

ADOMIAN DECOMPOSITION METHOD FOR HEAT TRANSFER AND HEAT SOURCE IN MHD FLOW OVER A STRETCHING SHEET

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Abstract

In this paper, the boundary layer equation of heat transfer of a free convective MHD flow with temperature dependent heat source of a viscous incompressible fluid, over a stretching sheet is studied. Nonlinear governing partial differential equations have been transformed to a set of ordinary differential equations using similarity transformation. The set of ordinary differential equations are solved numerically by Adomian Decomposition Method (ADM). The results are presented graphically for governing parameters such as magnetic and heat source.

Keywords: Stretching sheet, MHD free convection, Adomian decomposition method (ADM)

1. INTRODUCTION

In many industrial applications, the problems related to forced convection in large pipes or on the surface of the turbo machine-blades can be reduced to an external boundary-layer problem over a flat plate or a wedge, as in chemical engineering processes like metallurgical process, polymer extrusion process involves cooling of a molten liquid being stretched into a cooling system[1] in such processes the mechanical properties of the penultimate product would mainly depend on two things, one is the rate of cooling liquid used and other is the rate of stretching. Some of the polymer fluid such as polyethylene oxide, polyisobutylene solution in cetane, have better electromagnetic properties, are recommended as their flow can be regulated by external magnetic fields. Furthermore, boundary layer flow over a stretching sheet also arises in many practical situations such as polymer extrusion process, drawing of annealing of copper wires, continuous stretching of rolling and manufacturing of plastic films and artificial fibers. Materials manufactured by extrusion process and heat treated materials traveling between a feed roll and windup rolls or on conveyer belts.

Also, the study of hydrodynamic flow and convection heat transfer has gained attention due to its vast applications in industry and its important bearings on several technological and natural processes. The study of the flow and heat transfer in fluid past a porous surface has attracted the interest of many scientific investigators in view of its applications in engineering practice, particularly in chemical industries, such as the cases of boundary layer control, transpiration cooling and gaseous diffusion[2]. Hayat et al.[3] investigated the hydro magnetic oscillatory flow of a fluid bounded by a porous plate when the entire system rotates about axis normal to the plate and the result showed that the

flow field is appreciably influenced by the material parameter of the second grade fluid, applied magnetic field, the imposed frequency, rotation, suction and blowing parameters. Yurusoy and Pakdemirli [4] examine the exact solution of boundary layer equations of a non-Newtonian fluid over a stretching sheet by the method of lie group analysis and they found that the boundary layer thickness increases when the non-Newtonian behavior increases.

sakiadis [5] investigated the boundary layer flow over a continuous solid surface moving with constant speed. Crane [6] extended the work of Sakiadis to that of an extensible surface and presented an analytical solution for the boundary layer flow of an incompressible liquid caused solely by the linear stretching of an elastic flat sheet which moves in its own plane with velocity proportional to the distance from the fixed point. Tsou et al. [7] reported both analytical experimental results for the flow and heat transfer aspect arising in stretching sheet problem vertical porous plate. Makinde and Gbolagade [1] studied the numerical solution of flow over a stretching sheet with magnetic field and uniform heat source.

In this paper, we present Adomian Decomposition Method for the solution of heat transfer and temperature dependent heat source in MHD free convection flow over a stretching sheet

2.A GENERAL DESCRIPTION OF ADM

A general nonlinear differential equation will be used for simplicity, we consider

$$Fu = f \tag{1}$$

Where F is a nonlinear differential operator, y and u are function of η . (1) can be written as

$$Lu + Ru + Nu = f \tag{2}$$

where L is an operator representing the linear portion of F where is easily invertible, R is a linear operator for the remainder of the linear portion, and N is a nonlinear operator representing the nonlinear terms in F . Applying the inverse operator L^{-1} , the equation then becomes

$$L^{-1}Lu = L^{-1}f - L^{-1}Ru - L^{-1}Nu \tag{3}$$

Since F was taken to be a differential operator and L is linear, L^{-1} would represent an integration and with any given initial or boundary conditions, $L^{-1}Lu$ will give an equation for u incorporating these conditions. This gives

$$u(\eta) = g(\eta) - L^{-1}Ru - L^{-1}Nu \tag{4}$$

Where $g(\eta)$ represents the function generated by integrating u and using the initial or boundary conditions. The assume that the unknown function can be written as an infinite series

$$u(\eta) = \sum_{n=0}^{\infty} u_n(\eta) \quad (5)$$

We set $u_0 = g(\eta)$ and the remaining terms are to be determined by recursive relationship. This is found by first decomposing the nonlinear term into series of Adomian polynomials, A_n . The nonlinear term is written as

$$Nu = \sum_{n=0}^{\infty} A_n \quad (6)$$

In order to determine the Adomian polynomials, a grouping parameter λ , is introduced. It should be noted that λ is not a “smallness parameter”[6].The series

$$u(\lambda) = \sum_{n=0}^{\infty} \lambda^n y_n \quad (7)$$

$$Nu(\lambda) = \sum_{n=0}^{\infty} \lambda^n A_n \quad (8)$$

are established. Then A_n can be determined by

$$A_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} Nu(\lambda) \Big|_{\lambda=0} \quad (9)$$

Substituting (5),(6) and (9) in (4),we obtain

$$\sum_{n=0}^{\infty} u_n = u_0 - L^{-1} \sum_{n=0}^{\infty} Ru_n - L^{-1} \sum_{n=0}^{\infty} A_n \quad (10)$$

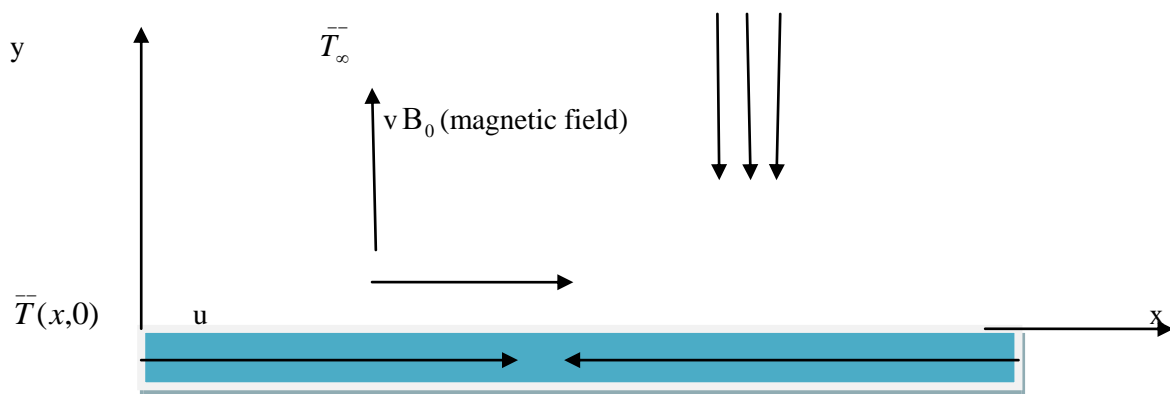
the recursive relationship is found to be

$$u_0 = g(\eta) \quad (11)$$

$$u_{n+1} = -L^{-1} \sum_{n=0}^{\infty} R u_n - L^{-1} \sum_{n=0}^{\infty} A_n \quad (12)$$

3. MATHEMATICAL FORMULATION OF PROBLEM

Consider a steady two-dimensional flow of an incompressible, electrically conducting viscous fluid past a flat, permeable stretching sheet. A uniform magnetic field is applied perpendicular to the stretching sheet. The flow takes place under buoyancy in the presence of radiation and temperature dependent heat source. The x-axis is taken in the direction along which the stretching sheet is set to motion and the y-axis is taken perpendicular to it. The flow is generated by the action of two equal and opposite forces along the x-axis and the sheet is stretched in such a way that the velocity at any instant is proportional to the distance from the origin (x=0). With these assumptions the boundary layer equations governing the flow are given by :



$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \quad (13)$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} - \sigma H_0^2 u \quad (14)$$

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} + Q(T - T_\infty) \quad (15)$$

with boundary conditions

$$u = 0, v = 0, T = T_w \text{ at } y = 0$$

$$u \rightarrow 0, T \rightarrow T_\infty \quad \text{as } y \rightarrow \infty \quad (16)$$

We introduce the stream function ψ defined by

$$u = \frac{\partial \psi}{\partial y}, v = -\frac{\partial \psi}{\partial x} \quad (17)$$

and a dimensionless stream function $f(\eta)$ to obtain velocity profile by

$$\psi = u_\infty \left(\frac{\nu x}{u_\infty} \right)^{\frac{1}{2}} f(\eta) \quad (18)$$

$$\text{Where } \eta = y \left(\frac{u_\infty}{\nu x} \right)^{\frac{1}{2}}$$

η being the similarity variable, $\nu = \frac{\mu}{\rho}$ kinematic viscosity.

Using the transformation

$$u = u_\infty f'(\eta), v = \frac{1}{2} \left(\frac{u_\infty \nu}{x} \right)^{\frac{1}{2}} (\eta f'(\eta) - f(\eta)) \quad (19)$$

and the dimension function for temperature

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \quad (20)$$

the continuity equation is identically satisfied and the momentum and heat transfer equations reduce to

$$f''' + \frac{1}{2} f f'' - M f' = 0 \quad (21)$$

$$\theta'' + \frac{1}{2} P_r f \theta' - \alpha \theta = 0 \quad (22)$$

The transformed boundary conditions are

$$f = 0, f' = 0, \theta = 0 \text{ at } \eta = 0$$

$$f' = 1, \theta = 0 \text{ as } \eta \rightarrow \infty \quad (23)$$

where

$$P_r = \frac{\mu C \rho}{k} \text{ the Prandtl Number} \quad M = \frac{\sigma H_0^2 x}{\rho u_\infty} \text{ the Number Hartmann}$$

$$\alpha = \frac{Qx}{ku_\infty} \text{ the Source Heat Parameter} \quad (24)$$

4. ADOMIAN DECOMPOSITION SOLUTION

Applying ADM to the equations (21) and (22), we obtain

$$f = f(0) + \eta f'(0) + \frac{\eta^2}{2} f''(0) + L^{-1} M f' + L^{-1} \left(-\frac{1}{2} f f'' \right) \quad (25)$$

$$\theta = \theta(0) + \eta \theta'(0) + L^{-1} \alpha \theta + L^{-1} \left(-\frac{1}{2} P_r f \theta' \right) \quad (26)$$

Let $\sigma = f''(0)$, $\beta = \theta'(0)$, where σ and β were obtained numerically [7].

One-term approximation is evaluated via boundary conditions (23), as follows

$$f(0) = 0 \quad \theta(0) = 1$$

$$f'(0) = 0 \quad \theta'(\eta) = \beta$$

$$f''(0) = \sigma \quad (27)$$

Substituting (27) in (25) and (26), we obtain

$$f = \frac{\eta^2}{2} \sigma + L^{-1} M f' + L^{-1} \left(-\frac{1}{2} f f'' \right) \quad (28)$$

$$\theta = 1 + \eta \beta + L^{-1} \alpha \theta + L^{-1} \left(-\frac{1}{2} P_r f \theta' \right) \quad (29)$$

Adomian polynomials, A_n, B_m are representing the nonlinear terms $\left(-\frac{1}{2} f f'' \right)$ and $\left(-\frac{1}{2} P_r f \theta' \right)$, the

corresponding Adomian polynomials are evaluated by (9),

$$A_0 = \frac{1}{2} \sigma^2 \eta^5$$

$$B_0 = \frac{1}{6} \alpha \beta \eta^3 + \frac{1}{2} \alpha \eta^2 - \frac{1}{48} P_r \sigma \beta \eta^4$$

$$A_1 = \frac{1}{24} M \sigma^2 \eta^4 - \frac{1}{240} \sigma^2 \eta^5 + \frac{1}{2} \sigma \eta^2 \left(\frac{1}{2} M \sigma \eta^2 - \frac{1}{12} \sigma^2 \eta^3 \right)$$

$$B_1 = \left(\frac{1}{24} M \sigma \eta^4 - \frac{1}{240} \sigma^2 \eta^5 \right) \beta + \frac{1}{2} \sigma \eta^2 \left(\frac{1}{2} \alpha \beta \eta^2 + \alpha \eta - \frac{1}{2} P_r \sigma \beta \eta^3 \right)$$

$$A_2 = \frac{1}{2} \alpha \eta^2 \left(-\frac{1}{30} M \sigma^2 \eta^5 + \frac{1}{24} M^2 \sigma \eta^4 + \frac{11}{288} \sigma^3 \eta^6 \right) + \left(\frac{1}{24} M \sigma \eta^4 - \frac{1}{240} \sigma^2 \eta^5 \right) \left(\frac{1}{2} M \sigma \eta^2 - \frac{1}{12} \sigma^2 \eta^3 \right) \\ + \left(-\frac{1}{1260} M \sigma^2 \eta^7 + \frac{1}{720} M^2 \sigma \eta^6 + \frac{11}{161280} \sigma^3 \eta^8 \right) \sigma$$

$$B_2 = 2\beta \left(-\frac{1}{1260} M \sigma^2 \eta^7 + \frac{1}{720} M^2 \sigma \eta^6 + \frac{11}{161280} \sigma^3 \eta^8 \right) + \left(\frac{1}{24} M \sigma \eta^4 - \frac{1}{240} \sigma^2 \eta^5 \right) \left(\frac{1}{2} \alpha \beta \eta^2 + \alpha \eta - \frac{1}{12} P_r \sigma \beta \eta^3 \right)$$

Therefore,

$$f(\eta) = \frac{1}{2} \sigma \eta^2 + \frac{1}{24} M \sigma \eta^4 - \frac{1}{240} \sigma^2 \eta^5 - \frac{1}{1260} M \sigma^2 \eta^7 + \frac{1}{720} M^2 \sigma \eta^6 + \frac{1}{161280} \sigma^3 \eta^8 + \frac{3}{179200} M \sigma^3 \eta^{10} \\ - \frac{1}{241920} M^2 \sigma^2 \eta^9 + \frac{1}{40320} M^3 \sigma \eta^8 - \frac{5}{4257792} \sigma^4 \eta^{11}$$

$$\theta(\eta) = 1 + \eta \beta + \frac{1}{6} \sigma \beta \eta^3 + \frac{1}{2} \sigma \eta^2 - \frac{1}{80} P_r \sigma \beta \eta^4 - \frac{1}{1440} \alpha \sigma \beta P_r \eta^6 + \frac{1}{20} \sigma^2 \beta \eta^5 + \frac{1}{24} \sigma^2 \eta^4 - \frac{3}{84} P_r \\ \left(-\frac{1}{240} \sigma^2 \beta - \frac{1}{24} \sigma^2 P_r \beta \right) \eta^7 - \frac{1}{60} P_r \left(-\frac{1}{240} M \sigma \beta + \frac{1}{4} \alpha \sigma \beta \right) \eta^6 - \frac{1}{80} P_r \alpha \sigma \eta^5$$

5. DISCUSSIONS AND RESULTS

The velocity and temperature distribution are discussed with reference to variations in Hartmann number M and heat source parameter α . The velocity and temperature profiles are drawn for different sets of values. The velocity is plotted in Figure 1, for different values of the governing parameter M . The horizontal axis is taken for variations in $\eta = 0$ to $\eta = 3.7$ and the vertical axis is taken for f . The velocity increases with the increase of M . The temperature θ is drawn in Figure 2, for different values

of the parameter α . The profiles for the function θ rapidly fall from $\eta = 0$ to $\eta = 3.7$. It is noted that θ decreases as increase.

6.CONCLUSION

In this paper, Adomian Decomposition Method has been successfully applied to heat transfer and temperature dependent heart source in MHD free convection flow over a stretching sheet problem with specified boundary conditions for momentum and energy equations. Also, the velocity and temperature profiles were obtained as a function of η . Using ADM method, an attempt has been made to show the effect of Hartmann number M in the velocity profile and heart source parameter in the temperature profile. It may be concluded ADM provides efficient alternative tools for solving nonlinear ordinary differential equation.

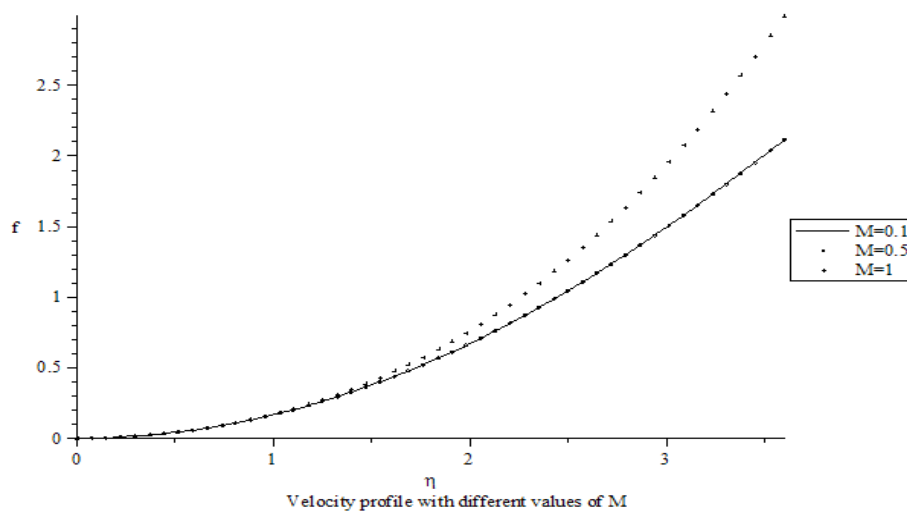


Figure 1

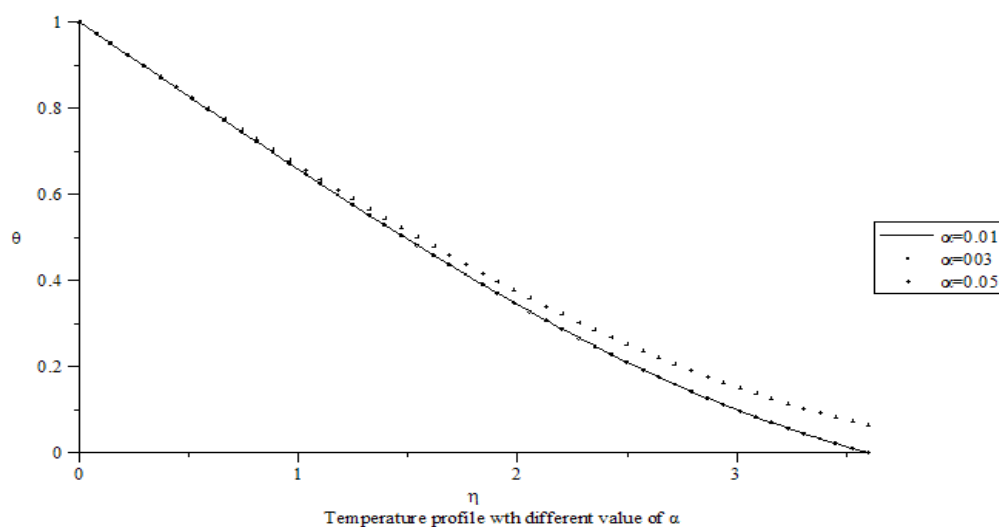


Figure 2

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