{(6)} < (b'_{33})^{(6)} \quad 18$$

Following the same procedure outlined in the previous section , the general solution of the governing equations is $\alpha'_i T_i + \beta'_i T_i + \gamma'_i T_i = C'_i e_i^{\lambda'_i t}$, $i = 32,33,34$ where $C'_{32}, C'_{33}, C'_{34}$ are arbitrary constant coefficients and $\alpha'_{32}, \alpha'_{33}, \alpha'_{34}, \gamma'_{32}, \gamma'_{33}, \gamma'_{34}$ corresponding multipliers to the characteristic roots of the system

MANIFESTATION OF WAVE EFFECTS CONCOMITANT AND CONSUBTANTIATORY TO UNCERTAINTY IN POSITION OF PHOTON IN THE AUGMENTED DOUBLE SLIT EXPERIMENT WITH ATTACHED DETECTORS

We will denote

By $T_i(t), i = 32,33,34$, the three parts of the **manifestation of wave effects concomitant and consubtantiatory to uncertainty in position of photon in the augmented double slit experiment with attached detectors** analogously to the G_i

By $(a_i'')^{(6)}(T_{33},t)$ $(T_{33} \geq 0, t \geq 0)$,the contribution of the **manifestation of wave effects concomitant and consubtantiatory to uncertainty in position of photon in the augmented double slit experiment with attached detectors**

By $(-b_i'')^{(6)}(G_{32}, G_{33}, G_{34}, t) = -(b_i'')^{(6)}((G_{35}), t)$, the contribution in category 3

GOVERNING EQUATIONS:

The differential system of this model is now

$$\frac{dG_{32}}{dt} = (a_{32})^{(6)}G_{33} - \left[(a_{32}')^{(6)} + (a_{32}'')^{(6)}(T_{33},t)\right]G_{32}$$ 19

$$\frac{dG_{33}}{dt} = (a_{33})^{(6)}G_{32} - \left[(a_{33}')^{(6)} + (a_{33}'')^{(6)}(T_{33},t)\right]G_{33}$$ 20

$$\frac{dG_{34}}{dt} = (a_{34})^{(6)}G_{33} - \left[(a_{34}')^{(6)} + (a_{34}'')^{(6)}(T_{33},t)\right]G_{34}$$ 21

$$\frac{dT_{32}}{dt} = (b_{32})^{(6)}T_{33} - \left[(b_{32}')^{(6)} - (b_{32}'')^{(6)}((G_{35}),t)\right]T_{32}$$ 22

$$\frac{dT_{33}}{dt} = (b_{33})^{(6)}T_{32} - \left[(b_{33}')^{(6)} - (b_{33}'')^{(6)}((G_{35}),t)\right]T_{33}$$ 23

$$\frac{dT_{34}}{dt} = (b_{34})^{(6)}T_{33} - \left[(b_{34}')^{(6)} - (b_{34}'')^{(6)}((G_{35}),t)\right]T_{34}$$ 24

$+(a_{32}'')^{(6)}(T_{33},t) =$ **First augmentation factor**

$-(b_{32}'')^{(6)}((G_{35}),t) =$ **First detritions factor**

Where we suppose

A. $(a_i)^{(6)},(a_i')^{(6)},(a_i'')^{(6)},(b_i)^{(6)},(b_i')^{(6)},(b_i'')^{(6)} > 0, \; i,j = 32,33,34$

B. The functions $(a_i'')^{(6)},(b_i'')^{(6)}$ are positive continuous increasing and bounded.

<u>Definition of</u> $(p_i)^{(6)}, \; (r_i)^{(6)}$:

$$(a_i'')^{(6)}(T_{33},t) \leq (p_i)^{(6)} \leq (\hat{A}_{32})^{(6)}$$ 25

$$(b_i'')^{(6)}(G_{35},t) \leq (r_i)^{(6)} \leq (b_i')^{(6)} \leq (\hat{B}_{32})^{(6)}$$ 26

C. $\lim_{T_2 \to \infty}(a_i'')^{(6)}(T_{33},t) = (p_i)^{(6)}$ 27

$\lim_{G\to\infty}(b_i'')^{(6)}((G_{35}),t) = (r_i)^{(6)}$ 28

Definition of $(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}$:

Where $\boxed{(\hat{A}_{32})^{(6)}, (\hat{B}_{32})^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}}$ are positive constants and $\boxed{i = 32,33,34}$

They satisfy Lipschitz condition: 29

$$|(a_i'')^{(6)}(T_{33}',t) - (a_i'')^{(6)}(T_{33},t)| \leq (\hat{k}_{32})^{(6)}|T_{33} - T_{33}'|e^{-(\hat{M}_{32})^{(6)}t}$$

$$|(b_i'')^{(6)}((G_{35})',t) - (b_i'')^{(6)}((G_{35}),(T_{35}))| < (\hat{k}_{32})^{(6)}||(G_{35}) - (G_{35})'||e^{-(\hat{M}_{32})^{(6)}t}$$ 30

With the Lipschitz condition, we place a restriction on the behavior of functions $(a_i'')^{(6)}(T_{33}',t)$ and $(a_i'')^{(6)}(T_{33},t)$. (T_{33}',t) And (T_{33},t) are points belonging to the interval $[(\hat{k}_{32})^{(6)}, (\hat{M}_{32})^{(6)}]$. It is to be noted that $(a_i'')^{(6)}(T_{33},t)$ is uniformly continuous. In the eventuality of the fact, that if $(\hat{M}_{32})^{(6)} = 1$ then the function $(a_i'')^{(6)}(T_{33},t)$, the **first augmentation coefficient** would be continuous.

Definition of $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$: 31

$(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}$, are positive constants

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}, \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$$

Definition of $(\hat{P}_{32})^{(6)}, (\hat{Q}_{32})^{(6)}$:

D. There exists two constants $(\hat{P}_{32})^{(6)}$ and $(\hat{Q}_{32})^{(6)}$ which together with $(\hat{M}_{32})^{(6)}, (\hat{k}_{32})^{(6)}, (\hat{A}_{32})^{(6)}$ and and $(\hat{B}_{32})^{(6)}$ the constants $(a_i)^{(6)}, (a_i')^{(6)}, (b_i)^{(6)}, (b_i')^{(6)}, (p_i)^{(6)}, (r_i)^{(6)}, i = 32,33,34,$ satisfy the inequalities

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(a_i)^{(6)} + (a_i')^{(6)} + (\hat{A}_{32})^{(6)} + (\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$ 32

$$\frac{1}{(\hat{M}_{32})^{(6)}}[(b_i)^{(6)} + (b_i')^{(6)} + (\hat{B}_{32})^{(6)} + (\hat{Q}_{32})^{(6)}(\hat{k}_{32})^{(6)}] < 1$$ 33

Theorem 1: if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions

Definition of $G_i(0), T_i(0)$:

$$G_i(t) \leq (\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}t} \quad , \quad \boxed{T_i(0) = T_i^0 > 0}$$

Proof:

Consider operator $\mathcal{A}^{(6)}$ defined on the space of sextuples of continuous functions $G_i, T_i: \mathbb{R}_+ \to \mathbb{R}_+$ which satisfy

$G_i(0) = G_i^0\ ,\ T_i(0) = T_i^0\ ,\ G_i^0 \le (\ \hat{P}_{32}\ )^{(6)}\ ,T_i^0 \le (\ \hat{Q}_{32}\ )^{(6)},$ 34

$0 \le G_i(t) - G_i^0 \le (\ \hat{P}_{32}\ )^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ 35

$0 \le T_i(t) - T_i^0 \le (\ \hat{Q}_{32}\ )^{(6)} e^{(\hat{M}_{32})^{(6)} t}$ 36

By 37

$$\bar{G}_{32}(t) = G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} G_{33}(s_{(32)}) - \left((a'_{32})^{(6)} + a''_{32})^{(6)} \left(T_{33}(s_{(32)}), s_{(32)} \right) \right) G_{32}(s_{(32)}) \right] ds_{(32)}$$

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$$\bar{G}_{33}(t) = G_{33}^0 + \int_0^t \left[(a_{33})^{(6)} G_{32}(s_{(32)}) - \left((a'_{33})^{(6)} + (a''_{33})^{(6)} \left(T_{33}(s_{(32)}), s_{(32)} \right) \right) G_{33}(s_{(32)}) \right] ds_{(32)}$$

$$\bar{G}_{34}(t) = G_{34}^0 + \int_0^t \left[(a_{34})^{(6)} G_{33}(s_{(32)}) - \left((a'_{34})^{(6)} + (a''_{34})^{(6)} \left(T_{33}(s_{(32)}), s_{(32)} \right) \right) G_{34}(s_{(32)}) \right] ds_{(32)}$$ 39

$$\bar{T}_{32}(t) = T_{32}^0 + \int_0^t \left[(b_{32})^{(6)} T_{33}(s_{(32)}) - \left((b'_{32})^{(6)} - (b''_{32})^{(6)} \left(G(s_{(32)}), s_{(32)} \right) \right) T_{32}(s_{(32)}) \right] ds_{(32)}$$ 40

$$\bar{T}_{33}(t) = T_{33}^0 + \int_0^t \left[(b_{33})^{(6)} T_{32}(s_{(32)}) - \left((b'_{33})^{(6)} - (b''_{33})^{(6)} \left(G(s_{(32)}), s_{(32)} \right) \right) T_{33}(s_{(32)}) \right] ds_{(32)}$$ 41

$$\bar{T}_{34}(t) = T_{34}^0 + \int_0^t \left[(b_{34})^{(6)} T_{33}(s_{(32)}) - \left((b'_{34})^{(6)} - (b''_{34})^{(6)} \left(G(s_{(32)}), s_{(32)} \right) \right) T_{34}(s_{(32)}) \right] ds_{(32)}$$ 42

Where $s_{(32)}$ is the integrand that is integrated over an interval $(0, t)$

(a) The operator $\mathcal{A}^{(6)}$ maps the space of functions satisfying 34,35,36 into itself .Indeed it is obvious that

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$$G_{32}(t) \le G_{32}^0 + \int_0^t \left[(a_{32})^{(6)} \left(G_{33}^0 + (\ \hat{P}_{32}\ )^{(6)} e^{(\hat{M}_{32})^{(6)} s_{(32)}} \right) \right] ds_{(32)} = \left(1 + (a_{32})^{(6)} t \right) G_{33}^0 + \frac{(a_{32})^{(6)} (\ \hat{P}_{32}\ )^{(6)}}{(\hat{M}_{32})^{(6)}} \left(e^{(\hat{M}_{32})^{(6)} t} - 1 \right)$$

From which it follows that

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$$(G_{32}(t) - G_{32}^0) e^{-(\hat{M}_{32})^{(6)} t} \le \frac{(a_{32})^{(6)}}{(\hat{M}_{32})^{(6)}} \left[\left((\ \hat{P}_{32}\ )^{(6)} + G_{33}^0 \right) e^{\left(-\frac{(\hat{P}_{32})^{(6)} + G_{33}^0}{G_{33}^0} \right)} + (\ \hat{P}_{32}\ )^{(6)} \right]$$

(G_i^0) is as defined in the statement of theorem 1

Analogous inequalities hold also for $G_{33}\ ,G_{34}, T_{32}, T_{33}, T_{34}$

It is now sufficient to take $\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\ , \frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}} < 1$ and to choose $(\ \hat{P}_{32}\ )^{(6)}$ and $(\ \hat{Q}_{32}\ )^{(6)}$ large to have

$$\frac{(a_i)^{(6)}}{(\hat{M}_{32})^{(6)}} \left[(\ \hat{P}_{32})^{(6)} + \left((\ \hat{P}_{32}\ )^{(6)} + G_j^0 \right) e^{-\left(\frac{(\hat{P}_{32})^{(6)} + G_j^0}{G_j^0} \right)} \right] \le (\ \hat{P}_{32}\ )^{(6)}$$

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$$\frac{(b_i)^{(6)}}{(\hat{M}_{32})^{(6)}}\left[((\hat{Q}_{32})^{(6)}+T_j^0)e^{-\left(\frac{(\hat{Q}_{32})^{(6)}+T_j^0}{T_j^0}\right)}+(\hat{Q}_{32})^{(6)}\right]\leq(\hat{Q}_{32})^{(6)}$$ 46

In order that the operator $\mathcal{A}^{(6)}$ transforms the space of sextuples of functions G_i, T_i satisfying 34,35,36 into itself
The operator $\mathcal{A}^{(6)}$ is a contraction with respect to the metric

$$d\left(((G_{35})^{(1)},(T_{35})^{(1)}),((G_{35})^{(2)},(T_{35})^{(2)})\right)=$$
$$\sup_i\{\max_{t\in\mathbb{R}_+}|G_i^{(1)}(t)-G_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t},\max_{t\in\mathbb{R}_+}|T_i^{(1)}(t)-T_i^{(2)}(t)|e^{-(\hat{M}_{32})^{(6)}t}\}$$ 47

Indeed if we denote

Definition of $(\widetilde{G_{35}}),(\widetilde{T_{35}})$: $\left((\widetilde{G_{35}}),(\widetilde{T_{35}})\right)=\mathcal{A}^{(6)}((G_{35}),(T_{35}))$ 48
It results

$$|\tilde{G}_{32}^{(1)}-\tilde{G}_i^{(2)}|\leq\int_0^t(a_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}ds_{(32)}+$$
$$\int_0^t\{(a'_{32})^{(6)}|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{-(\hat{M}_{32})^{(6)}s_{(32)}}+$$
$$(a''_{32})^{(6)}(T_{33}^{(1)},s_{(32)})|G_{32}^{(1)}-G_{32}^{(2)}|e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}-$$
$$G_{32}^{(2)}|(a''_{32})^{(6)}(T_{33}^{(1)},s_{(32)})-(a''_{32})^{(6)}(T_{33}^{(2)},s_{(32)})|\ e^{-(\hat{M}_{32})^{(6)}s_{(32)}}e^{(\hat{M}_{32})^{(6)}s_{(32)}}\}ds_{(32)}$$ 49

Where $s_{(32)}$ represents integrand that is integrated over the interval $[0,t]$

From the hypotheses on 25,26,27,28 and 29 it follows
$$|(G_{35})^{(1)}-(G_{35})^{(2)}|e^{-(\hat{M}_{32})^{(6)}t}\leq$$
$$\frac{1}{(\hat{M}_{32})^{(6)}}\left((a_{32})^{(6)}+(a'_{32})^{(6)}+(\hat{A}_{32})^{(6)}+(\hat{P}_{32})^{(6)}(\hat{k}_{32})^{(6)}\right)d\left(((G_{35})^{(1)},(T_{35})^{(1)};(G_{35})^{(2)},(T_{35})^{(2)})\right)$$ 50

And analogous inequalities for G_i *and* T_i. Taking into account the hypothesis (34,35,36) the result follows

Remark 1: The fact that we supposed $(a''_{32})^{(6)}$ and $(b''_{32})^{(6)}$ depending also on t can be considered as not conformal with the reality, however we have put this hypothesis ,in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by $(\hat{P}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}t}$ *and* $(\hat{Q}_{32})^{(6)}e^{(\hat{M}_{32})^{(6)}t}$ respectively of $\mathbb{R}_+$. 51

If instead of proving the existence of the solution on $\mathbb{R}_+$, we have to prove it only on a compact then it suffices to consider that $(a''_i)^{(6)}$ and $(b''_i)^{(6)}, i=32,33,34$ depend only on T_{33} and respectively on (G_{35})(*and not on t*) and hypothesis can replaced by a usual Lipschitz condition.

Remark 2: There does not exist any t where $G_i(t)=0$ *and* $T_i(t)=0$ 52

From 19 to 24 it results

$$G_i(t)\geq G_i^0 e^{\left[-\int_0^t\{(a'_i)^{(6)}-(a''_i)^{(6)}(T_{33}(s_{(32)}),s_{(32)})\}ds_{(32)}\right]}\geq 0$$

$$T_i(t)\geq T_i^0 e^{\left(-(b'_i)^{(6)}t\right)}>0\quad\text{for } t>0$$

Definition of $\left((\widehat{M}_{32})^{(6)}\right)_1, \left((\widehat{M}_{32})^{(6)}\right)_2$ and $\left((\widehat{M}_{32})^{(6)}\right)_3$: 53

Remark 3: if G_{32} is bounded, the same property have also G_{33} and G_{34} . indeed if

$G_{32} < (\widehat{M}_{32})^{(6)}$ it follows $\frac{dG_{33}}{dt} \leq \left((\widehat{M}_{32})^{(6)}\right)_1 - (a'_{33})^{(6)}G_{33}$ and by integrating

$G_{33} \leq \left((\widehat{M}_{32})^{(6)}\right)_2 = G_{33}^0 - 2(a_{33})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_1/(a'_{33})^{(6)}$

In the same way , one can obtain

$G_{34} \leq \left((\widehat{M}_{32})^{(6)}\right)_3 = G_{34}^0 - 2(a_{34})^{(6)}\left((\widehat{M}_{32})^{(6)}\right)_2/(a'_{34})^{(6)}$

If G_{33} or G_{34} is bounded, the same property follows for G_{32} , G_{34} and G_{32} , G_{33} respectively.

Remark 4: If G_{32} is bounded, from below, the same property holds for G_{33} and G_{34} . The proof is analogous with the preceding one. An analogous property is true if G_{33} is bounded from below. 54

Remark 5: If T_{32} is bounded from below and $\lim_{t\to\infty}((b_i'')^{(6)}((G_{35})(t),t)) = (b'_{33})^{(6)}$ then $T_{33} \to \infty$. 55

Definition of $(m)^{(6)}$ and ε_6 :

Indeed let t_6 be so that for $t > t_6$

$(b_{33})^{(6)} - (b_i'')^{(6)}\left((G_{35})(t),t\right) < \varepsilon_6, T_{32}(t) > (m)^{(6)}$

Then $\frac{dT_{33}}{dt} \geq (a_{33})^{(6)}(m)^{(6)} - \varepsilon_6 T_{33}$ which leads to

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{\varepsilon_6}\right)(1 - e^{-\varepsilon_6 t}) + T_{33}^0 e^{-\varepsilon_6 t}$ If we take t such that $e^{-\varepsilon_6 t} = \frac{1}{2}$ it results

$T_{33} \geq \left(\frac{(a_{33})^{(6)}(m)^{(6)}}{2}\right), \quad t = \log\frac{2}{\varepsilon_6}$ By taking now ε_6 sufficiently small one sees that T_{33} is unbounded. The same property holds for T_{34} if $\lim_{t\to\infty}(b''_{34})^{(6)}\left((G_{35})(t),t(t),t\right) = (b'_{34})^{(6)}$
We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 42

Behavior of the solutions of equation 37 to 42

Theorem 2: If we denote and define

Definition of $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$:

(a) $(\sigma_1)^{(6)}, (\sigma_2)^{(6)}, (\tau_1)^{(6)}, (\tau_2)^{(6)}$ four constants satisfying 56

$-(\sigma_2)^{(6)} \leq -(a'_{32})^{(6)} + (a'_{33})^{(6)} - (a''_{32})^{(6)}(T_{33},t) + (a''_{33})^{(6)}(T_{33},t) \leq -(\sigma_1)^{(6)}$ 57

$-(\tau_2)^{(6)} \leq -(b'_{32})^{(6)} + (b'_{33})^{(6)} - (b''_{32})^{(6)}\left((G_{35}),t\right) - (b''_{33})^{(6)}\left((G_{35}),t\right) \leq -(\tau_1)^{(6)}$ 58

Definition of $(\nu_1)^{(6)}, (\nu_2)^{(6)}, (u_1)^{(6)}, (u_2)^{(6)}, \nu^{(6)}, u^{(6)}$: 59

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(b) By $(\nu_1)^{(6)} > 0, (\nu_2)^{(6)} < 0$ and respectively $(u_1)^{(6)} > 0, (u_2)^{(6)} < 0$ the roots of the equations 60

$(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_1)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ and 61

Definition of $(\bar{\nu}_1)^{(5)},, (\bar{\nu}_2)^{(6)}, (\bar{u}_1)^{(6)}, (\bar{u}_2)^{(6)}$: 62

By $(\bar{\nu}_1)^{(6)} > 0, (\bar{\nu}_2)^{(6)} < 0$ and respectively $(\bar{u}_1)^{(6)} > 0, (\bar{u}_2)^{(6)} < 0$ the roots of the equations 63

$(a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)} = 0$ and $(b_{33})^{(6)}\left(u^{(6)}\right)^2 + (\tau_2)^{(6)}u^{(6)} - (b_{32})^{(6)} = 0$ 64

Definition of $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}, (\nu_0)^{(6)}$:- 65

(c) If we define $(m_1)^{(6)}, (m_2)^{(6)}, (\mu_1)^{(6)}, (\mu_2)^{(6)}$ by 66

$(m_2)^{(6)} = (\nu_0)^{(6)}\ (m_1)^{(6)} = (\nu_1)^{(5)},\ \boldsymbol{if}\ (\nu_0)^{(6)} < (\nu_1)^{(6)}$ 67

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\bar{\nu}_6)^{(6)},\ \boldsymbol{if}\ (\nu_1)^{(6)} < (\nu_0)^{(6)} < (\bar{\nu}_1)^{(6)},$ and $\boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$ 68

$(m_2)^{(6)} = (\nu_1)^{(6)}, (m_1)^{(6)} = (\nu_0)^{(6)},\ \boldsymbol{if}\ (\bar{\nu}_1)^{(6)} < (\nu_0)^{(6)}$ 69

and analogously

$(\mu_2)^{(6)} = (u_0)^{(6)}, (\mu_1)^{(6)} = (u_1)^{(6)},\ \boldsymbol{if}\ (u_0)^{(6)} < (u_1)^{(6)}$ 70

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (\bar{u}_1)^{(6)},\ \boldsymbol{if}\ (u_1)^{(6)} < (u_0)^{(6)} < (\bar{u}_1)^{(6)},$ and $\boxed{(u_0)^{(6)} = \frac{T_{32}^0}{T_{33}^0}}$ 71

$(\mu_2)^{(6)} = (u_1)^{(6)}, (\mu_1)^{(6)} = (u_0)^{(6)}, \boldsymbol{if}\ (\bar{u}_1)^{(6)} < (u_0)^{(6)}$ where $(u_1)^{(6)}, (\bar{u}_1)^{(6)}$

are defined by 59 and 66 respectively

Then the solution of 19,20,21,22,23 and 24 satisfies the inequalities 72

$$G_{32}^0 e^{\left((S_1)^{(6)} - (p_{32})^{(6)}\right)t} \leq G_{32}(t) \leq G_{32}^0 e^{(S_1)^{(6)}t}$$

where $(p_i)^{(6)}$ is defined by equation 25

$$\frac{1}{(m_1)^{(6)}} G_{32}^0 e^{\left((S_1)^{(6)} - (p_{32})^{(6)}\right)t} \leq G_{33}(t) \leq \frac{1}{(m_2)^{(6)}} G_{32}^0 e^{(S_1)^{(6)}t}$$ 73

$$\left(\frac{(a_{34})^{(6)} G_{32}^0}{(m_1)^{(6)}\left((S_1)^{(6)} - (p_{32})^{(6)} - (S_2)^{(6)}\right)}\left[e^{\left((S_1)^{(6)} - (p_{32})^{(6)}\right)t} - e^{-(S_2)^{(6)}t}\right] + G_{34}^0 e^{-(S_2)^{(6)}t} \leq G_{34}(t) \leq \right.$$
$$\left.\frac{(a_{34})^{(6)} G_{32}^0}{(m_2)^{(6)}\left((S_1)^{(6)} - (a'_{34})^{(6)}\right)}\left[e^{(S_1)^{(6)}t} - e^{-(a'_{34})^{(6)}t}\right] + G_{34}^0 e^{-(a'_{34})^{(6)}t}\right)$$ 74

$$\boxed{T_{32}^0 e^{(R_1)^{(6)}t} \leq T_{32}(t) \leq T_{32}^0 e^{\left((R_1)^{(6)} + (r_{32})^{(6)}\right)t}}$$ 75

$$\frac{1}{(\mu_1)^{(6)}} T_{32}^0 e^{(R_1)^{(6)}t} \le T_{32}(t) \le \frac{1}{(\mu_2)^{(6)}} T_{32}^0 e^{((R_1)^{(6)}+(r_{32})^{(6)})t}$$ 76

$$\frac{(b_{34})^{(6)}T_{32}^0}{(\mu_1)^{(6)}((R_1)^{(6)}-(b'_{34})^{(6)})}\left[e^{(R_1)^{(6)}t} - e^{-(b'_{34})^{(6)}t}\right] + T_{34}^0 e^{-(b'_{34})^{(6)}t} \le T_{34}(t) \le$$ 77

$$\frac{(a_{34})^{(6)}T_{32}^0}{(\mu_2)^{(6)}((R_1)^{(6)}+(r_{32})^{(6)}+(R_2)^{(6)})}\left[e^{((R_1)^{(6)}+(r_{32})^{(6)})t} - e^{-(R_2)^{(6)}t}\right] + T_{34}^0 e^{-(R_2)^{(6)}t}$$

Definition of $(S_1)^{(6)}, (S_2)^{(6)}, (R_1)^{(6)}, (R_2)^{(6)}$:-

Where $(S_1)^{(6)} = (a_{32})^{(6)}(m_2)^{(6)} - (a'_{32})^{(6)}$

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$$(S_2)^{(6)} = (a_{34})^{(6)} - (p_{34})^{(6)}$$

$$(R_1)^{(6)} = (b_{32})^{(6)}(\mu_2)^{(6)} - (b'_{32})^{(6)}$$

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$$(R_2)^{(6)} = (b'_{34})^{(6)} - (r_{34})^{(6)}$$

Proof : From 19,20,21,22,23,24 we obtain

$$\frac{d\nu^{(6)}}{dt} = (a_{32})^{(6)} - \left((a'_{32})^{(6)} - (a'_{33})^{(6)} + (a''_{32})^{(6)}(T_{33},t)\right) - (a''_{33})^{(6)}(T_{33},t)\nu^{(6)} - (a_{33})^{(6)}\nu^{(6)}$$ 80

Definition of $\nu^{(6)}$:- $\left|\nu^{(6)} = \frac{G_{32}}{G_{33}}\right|$

It follows

$$-\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_2)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right) \le \frac{d\nu^{(6)}}{dt} \le -\left((a_{33})^{(6)}\left(\nu^{(6)}\right)^2 + (\sigma_1)^{(6)}\nu^{(6)} - (a_{32})^{(6)}\right)$$ 81

From which one obtains

Definition of $(\bar{\nu}_1)^{(6)}, (\nu_0)^{(6)}$:-

(a) For $0 < \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}} < (\nu_1)^{(6)} < (\bar{\nu}_1)^{(6)}$

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$$\nu^{(6)}(t) \ge \frac{(\nu_1)^{(6)} - (C)^{(6)}(\nu_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}}{1+(C)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_0)^{(6)}\right)t\right]}} \quad , \quad \boxed{(C)^{(6)} = \frac{(\nu_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\nu_2)^{(6)}}}$$

it follows $(\nu_0)^{(6)} \le \nu^{(6)}(t) \le (\nu_1)^{(6)}$

In the same manner , we get 83

$$\nu^{(6)}(t) \le \frac{(\bar{\nu}_1)^{(6)} + (\bar{C})^{(6)}(\bar{\nu}_2)^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)} e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \quad , \quad \boxed{(\bar{C})^{(6)} = \frac{(\bar{\nu}_1)^{(6)}-(\nu_0)^{(6)}}{(\nu_0)^{(6)}-(\bar{\nu}_2)^{(6)}}}$$

From which we deduce $(\nu_0)^{(6)} \leq \nu^{(6)}(t) \leq (\bar{\nu}_1)^{(6)}$

(b) If $0 < (\nu_1)^{(6)} < (\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0} < (\bar{\nu}_1)^{(6)}$ we find like in the previous case, 84

$$(\nu_1)^{(6)} \leq \frac{(\nu_1)^{(6)}+(C)^{(6)}(\nu_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}}{1+(C)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\nu_1)^{(6)}-(\nu_2)^{(6)}\right)t\right]}} \leq \nu^{(6)}(t) \leq$$

$$\frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\bar{\nu}_1)^{(6)}$$

(c) If $0 < (\nu_1)^{(6)} \leq (\bar{\nu}_1)^{(6)} \leq \boxed{(\nu_0)^{(6)} = \frac{G_{32}^0}{G_{33}^0}}$, we obtain 85

$$(\nu_1)^{(6)} \leq \nu^{(6)}(t) \leq \frac{(\bar{\nu}_1)^{(6)}+(\bar{C})^{(6)}(\bar{\nu}_2)^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}}{1+(\bar{C})^{(6)}e^{\left[-(a_{33})^{(6)}\left((\bar{\nu}_1)^{(6)}-(\bar{\nu}_2)^{(6)}\right)t\right]}} \leq (\nu_0)^{(6)}$$

And so with the notation of the first part of condition (c) , we have

Definition of $\nu^{(6)}(t)$:-

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$$(m_2)^{(6)} \leq \nu^{(6)}(t) \leq (m_1)^{(6)}, \quad \boxed{\nu^{(6)}(t) = \frac{G_{32}(t)}{G_{33}(t)}}$$

In a completely analogous way, we obtain

Definition of $u^{(6)}(t)$:-

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$$(\mu_2)^{(6)} \leq u^{(6)}(t) \leq (\mu_1)^{(6)}, \quad \boxed{u^{(6)}(t) = \frac{T_{32}(t)}{T_{33}(t)}}$$

Now, using this result and replacing it in 19, 20,21,22,23, and 24 we get easily the result stated in the theorem.

Particular case :

If $(a_{32}'')^{(6)} = (a_{33}'')^{(6)}, then\ (\sigma_1)^{(6)} = (\sigma_2)^{(6)}$ and in this case $(\nu_1)^{(6)} = (\bar{\nu}_1)^{(6)}$ if in addition $(\nu_0)^{(6)} = (\nu_1)^{(6)}$ then $\nu^{(6)}(t) = (\nu_0)^{(6)}$ and as a consequence $G_{32}(t) = (\nu_0)^{(6)}G_{33}(t)$ **this also defines** $(\nu_0)^{(6)}$ **for the special case** .

Analogously if $(b_{32}'')^{(6)} = (b_{33}'')^{(6)}, then\ (\tau_1)^{(6)} = (\tau_2)^{(6)}$ and then
$(u_1)^{(6)} = (\bar{u}_1)^{(6)}$if in addition $(u_0)^{(6)} = (u_1)^{(6)}$ then $T_{32}(t) = (u_0)^{(6)}T_{33}(t)$ This is an important consequence of the relation between $(\nu_1)^{(6)}$ and $(\bar{\nu}_1)^{(6)}$, **and definition of** $(u_0)^{(6)}$

1. STATIONARY SOLUTIONS AND STABILITY

We can prove the following

Theorem 3: If $(a_i'')^{(6)} and\ (b_i'')^{(6)}$ are independent on t , and the conditions (with the notations 25,26,27,28)

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$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} < 0$

$(a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} + (a_{32})^{(6)}(p_{32})^{(6)} + (a'_{33})^{(6)}(p_{33})^{(6)} + (p_{32})^{(6)}(p_{33})^{(6)} > 0$

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} > 0$,

$(b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - (b'_{32})^{(6)}(r_{33})^{(6)} - (b'_{33})^{(6)}(r_{33})^{(6)} + (r_{32})^{(6)}(r_{33})^{(6)} < 0$

$with\ (p_{32})^{(6)}, (r_{33})^{(6)}$ as defined by equation 25 are satisfied , then the system

$(a_{32})^{(6)}G_{33} - [(a'_{32})^{(6)} + (a''_{32})^{(5)}(T_{33})]G_{32} = 0$ 89

$(a_{33})^{(6)}G_{32} - [(a'_{33})^{(6)} + (a''_{33})^{(5)}(T_{33})]G_{33} = 0$ 90

$(a_{34})^{(6)}G_{33} - [(a'_{34})^{(6)} + (a''_{34})^{(5)}(T_{33})]G_{34} = 0$ 91

$(b_{32})^{(6)}T_{33} - [(b'_{32})^{(6)} - (b''_{32})^{(6)}(G_{35})]T_{32} = 0$ 92

$(b_{33})^{(6)}T_{32} - [(b'_{33})^{(6)} - (b''_{33})^{(6)}(G_{35})]T_{33} - 0$ 93

$(b_{34})^{(6)}T_{33} - [(b'_{34})^{(6)} - (b''_{34})^{(6)}(G_{35})]T_{34} = 0$ 94

has a unique positive solution , which is an equilibrium solution for the system (19 to 24)

Proof:

(a) Indeed the first two equations have a nontrivial solution G_{32}, G_{33} if

$$F(T_{35}) = (a'_{32})^{(6)}(a'_{33})^{(6)} - (a_{32})^{(6)}(a_{33})^{(6)} - (a'_{32})^{(6)}(a''_{33})^{(6)}(T_{33}) + (a'_{33})^{(6)}(a''_{32})^{(6)}(T_{33}) + (a''_{32})^{(6)}(T_{33})(a''_{33})^{(6)}(T_{33}) = 0$$ 95

Definition and uniqueness of T^*_{33} :-

After hypothesis $f(0) < 0, f(\infty) > 0$ and the functions $(a''_i)^{(6)}(T_{33})$ are being increasing, it follows that there exists a unique T^*_{33} for which $f(T^*_{33}) = 0$. With this value , we obtain from the three first equations

$$G_{32} = \frac{(a_{32})^{(6)}G_{33}}{[(a'_{32})^{(6)} + (a''_{32})^{(6)}(T^*_{33})]} \quad , \quad G_{34} = \frac{(a_{34})^{(6)}G_{33}}{[(a'_{34})^{(6)} + (a''_{34})^{(6)}(T^*_{33})]}$$ 96

(b) By the same argument, the equations 92,93 admit solutions G_{32}, G_{33} if

$$\varphi(G_{35}) = (b'_{32})^{(6)}(b'_{33})^{(6)} - (b_{32})^{(6)}(b_{33})^{(6)} - [(b'_{32})^{(6)}(b''_{33})^{(6)}(G_{35}) + (b'_{33})^{(6)}(b''_{32})^{(6)}(G_{35})] + (b''_{32})^{(6)}(G_{35})(b''_{33})^{(6)}(G_{35}) = 0$$ 97

Where in $(G_{35})(G_{32}, G_{33}, G_{34}), G_{32}, G_{34}$ must be replaced by their values from 96. It is easy to see that φ is a decreasing function in G_{33} taking into account the hypothesis $\varphi(0) > 0 , \varphi(\infty) < 0$ it follows that there exists a

unique G_{33}^{*} such that $\varphi(G^{*}) = 0$
Finally we obtain the unique solution of 89 to 94

G_{33}^{*} given by $\varphi((G_{35})^{*}) = 0$, T_{33}^{*} given by $f(T_{33}^{*}) = 0$ and

$$G_{32}^{*} = \frac{(a_{32})^{(6)}G_{33}^{*}}{[(a'_{32})^{(6)}+(a''_{32})^{(6)}(T_{33}^{*})]} \quad , \quad G_{34}^{*} = \frac{(a_{34})^{(6)}G_{33}^{*}}{[(a'_{34})^{(6)}+(a''_{34})^{(6)}(T_{33}^{*})]}$$ 98

$$T_{32}^{*} = \frac{(b_{32})^{(6)}T_{33}^{*}}{[(b'_{32})^{(6)}-(b''_{32})^{(6)}((G_{35})^{*})]} \quad , \quad T_{34}^{*} = \frac{(b_{34})^{(6)}T_{33}^{*}}{[(b'_{34})^{(6)}-(b''_{34})^{(6)}((G_{35})^{*})]}$$ 99

Obviously, these values represent an equilibrium solution of foregoing system (19-24).

ASYMPTOTIC STABILITY ANALYSIS

__**Theorem 4:**__ If the conditions of the previous theorem are satisfied and if the functions $(a''_i)^{(6)}$ *and* $(b''_i)^{(6)}$ Belong to $C^{(6)}(\mathbb{R}_+)$ then the above equilibrium point is asymptotically stable.

__**Proof:**__ Denote

__**Definition of**__ $\mathbb{G}_i, \mathbb{T}_i$:-

$$G_i = G_i^{*} + \mathbb{G}_i \qquad , T_i = T_i^{*} + \mathbb{T}_i$$ 100

$$\frac{\partial(a''_{32})^{(6)}}{\partial T_{33}}(T_{33}^{*}) = (q_{32})^{(6)} \quad , \quad \frac{\partial(b''_i)^{(6)}}{\partial G_j}((G_{35})^{*}) = s_{ij}$$ 101

Then taking into account equations 89 to 94 and neglecting the terms of power 2, we obtain

$$\frac{d\mathbb{G}_{32}}{dt} = -((a'_{32})^{(6)} + (p_{32})^{(6)})\mathbb{G}_{32} + (a_{32})^{(6)}\mathbb{G}_{33} - (q_{32})^{(6)}G_{32}^{*}\mathbb{T}_{33}$$ 102

$$\frac{d\mathbb{G}_{33}}{dt} = -((a'_{33})^{(6)} + (p_{33})^{(6)})\mathbb{G}_{33} + (a_{33})^{(6)}\mathbb{G}_{32} - (q_{33})^{(6)}G_{33}^{*}\mathbb{T}_{33}$$ 103

$$\frac{d\mathbb{G}_{34}}{dt} = -((a'_{34})^{(6)} + (p_{34})^{(6)})\mathbb{G}_{34} + (a_{34})^{(6)}\mathbb{G}_{33} - (q_{34})^{(6)}G_{34}^{*}\mathbb{T}_{33}$$ 104

$$\frac{d\mathbb{T}_{32}}{dt} = -((b'_{32})^{(6)} - (r_{32})^{(6)})\mathbb{T}_{32} + (b_{32})^{(6)}\mathbb{T}_{33} - \sum_{j=32}^{34}(s_{(32)(j)}T_{32}^{*}\mathbb{G}_j)$$ 105

$$\frac{d\mathbb{T}_{33}}{dt} = -((b'_{33})^{(6)} - (r_{33})^{(6)})\mathbb{T}_{33} + (b_{33})^{(6)}\mathbb{T}_{32} - \sum_{j=32}^{34}(s_{(33)(j)}T_{33}^{*}\mathbb{G}_j)$$ 106

$$\frac{d\mathbb{T}_{34}}{dt} = -((b'_{34})^{(6)} - (r_{34})^{(6)})\mathbb{T}_{34} + (b_{34})^{(6)}\mathbb{T}_{33} - \sum_{j=32}^{34}(s_{(34)(j)}T_{34}^{*}\mathbb{G}_j)$$ 107

The characteristic equation of this system is

$$((\lambda)^{(6)} + (b'_{34})^{(6)} - (r_{34})^{(6)})\{((\lambda)^{(6)} + (a'_{34})^{(6)} + (p_{34})^{(6)})$$
$$\left[\left(((\lambda)^{(6)} + (a'_{32})^{(6)} + (p_{32})^{(6)})(q_{33})^{(6)}G_{33}^{*} + (a_{33})^{(6)}(q_{32})^{(6)}G_{32}^{*}\right)\right]$$
$$\left(((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(33)}T_{33}^{*} + (b_{33})^{(6)}s_{(32),(33)}T_{33}^{*}\right)$$
$$+\left(((\lambda)^{(6)} + (a'_{33})^{(6)} + (p_{33})^{(6)})(q_{32})^{(6)}G_{32}^{*} + (a_{32})^{(6)}(q_{33})^{(6)}G_{33}^{*}\right)$$
$$\left(((\lambda)^{(6)} + (b'_{32})^{(6)} - (r_{32})^{(6)})s_{(33),(32)}T_{33}^{*} + (b_{33})^{(6)}s_{(32),(32)}T_{32}^{*}\right)$$
$$\left(((\lambda)^{(6)})^2 + ((a'_{32})^{(6)} + (a'_{33})^{(6)} + (p_{32})^{(6)} + (p_{33})^{(6)})(\lambda)^{(6)}\right)$$
$$\left(((\lambda)^{(6)})^2 + ((b'_{32})^{(6)} + (b'_{33})^{(6)} - (r_{32})^{(6)} + (r_{33})^{(6)})(\lambda)^{(6)}\right)$$

$$+\left(\left((\lambda)^{(6)}\right)^2+\left((a'_{32})^{(6)}+(a'_{33})^{(6)}+(p_{32})^{(6)}+(p_{33})^{(6)}\right)(\lambda)^{(6)}\right)(q_{34})^{(6)}G_{34}$$
$$+\left((\lambda)^{(6)}+(a'_{32})^{(6)}+(p_{32})^{(6)}\right)\left((a_{34})^{(6)}(q_{33})^{(6)}G^{*}_{33}+(a_{33})^{(6)}(a_{34})^{(6)}(q_{32})^{(6)}G^{*}_{32}\right) \quad 108$$
$$\left(\left((\lambda)^{(6)}+(b'_{22})^{(6)}-(r_{22})^{(6)}\right)s_{(22),(24)}T^{*}_{22}+(b_{22})^{(6)}s_{(22),(24)}T^{*}_{22}\right)\Big\} = 0$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

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