Propagation of Rayleigh waves in non-homogeneous orthotropic elastic media under the effect of magnetic field

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Abstract

The influence of magnetic field on the propagation of Rayleigh waves in an inhomogeneous, orthotropic elastic solid medium has been discussed. The method of separation of variable is used to find the frequency equation of the surface waves. The obtained dispersion equations are in agreement with the classical results when magnetic field and non-homogeneity are neglected

Keywords: Inhomogeneity, Orthotropic elastic solid, field, Magnetic field.

Nomenclature

$\vec{E}$ is the electric intensity,

$\vec{B}$ is magnetic field induction,

μ_0 is permeability of vacuum,

ε_0 is permittivity of vacuum,

μ_e is the magnetic permeability of the medium,

$\vec{J}$ is the current density,

σ^o is the conductivity of the material

$\vec{V} = \frac{\overrightarrow{\partial v}}{\partial t}$ is velocity of conductor,

ρ is the density of the material,

g is the earth ,

σ_{ij} is the stress component,

$\vec{u}$ is the component of displacement vector,

$\vec{H}_i$ is the perturbed magnetic field,

$\vec{H}$ is magnetic field intensity,

$\vec{H}_0$ is the initial magnetic field intensity along z-axis

$\vec{F}$ is the Lorentz's force,

C_{ij} is elastic constant,

t is the time.

Introduction

The theory of elasticity is an approximation to the stress-strain behavior of real materials. An ideal elastic material regains its original configuration on the removal of deforming force. Therefore an ideal ''elastic wave'' is that wave which propagates through a material in such a way that the particles oscillates about their mean positions without causing any change.

Bromwich [1] was the first who taken the case of in wave propagation through elastic solid media. Love[2] investigated the influence of Rayleigh wave. Many researchers such as Biot[3] studied the effect of and initial stress on Rayleigh waves, De and Sengupta[4] considered problems of elastic waves under the effect of field, Sengupta and Acharya[5] discussed the influence of on the propagation of waves in a magnetoelastic layer. Sharma and Kaur [6] studied Rayleigh waves in rotating thermoelastic solids with voids. Chattopadhyay *et al.* [7, 8] studied the propagation of G-type seismic waves in viscoelastic medium, they also discussed the effect of point source, and heterogeneity on the propagation of SH- waves. Abd-Alla and Ahmed [9] studied the Rayleigh waves in an orthotropic magneto-elastic medium under field and initial stress. Recently, Love waves in a non-homogeneous elastic media, Rayleigh waves in a non-homogeneous granular media, Stoneley, Rayleigh and Love waves in viscoelastic media, Love waves in a non-homogeneous orthotropic layer under 'P' overlying semi-infinite non-homogeneous medium were studied by Kakar et al. [10, 11, 12, 13].

In the present study, the influence magnetic field on the propagation of Rayleigh type waves in a non-homogeneous, orthotropic elastic solid medium has been discussed. The dispersion equation so obtained is in well agreement with the corresponding classical results.

2. Formulation of the problem and basic equations

The problem is dealing with magnetoelasticity. Therefore the basic equations will be electromagnetism and elasticity. The Maxwell equations of the electromagnetic field in a vacuum (in the absence of displacement current), are

$$\vec{\nabla}\cdot\vec{E}=0, \tag{1a}$$

$$\vec{\nabla}\cdot\vec{B}=0, \tag{1b}$$

$$\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}, \tag{1c}$$

$$\vec{\nabla}\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}. \tag{1d}$$

The current displacement vector and electric field are related as

$$\vec{J}=\sigma^{o}\vec{E}, \tag{2a}$$

If the conductor is moving with velocity $\vec{V}$ in applied magnetic field, then

$$\vec{J}=\sigma^{o}(\vec{E}+\vec{V}\times\vec{B})=\sigma^{o}(\vec{E}+\overrightarrow{\frac{\partial v}{\partial t}}\times\vec{B}). \tag{2b}$$

The electromagnetic wave equation through a vacuum is given by

$$\left(\nabla^2-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\right)\vec{\mathrm{E}} = 0, \tag{3a}$$

$$\left(\nabla^2-\mu_0\epsilon_0\frac{\partial^2}{\partial t^2}\right)\vec{\mathrm{B}} = 0. \tag{3b}$$

Let us consider an orthotropic, non-homogeneous elastic solid along x-direction further it is also under the influence of magnetic field. Here we consider Oxyz Cartesian coordinates system where O be any point on the plane boundary and Oz be normal to the medium and Rayleigh wave propagation is taken in the positive direction of x-axis. It is also assumed that at a great distance from center of disturbance, the wave propagation is two dimensional and is polarized in (x, z) plane. So, displacement components along x and z direction i.e. u and w are non-zero while v = 0. Also it is assumed that wave is surface wave as the disturbance is extensively confined to the boundary.
Also it is assumed that wave is surface wave as the disturbance is extensively confined to the boundary. Let ρ be the density of the material medium.
The value of magnetic field intensity is

$$\vec{\mathrm{H}}(0,0,\mathrm{H})=\vec{\mathrm{H}}_0+\vec{\mathrm{H}}_i \tag{4}$$

Equations governing the propagation of small elastic disturbances in a perfectly conducting elastic solid having electromagnetic force $\vec{F}=\left(\vec{J}\times\vec{\mathrm{B}}\right)$ (the Lorentz force, $\vec{J}$ is the current density and $\vec{\mathrm{B}}$ being magnetic induction vector) as the only body force are (Biot [3])

$$\sigma_{11,x}+\sigma_{12,y}+\sigma_{13,z}+F_x=\rho u_{,tt}$$

$$\sigma_{12,x}+\sigma_{22,y}+\sigma_{23,z}+F_y=\rho v_{,tt},$$

$$\sigma_{13,x}+\sigma_{23,y}+\sigma_{33,z}+F_z=\rho w_{,tt} \tag{5}$$

where u, v, w are displacement components in x, y and z direction and w_x, w_y, w_z are rotational components and are given by

$$w_x=\frac{1}{2}(w_{,y}-v_{,z}),\qquad w_y=\frac{1}{2}(u_{,z}-w_{,x}),\qquad w_z=\frac{1}{2}(v_{,x}-u_{,y}). \tag{6}$$

Further dynamical Eq. (5) in (x, z) directions are given by

$\sigma_{11},x + \sigma_{12},y + \sigma_{13},z + F_x = \rho u_{,tt}$

$\sigma_{13},x + \sigma_{23},y + \sigma_{33},z + F_z = \rho w_{,tt}$ (7)

where stress components are given by

$\sigma_{11} = C_{11}\, u_1,x + C_{13}\, u_3,z$,

$\sigma_{33} = C_{31}\, u_1,x + C_{33}\, u_3,z$,

$\sigma_{13} = C_{44}\, u_1,z + u_3,x$, (8)

where C_{ij} are elastic constants. Since the problem is treated in two-dimensions (x, z), therefore $C_{12}=C_{22}=C_{23}=0$

Let us take the assumption that $C_{44} = \frac{1}{2}(C_{11} - C_{13})$.

Substituting Eq. (8) in Eq. (7) ; we have

$C_{11}\,(2u_1,xx + u_1,zz + u_3,xz) + C_{13}\,(u_3,xz - u_1,zz) + (u_1,z + u_3,x)(C_{11} - C_{13}),z + 2u_1,x\, C_{11},x + 2u_3,z\, C_{13},x + 2\mu_e H_0^2 (u_{,xx} + w_{,xz}) = \rho u_{,tt}$ (9)

$C_{11}\,(u_1,xz + u_3,xx) + C_{13}\,(u_1,xz - u_3,xx) + 2\,C_{33}\, u_3,zz + 2\mu_e H_0^2 (u_{,xx} + w_{,xz}) + (u_1,z + u_3,x)(C_{11} - C_{13}),x + u_1,x\, C_{13},z + u_3,z\, C_{33},z = \rho w_{,tt}$. (10)

Now we assume the non-homogeneity for the elastic half space and density are given by

$C_{ij} = \alpha_{ij}\, e^{mz},\ \rho = \rho_0\, e^{mz}$, (11)

where $\lambda_{i,j}$, ρ_0, and m are constants.

Substituting Eq. (11) in Eq. (9) and in Eq. (10), we get

$e^{mz}\, \alpha_{11}\,(2u_1,xx + u_1,zz + u_3,xz) + \alpha_{13}\,(u_3,xz - u_1,zz)\, e^{mz} + (u_1,z + u_3,x)(\alpha_{11} - \alpha_{13})\, m\, e^{mz} + 2\mu_e H_0^2 (u_{,xx} + w_{,xz}) = \rho u_{,tt}$ (12)

$\alpha_{11}\,(u_1,xz + u_3,xx) + (\alpha_{13})(u_1,xz) - (\alpha_{13})\, u_3,xx + 2\alpha_{33}\, u_3,zz + 2\mu_e H_0^2 (u_{,xx} + w_{,xz}) + 2\,\alpha_{13}\, m\, u_1,x + 2\alpha_{33}\, m u_3,z = \rho w_{,tt}$. (13)

To investigate the surface wave propagation along Ox, we introduce displacement potentials in terms of displacements components are given by

$u = \phi,x - \psi,z\ ;\ w = \phi,z + \psi,x$ (14)

Introducing Eq. (14) in Eqs (13) and (12) we get

$$2\,(\alpha_{11}+\mu_e H_0^2)\,\nabla^2\phi + m\,(\alpha_{11}-\alpha_{13})\,(2\phi_{,z}+\psi_{,x}) = 2\rho_0\phi_{,tt}, \quad (15)$$

$$(\alpha_{11}-\alpha_{13})\,\nabla^2\psi - m\,(\alpha_{11}-\alpha_{13})\,\psi_{,z} = 2\rho_0\psi_{,tt}, \quad (16)$$

and

$$\alpha_{11}\,\phi_{,xx} + \alpha_{33}\,\phi_{,zz} - 2\alpha_{13}\,m\,\psi_{,x} + 2\alpha_{33}\,m\phi_{,z} = 2\rho_0\psi_{,tt} \quad (17)$$

$$(\alpha_{11}-\alpha_{13}+\mu_e H_0^2)\,\psi_{,xx} + (2\,\alpha_{33}-\alpha_{13}-\alpha_{11})\,\psi_{,zz} + (2\alpha_{13}\,m)\,\phi_{,x} + 2\alpha_{33} m\,\psi_{,z} = 2\rho_0\phi_{,tt}. \quad (18)$$

where $\nabla^2 = \dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial z^2}$.

Since the velocity of waves are different in x and z direction. Now Eq. (15) and Eq. (16) represent the compressive wave along x and z-direction while Eq. (17) and Eq. (18) represents the shear waves along these directions. Since we consider the propagation of Rayleigh waves in x-direction, therefore we consider only Eq. (15) and Eq. (18).

To solve Eq. (18) and Eq. (15) we introduce

$$\phi\,(x, y, z) = f\,(z)\; e^{i\alpha(x-ct)},$$

$$\psi\,(x, y, z) = h(z)\; e^{i\alpha(x-ct)}. \quad (19)$$

putting Eq. (19) in Eq. (15) and Eq. (18) we get

$$f_{,zz} + A f_{,z} + Bf + Ch = 0, \quad (20)$$

$$h_{,zz} + A'h_{,z} + B'h + C'f = 0, \quad (21)$$

where

$$A = \frac{m\,(\alpha_{11}-\alpha_{13})}{\alpha_{11}+\mu_e H_0^2},\; B = \frac{\alpha^2\left(\rho_0 c^2 - \alpha_{11}\right)}{\alpha_{11}+\mu_e H_0^2},$$

$$C = \frac{\left[m\,(\alpha_{11}-\alpha_{13})\right] i\alpha}{2\left(\alpha_{11}+\mu_e H_0^2\right)},$$

$$A' = \frac{2m\alpha_{33}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}},\; B' = \frac{\alpha^2\left(2c^2\rho_0 - \alpha_{11}+\alpha_{13}\right)}{2\alpha_{33}-\alpha_{11}-\alpha_{13}},$$

$$C' = \frac{\left(2\rho_0 g + 2m\alpha_{13}\right) i\alpha}{2\alpha_{33}-\alpha_{11}-\alpha_{13}}. \quad (22)$$

Now Eq. (20) and Eq. (21) have exponential solution in order that f (z) and h (z) describe surface waves and also they varnish as z $\to \infty$ hence Eq. (15) takes the form,

$$\phi\,(x, z, t) = \left[C_1 e^{-\lambda_1 z} + C_2 e^{-\lambda_2 z}\right] e^{i\alpha(x-ct)},$$

$$\psi\,(x, z, t) = \left[C_3 e^{-\lambda_1 z} + C_4 e^{-\lambda_2 z}\right] e^{i\alpha(x-ct)}, \qquad (23)$$

where C_1, C_2, C_3, C_4 are arbitrary constants and λ_1, λ_2 are the roots of the equation

$$\lambda^4 + \left[\frac{m(\alpha_{11}-\alpha_{13})}{\alpha_{11}+\mu_e H_0^2} + \frac{2m\alpha_{33}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}}\right]\lambda^3$$
$$+\ \alpha^2\left[\frac{2\rho_0 c^2-\alpha_{11}+\alpha_{13}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}} + \frac{\rho_0 c^2-\alpha_{11}}{\alpha_{11}+\mu_e H_0^2}\right]\lambda^2$$
$$+\ m\alpha^2\left[\frac{(\alpha_{11}-\alpha_{33})(2\rho_0 c^2-\alpha_{11}+\alpha_{13}) + (\rho_0 c^2-\alpha_{11})\,2\alpha_{33}}{(\alpha_{11}+\mu_e H_0^2)(2\alpha_{33}-\alpha_{11}-\alpha_{13})}\right]\lambda$$
$$+\left[\frac{\alpha^4(\rho_0 c^2-\alpha_{11})(2\rho_0 c^2-\alpha_{11}+\alpha_{13})}{(\alpha_{11}+P_0+\mu_e H_0^2)(2\alpha_{33}-\alpha_{11}-\alpha_{13})}\right.$$
$$\left. +\alpha^2\,\frac{\{m(\alpha_{11}-\alpha_{13})\}\,(2\rho_0 g+2m\alpha_{13})}{2(\alpha_{11}+\mu_e H_0^2)(2\alpha_{33}-\alpha_{11}-\alpha_{13})}\right] = 0. \qquad (24)$$

Here we consider only real roots of Eq. (24). Now the constants C_1, C_2 and C_3, C_4 are related by the Eq (20) and Eq. (21).

By equating the co-efficients of $e^{-\lambda_1 z}$ and $e^{-\lambda_2 z}$ to zero, Eq. (20) gives,

$$C_3 = \gamma_1 C_1,\ C_4 = \gamma_2 C_2, \qquad (25)$$

where

$$\gamma_j = \frac{2i[(\alpha_{11} ?+ \mu_e H_0^2\ \lambda_j^{\,2}\ ?\ \mathrm{m}\,(\alpha_{11}\ 爾\ \alpha_{13})\lambda_j ?+\ \rho_0\ 爾\ \ \alpha_{11})]}{\alpha\left[m(\alpha_{11}-\alpha_{13})\right]}$$

$$(j = 1,\ 2.) \qquad (26)$$

3. Boundary Conditions

The plane z = 0 is free from stresses i.e. $\sigma_{13} = \sigma_{33} = 0$ at z = 0, (27)
where

$$\sigma_{13} = \frac{1}{2}(\alpha_{11} - \alpha_{13})\,[2\,\phi_{,xz} - \psi_{,zz} + \psi_{,xx}]\,e^{mz}, \qquad (28)$$

$$\sigma_{33} = \alpha_{31}\,[\phi_{,xx} - \psi_{,xz}]\,e^{mz} + \alpha_{33}[\phi_{,zz} + \psi_{,zx}]\,e^{mz}. \qquad (29)$$

Introducing Eq. (28) and Eq. (29) in Eq. (27) we have

$$C_1 (2\lambda_1 i\alpha + \gamma_1\lambda_1^2 + \alpha^2\gamma_1) + C_2 [2 \lambda_2 i\alpha + \gamma_2\lambda_2^2 + \alpha^2 \gamma_2] = 0, \quad (30)\ (33)$$

$$C_1 [-\alpha^2 \lambda_{13} + \lambda_1^2 \alpha_{33} - \alpha_1\gamma_1 i\alpha (\alpha_{33} - \alpha_{13})] + C_2 [-\alpha^2\lambda_{13} + \lambda_2^2 \alpha_{33} - \lambda_1 \gamma_1 i\alpha (\alpha_{33} - \alpha_{13})] = 0. \quad (31)$$

Eliminating C_1 and C_2 from Eq. (30) and Eq. (31) ; we have

$$[2\lambda_1 i\alpha + \gamma_1\lambda_1^2 + \alpha^2\gamma_1] [-\alpha^2\lambda_{13} + \lambda_2^2 \alpha_{33} - \lambda_1 \gamma_1 i\alpha (\alpha_{33} - \alpha_{13})]$$
$$- [2 \lambda_2 i\alpha + \gamma_2\lambda_2^2 + \alpha^2 \gamma_2] [-\alpha^2 \lambda_{13} + \lambda_1^2 \alpha_{33} - \alpha_1\gamma_1 i\alpha (\alpha_{33} - \alpha_{13})] = 0, \quad (32)$$

where γ_j (j = 1, 2) are given by Eq. (26) and λ_j (j = 1, 2) are roots of Eq. (24).

Now Eq (32) gives the wave velocity equation for Rayleigh waves in a non-homogeneous elastic half space of orthotropic material under magnetic field. From Eq. (32), it follows that Rayleigh waves depends magnetic field and non-homogeneous character of the medium and nature of the material.

From Eq. (32), we conclude that if α is large i.e. length of wave i.e. $\frac{2\pi}{\alpha}$ is small then , magnetic field and have small effects on Rayleigh waves in non-homogeneous orthotropic half space and if α is small i.e. $\frac{2\pi}{\alpha}$ is large then , magnetic field and plays a vital role for finding out the wave velocity c.

When the medium is isotropic, Eq. (32) becomes

$$[2\lambda_1 i\alpha + \gamma_1\lambda_1^2 + \alpha^2\gamma_1] [K_1^2 (\lambda_2^2 - \alpha^2) + 2 K_2^2 (1 - i\alpha \gamma_2\lambda_2)]$$
$$- [2 \lambda_2 i\alpha + \gamma_2\lambda_2^2 + \alpha^2 \gamma_2] [K_1^2 (\lambda_1^2 - \alpha^2) + 2 K_2^2 (1 - i\alpha \gamma_1\lambda_1)] = 0, \quad (33)$$

where $K_1^2 = \frac{\lambda+2\mu}{\rho}$, $K_2^2 = \frac{\mu}{\rho}$, (λ, μ are Lame's constants). (34)

Eq. (34) determines the Rayleigh waves in a non-homogeneous isotropic elastic solid under the magnetic field.

When initial magnetic field are absent i.e. $H_0=0$ then Eq. (33) reduces to,

$$[2\lambda_1 i\alpha + \gamma_1\lambda_1^2 + \alpha^2\gamma_1] [K_1^2 (\lambda_2^2 - \alpha^2) + 2 K_2^2 (1 - i\alpha \gamma_2\lambda_2)]$$
$$- [2 \lambda_2 i\alpha + \gamma_2\lambda_2^2 + \alpha^2 \gamma_2] [K_1^2 (\lambda_1^2 - \alpha^2) + 2 K_2^2 (1 - i\alpha \gamma_1\lambda_1)] = 0, \quad (35)$$

where $K_1^2 = \frac{\lambda+2\mu}{\rho}$, $K_2^2 = \frac{\mu}{\rho}$.

Eq. (35) determines the Rayleigh surface waves in non-homogeneous isotropic elastic solid under the influence of which is similar to corresponding classical result given by Das *et al.*

When magnetic field is absent, we get same velocity equation for Rayleigh waves in non-homogeneous elastic solid as eq (32) with

$$\gamma_j = \frac{2i[(\alpha_{11})\lambda_j^2 \text{ ? } m(\alpha_{11} \text{爾} \alpha_{13})\lambda_j \text{ ? } + \rho_0 \text{藊爾} \alpha_{11})]}{[\alpha m(\alpha_{11} - \alpha_{13})]} \ ; j = 1, 2,$$

where λ_1, λ_2 are roots of the equation

$$\lambda^4 + \left[\frac{m(\alpha_{11} - \alpha_{13})}{\alpha_{11}} + \frac{2m\alpha_{33}}{2\alpha_{33} - \alpha_{11} - \alpha_{13}}\right]\lambda^3$$

$$+\alpha^2\left[\frac{2\rho_0c^2-\alpha_{11}+\alpha_{13}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}}+\frac{\rho_0c^2-\alpha_{11}}{\alpha_{11}}\right]\lambda^2$$

$$+\ m\alpha^2\left[\frac{(\alpha_{11}-\alpha_{33})\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}\right)+\left(\rho_0c^2-\alpha_{11}\right)2\alpha_{33}}{\alpha_{11}\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}\right)}\right]\lambda$$

$$+\left[\frac{\alpha^4\left(\rho_0c^2-\alpha_{11}\right)\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}\right)}{\alpha_{11}\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}\right)}\right.$$

$$\left.+\alpha^2\ \frac{\{m(\alpha_{11}-\alpha_{13})\}\ (m\alpha_{13})}{(\alpha_{11})\ (2\alpha_{33}-\alpha_{11}-\alpha_{13})}\right]=0. \qquad (36)$$

When $H_0=0$

we get, velocity equation for Rayleigh waves is similar to Eq. (32) with

$$\gamma j=\frac{2i[(\alpha_{11}\lambda_j{}^2\ ?\ \ \mathrm{m}\ (\alpha_{11}\ \text{爾}\alpha_{13})\lambda_j\ ?\!+\ \rho_0\ \text{霥爾}\alpha_{11})]}{\alpha\left[m\left(\alpha_{11}-\alpha_{13}\right)\right]}\quad ;\ j=1,\ 2,$$

where λ_1, λ_2 are roots of the equation

$$\lambda^4+\left[\frac{m\left(\alpha_{11}-\alpha_{13}\right)}{\alpha_{11}}+\frac{2m\alpha_{33}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}}\right]\lambda^3$$

$$+\ \alpha^2\left[\frac{2\rho_0c^2-\alpha_{11}+\alpha_{13}}{2\alpha_{33}-\alpha_{11}-\alpha_{13}}+\frac{\rho_0c^2-\alpha_{11}}{\alpha_{11}}\right]\lambda^2$$

$$+\ m\alpha^2\left[\frac{(\alpha_{11}-\alpha_{33})\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}\right)+\left(\rho_0c^2-\alpha_{11}\right)2\alpha_{33}}{\alpha_{11}\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}\right)}\right]\lambda$$

$$+\left[\frac{\alpha^4\left(\rho_0c^2-\alpha_{11}\right)\left(2\rho_0c^2-\alpha_{11}+\alpha_{13}\right)}{\alpha_{11}\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}\right)}+\alpha^2\ \frac{\{m(\alpha_{11}-\alpha_{13})\}\ (2m\alpha_{13})}{2\alpha_{11}\left(2\alpha_{33}-\alpha_{11}-\alpha_{13}\right)}\right]=0. \qquad (37)$$

When the non-homogeneity of the material, $H_0=0$, and field are absent further medium is initially unstressed and isotropic, Eq. (32) reduces to,

$$4\sqrt{\left(1-\frac{c^2}{K_1{}^2}\right)\left(1-\frac{c^2}{K_2{}^2}\right)}=\left(2-\frac{c^2}{K_2{}^2}\right)^2, \qquad (38)$$

where $K_1{}^2=\dfrac{\lambda+2\mu}{\rho}$, $K_2{}^2=\dfrac{\mu}{\rho}$.

Eq. (38) is similar to the equation given by Rayleigh.

4. Conclusions

1. Equation (32) represents the wave velocity equation for the Rayleigh waves in a non-homogeneous, orthotropic elastic solid medium under the influence of magnetic field.
2. It also depends upon the wave number and confirming that waves are dispersive. Moreover, the dispersion equation contains terms involving magnetic field and non-homogeneity, so the phase velocity 'c' not only depends upon magnetic field but also on the non-homogeneity of the material medium.

3. The explicit solutions of this wave velocity equation cannot be determined by analytical methods. However, these equations can be solved with the help of numerical method, by a suitable choice of physical parameters involved in medium.

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