Assisting Basic Six Pupils of Presbyterian Women's College of Education Demonstration School, Aburi-Akuapem, To Discover the Product of Two Multi-Digit Multiplicands Using the Lattice

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Abstract

An action research was performed with the aim of solving problems involving two multi-digit multiplicands using the lattice to give pupils' conceptual understanding of multiplication in basic six at the Presbyterian Women's College of Education Demonstration School. Twenty-five pupils out of fifty (50) were selected using The simple random sampling technique from a combined class of Six 'A' and Six 'B' who have difficulties in answering questions under multiplication involving two multi-digit numbers, after writing a teacher made test (Pre-Test). The Pre- Test produced 16% pass mark as compared to the post-test which gave 88% pass mark after the intervention. The discovery method of teaching was used in the intervention procedure to find the multiplication of multi-digit numbers. Pupils were directed to use the lattice to perform activities which had the potential of developing their conceptual understanding of the topic. The data collected indicated that some pupils improved their understanding but some pupils needed more time to re-learn the subject. This research only presents some practical solutions proposed within the time available.

Keywords: Action Research, Pre- Test, post-test, Multi-Digit Numbers

1. Introduction

It is commonly agreed that learning with understanding is more desirable than learning by rote. Understanding is described in terms of the way information is represented and structured in the memory. A mathematical idea or procedure or fact is understood if it is a part of an internal network, and the degree of understanding is determined by the number and the strength of the connections between ideas. When a student learns a piece of mathematical knowledge without making connections with items in his or her existing networks of internal knowledge, he or she is learning without understanding (Onyebuchi, 2009).

Narh (2011) asserts that if the educational structure cannot give then pupils cannot have the requisite materials and technical know-how needed to contribute their quota towards the development of Ghana. As such Mathematics and the foundations of Mathematics is one of the central subjects within the list of foundation subjects that form the core curriculum in the educational structures for basic and secondary education in most countries throughout the world. The subject requires a high level of understanding which occupies a confidential position in the school curriculum because an individual's ability to cope with it improves one's chances of basic progression. This position was attained ever since it was made to replace classical language such as Latin or Greek which was used as screening devices for entry to higher education levels and professions prior to the early half of the twentieth century (Howson and Wilson, 1989) as cited in Mereku (2000) and Narh (2011).

The importance of mathematics can be seen from its application in our daily lives and its role in technology. No other subject forms a strong binding force among various branches of science than mathematics and without it; knowledge of science often remains superficial Moyer, (2001) as cited in Narh (2011). In addition to that, there has been unimpressive performance in mathematics over the past years as evidenced in National Education

Assessment and Trends in International Mathematics and Science Study (TIMSS) report. (Anamuah-Mensah, Mereku and Ghartey-Ampiah, 2008). In other words, the effective use of the principles in mathematics, the necessary concepts and skills cannot be acquired and applied by pupils if they do not have a good foundation in mathematics.

A mathematics curriculum framework released by the US National Council of Teachers of Mathematics (NCTM, 2000) offers a research-based description of what is involved for students to learn mathematics with understanding. The approach is based on "how learners learn, not on "how to teach", and it should enable mathematics teachers to see mathematics from the standpoint of the learner as he progresses through the various stages of cognitive development. The focus in the present study is to try to find out what aspects of the process of teaching and learning seem to be important in enabling students to grow, develop and achieve. The attention here is on the learner and the nature of the learning process.

Mathematics is a symbolic language in which problem-situations and the solutions found are expressed. The systems of mathematical symbols have a communicative function and an instrumental role. Mathematics is a logically organized conceptual system. Once a mathematical object has been accepted as a part of this system, it can also be considered as a textual reality and a component of the global structure. It may be handled as a whole to create new mathematical objects, widening the range of mathematical tools at the same time, introducing new restrictions in mathematical work and language.

Over the past few years, teaching multiplication largely meant helping children learn their times tables. Learning multiplication was ALL about memorization and rote without any understanding. However, after years of pure mathematics research, the focus is now on helping children to see a bigger picture of multiplication, like when it's used, why it's more useful than counting, what the different real world applications are, etc. That's generally the more difficult aspect of teaching multiplication (ie: ensuring that real world applications are discussed in your multiplication lessons.

Multiplication is denoted by the cross symbol "×" is one of the basic mathematical operation of arithmetic, the others being addition, subtraction and division. Multiplication is a binary operation that operates on a pair of numbers to produce another number. Given a pair of numbers a and b called factors, multiplication assigns them a value $a \times b = c$, called their product.

The Primary School mathematics syllabus (CRDD, 2007) for instance, has some of its objectives as: socialize; adjust to and handle number words; perform number operations; make use of appropriate strategies of calculation; Recognize and use functions, formulae, equations and inequalities; use graphical representation of equations and inequalities; identify /recognize the use of the arbitrary standards of measurements; identify solid shapes in daily life; Manipulate learning materials to enhance understanding of concepts and skills and Collect, analyze and interpret data and operations.

During the researcher's interaction with basic six pupils of the PWCE demonstration school, it was clear that pupils had difficulties in the multiplication of multi-digit numbers. This was evident in the outcome of the first term examination for the 2007/2008 academic year. In addition to that, a diagnostic test the researchers conducted for pupils (see appendix A) revealed that out of the thirty-five (35) pupils who took the test only ten (10) answered all the questions correctly. This outcome showed that most of the students could not answer the questions on the multiplication of multi-digit numbers, even though they made the attempt. This result confirmed the researcher's earlier findings.

It is based on these findings that the researchers has decided to use the guided discovery method to assist basic six pupils of PWCE demonstration school to overcome difficulties in the multiplication of multi-digit numbers. We anticipate that the outcome of this research will contribute greatly in informing teachers on adapting varying teaching strategies in facilitating qualitative understanding, suggesting to teachers, pragmatic ways of developing students' attitudes towards the learning of mathematics at all levels.

2. Literature Review

2.1. Concepts

A concept is a mental image, generalization, of certain characteristics and aspects that make up an item. In other words, Concepts are mental categories for objects, events, or ideas that have a common set of features".

Concepts are used to describe all examples of items under that category and separate them from non-examples. Alessi and Trollop (2001) explained that an appropriate teaching design is to first teach relevant features by stating a definition of the concept in terms of these features.

Concept learning encompasses learning how to differentiate and categorize things. It also involves recall of instances, combination of new examples and sub-categorization. Concept formation is not related to simple recall, it must be constructed.

Drah, Adu & Mereku (2014) cited that by using concepts, students are "freed from the control of specific stimuli in the environment" thereby enabling them to learn by means of verbal instruction presented orally or as printed matter. As such a formal definition of concepts helps us to form an image of the concept, but do not guarantee the understanding of the concept. It happens that the moment learners have formed their concept images or their subjective conceptions of mathematical concepts the definitions become unnecessary. Pupils have an intent to interpret the mathematical concepts operationally as processes even if the concepts in teaching of mathematics were introduced structurally, by using definitions (Sfard, 1989).

Practical knowledge refers to computational skills and knowledge of measures and actions for identifying mathematical components, algorithms and definitions. Conceptual knowledge refers to knowledge of the underlying structure of mathematics. This is knowledge rich in relationships which involves the understanding of mathematical concepts, definitions and fact knowledge. In other words, practical and conceptual knowledge are essential aspects of mathematical understanding. Mathematical understanding must include teaching in both practical and conceptual knowledge.

2.2. The concept of multiplication

The concept of multiplication has been defined by great mathematicians, tutors and students a like as repeated addition. This is evident in the mathematical operation 8 x 6 = 8 + 8 + 8 +8 +8 + 8. In effect, multiplication is adding a number on a number of occasions hence bringing the idea of repeated addition.

Report on the assessment of the two hypotheses through an intervention method brought out the conclusion that multiplication is based on two alternative hypotheses. The first hypotheses suggests that the concept of multiplication is grounded on the understanding of repeated addition, whiles the second proposes that repeated addition is only a calculation procedure and that the understanding of multiplication has its roots in the schema of the correspondence These hypotheses have been offered to explain the origin of the concept of multiplication in children's reasoning. (Park & Nunes, 2001),.

Keith (2012) argues that multiplication as repeated addition does not work with all numbers. He explains that with positive integers, it is perfect but not successful with negative integers. For example, 6 x -3 makes sense to stretch -3 out 6 times, but the idea of -3 x 6 breaks down because it is not possible to stretch something out three times much more to talk of -3 x -6. If one would stretch to the left on the number chart, there is a consistency. But there is confusion within -3 x -6 in that how does one stretch -6 a total of -3 times?

In addition to that Keith explains that ''multiplication as scaling'' in terms of resizing, by comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication; and explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence $a/b = (n \times a)/(n \times b)$ to the effect of multiplying a/b by 1. The identity property of multiplication tells us that a number x 1 has a product equal to the number (itself). For example, 5 x 1 = 5. Thus, when we multiply a number by a fraction that is less than 1 our product has to be less than the number we are multiplying the fraction by. For example, 8 x 1/4 = 2. The product (2) is less than the original factor 8 because the second factor (1/4) is less than 1. Conversely, when we multiply a number by a number greater than 1 (including fractions/mixed numbers) our product is greater than the original factor. For example, 8 x 5/8 = 5. The product (5) is greater than the first factor (8) because the second product is greater than 1.

2.3 Perspectives in Teaching

Discovery learning

Brunner (1967), who is one of the proponents of discovery learning explains that this theory:

- Actively engages students in the learning process
- Motivates students to participate
- Encourages autonomy and independence
- Promotes the development of creativity and problem-solving skills
- Provides a individualized learning experience.

According to Spencer (1999), key features of guided discovery learning are:

- A context and frame for student learning through the provision of learning outcomes
- Learners have responsibility for exploration of content necessary for understanding through self directed learning
- Study guides are used to facilitate and guide self directed learning
- Understanding is reinforced through application in problem oriented, task based, and work related experiences

Castronova (2002), also identifies five distinctiveness of discovery learning as:

- learning is active and pupils participate in hands-on and problem-solving activities rather than knowledge transfer.
- emphasizes the process instead of the end product, thus encouraging mastery and application.
- encourages the pupils to continue to search for solutions in the lessons learnt from failure within this model of instruction.
- feedback becomes an essential part of the learning process and that teamwork and discussion allows pupils to develop deeper understanding of the various concepts.
- satisfies natural human curiosity and promotes individual interests.

2.4 Action Research

Action research is a disciplined inquiry done by a teacher with the intent that the research will inform and change his or her practices in the future. It is carried out within the context of the teacher's environment with the students and at the school in which the teacher works on questions that deal with educational matters at hand. As a process it helps teachers to examine their own educational practices systematically using the techniques of research (Ferrance, 2000, p 1).

Fisher & Phelps, (2006), explains that Action Research is an applied scholarly paradigm resulting in action for a specific situation offering faculty instant benefits by improving his or her own teaching and providing explicit documentation for meeting their educational responsibilities as required by standards. It seeks to document the context, change processes, resultant learning and theorizing of faculty in developing their pedagogies.

2.5 Types of Action Research

There are different types of Action research. These include Individual teacher research, Collaborative action research, school wide action research as well as District wide action research.

Individual teacher research usually focuses on a single issue in the classroom. The teacher may be seeking solutions to problems of classroom management, instructional strategies, use of materials, or student learning. The problem is one that the teacher believes is evident in his or her classroom and one that can be addressed on an individual basis.

Collaborative action research may include as few as two teachers or a group of teachers and others interested in addressing a classroom or departmental issue. This issue may involve one classroom or a common problem shared by many classrooms. Teachers can come together to identify and diagnose a problem and develop a solution through collaboration. Collaboration amongst teachers is necessary if they are to tackle action research in the context of their working life in school and become more fully engaged in working on pupil learning and participation (Davis and Howes 2007).

School-wide research focuses on issues common to all in the school. For example, a school may have a concern about the lack of parental involvement in activities, and is looking for a way to reach more parents to involve them in meaningful ways. Or, the school may be looking to address its organizational and decision-making structures. Teams of staff from the school work together to narrow the question, gather and analyze the data, and decide on a plan of action. An example of action research for a school could be to examine their state test scores to identify areas that need improvement, and then determine a plan of action to improve student performance. District-wide research is a research in which a district may choose to address a problem common to several schools or the district.

2.6 Action research and Professional development

Action research helps improve the teacher's self esteem. Documented action research can be one source of solving problem. It also helps in developing the teacher's repertoire. Finally, the teacher can use action research he or she has done to help others whenever they have difficulties. In short, when we help others, we help ourselves. Hence Action Research makes the teacher a Reflective Practitioner and a Researcher.

3. Methodology

Introduction

This section highlights the methodology that was used in conducting the research. It gives a vivid description of the population, sample, sampling techniques as well as the type of instruments used in the data collection process.

3.1 Research Design

The researchers design for this study was an action research. It involves per- test, observation and post-test. Some variables to be considered in this research are interest and performance.

3.2 Population

The targeted population is basic six pupils of Presbyterian womens college of Education demonstration school in the Akuapem South District of Ghana. The rationale for the choice of population is that:

- (i) the researchers teach the class in question.
- (ii) the concept of multiplication is emphasized at that level of the basic education.
- (iii) Students should have a firm grasp of the concept at that level of education.

3.3 Sample size and Sampling Procedure

Twenty-five (25) students out of fifty (50) from the class were the selected sample. The sample was selected randomly from a group of pupils who had difficulties in answering questions under multi-digit multiplicands, after writing a teacher made test (pre test).

3.4 Research Instruments and their Description

Based on the topic under consideration and the population visa-vise the sample, the researchers found it appropriate to use diagnostic test (teacher made), observation and post- test for data collection. A sample of the diagnostic test items (see appendix B) were used to find out specific problems students face when answering questions on multiplication of multi-digit numbers. Test items were carefully structured to check ambiguities and inconsistencies. This was to ensure reliability and validity of the study, hence the relating of diagnostic test items to exercise 10.4 of the Mathematics made easy for primary schools book 6 (Sackitey & Agyedabi, 2012: 223).

3.5 Administration of Research Methodology

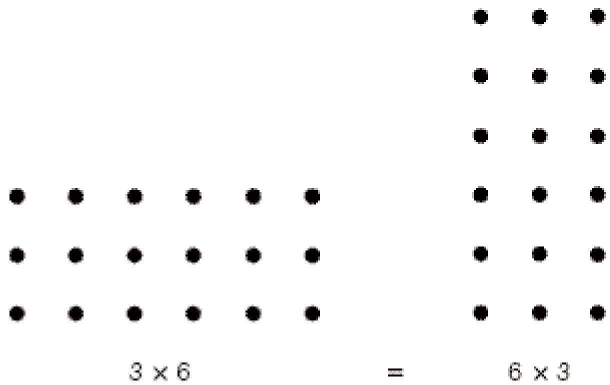
Introduction

The researchers in their quest to remedy the difficulties faced by pupils designed an intervention.

3.6 Intervention

Multiplication has certain fundamental properties that are of great importance in arithmetic. The Commutative Property of Multiplication states that changing the order in which two numbers are multiplied does not change the product. That is, for all numbers a and b, $a \times b = b \times a$.

The array model can be used to make this plausible. For example, because $3 \times 6 = 6 \times 3$, an array with 3 rows and 6 dots in each row has the same number of dots as an array with 6 rows and 3 dots in each row.



Another important property of multiplication is the Identity Property of Multiplication. It states that the product of any number and 1 is that number. That is, for all numbers a, $a \times 1 = 1 \times a = a$.

The Zero Property of Multiplication states that when a number is multiplied by zero, the product is zero. That is, for all numbers a, $a \times 0 = 0 \times a = 0$.

Using the pencil and paper, the common methods for multiplying require a multiplication table numbers of memorized or consulted products of small numbers (typically any two numbers from 0 to 9), however one method, the peasant multiplication algorithm, does not.

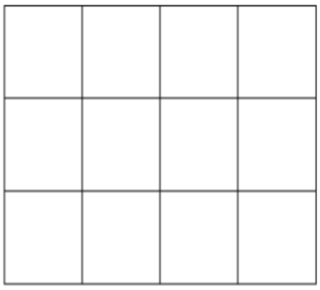
Lattice multiplication is a method of multiplying large numbers using a grid. It is algorithmically equivalent to regular long multiplication, but the lattice method breaks the multiplication process into smaller steps, which some students find easier. Digits to be carried are written within the grid, making them harder to miss.

3.7 Description

Step 1

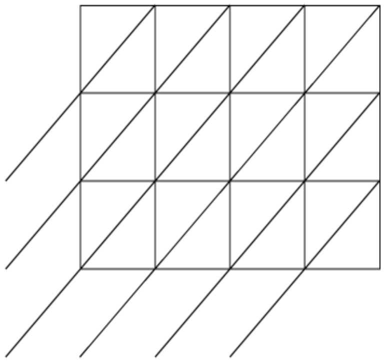
Setting up the lattice

Pupils are guided to draw a grid as shown below and each cell splitted diagonally. The grid that is constructed should have as many rows and columns as the multiplicand and the multiplier. The grid shown here is for multiplying a 4-digit number by a 3-digit number.



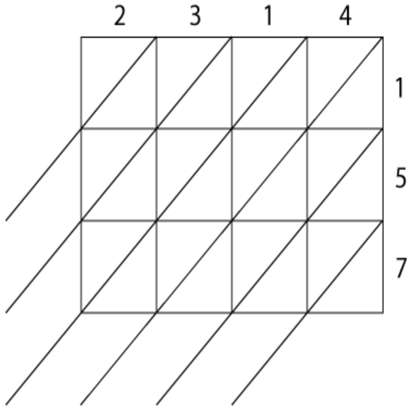
Step 2

Next, draw a diagonal through each box from upper right corner to lower left corner. Continue the line a short way past the grid.



Step 3

Teacher guides pupils to write one factor across the top and the other down the right side, lining up the digits with the boxes/cells.



Explain to pupils that the two multiplicands of the product to be calculated are written along the top and right side of the lattice respectively, with one digit per column across the top for the first multiplicand and one digit per row down the right side for the second multiplicand. Then each cell of the lattice is filled in with product of its column and row digit.

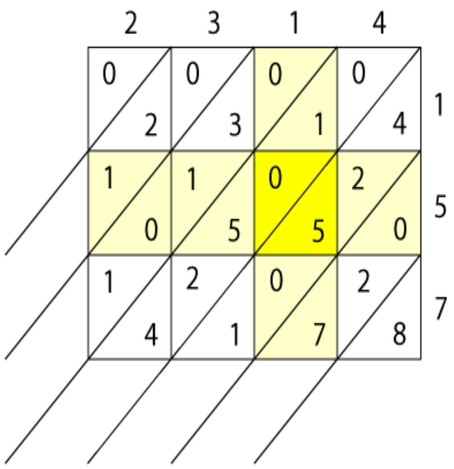
For example, if the column digit is 5 and the row digit is 2, then the answer 10 will be written in the cell, with the digit 1 above the diagonal and the digit 0 below the diagonal. On the other hand if the column digit is 8 and the row digit is 1, then the answer 8 will be written in the cell, with the digit 0 above the diagonal and the digit 8 below the diagonal (see picture for Step 2).

Pupils are made to discover that if the simple product lacks a digit in the tens place, simply fill in the tens place with a 0.

Step 4

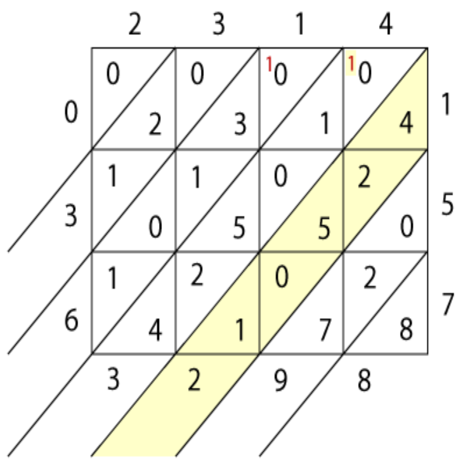
Teacher explains to pupils that the multiplication is performed by multiplying the digits at the head of each row and column. Fill in each square of the grid with the product of the digits above and to its right, recording the products so that the tens are in the upper (diagonal) half of the square and the ones are in the lower half. If the product does not have a tens digit, record a zero in that triangle.

In the example shown here, the highlighted row and column gives us 1 ×5=5, so we write 0 in the upper half of the square and 5 in the lower half.



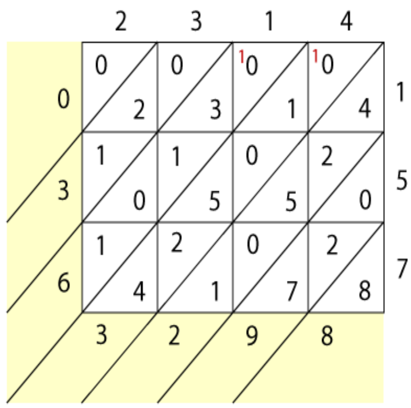
Step 5

Teacher guides pupils to add the numbers in the grid along the diagonals, starting from the lower right corner. Carry any tens into the top of the next diagonal. The highlighted diagonal gives us 4 + 2 + 5 + 1 = 12, so we write 2 at the bottom of the diagonal and carry the 1 to the top of the next diagonal to the left.



Step 6

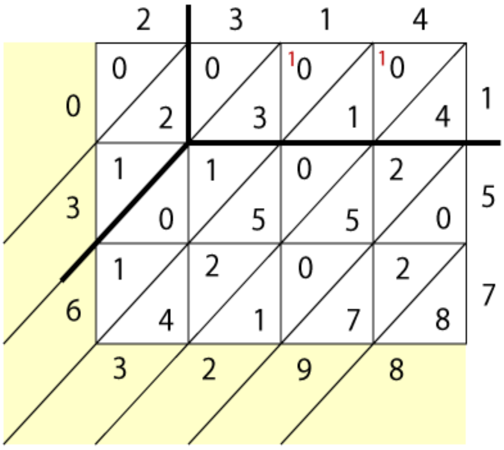
To find the answer, guide pupils to read the digits starting down the left of the grid and continuing across the bottom. Here, the answer to 2314 × 157 is 363,298.



3.8 Lattice Multiplication of Decimals

Multiplying numbers to more than a couple of decimal places by hand is tedious and error prone. The lattice technique can also be used to multiply decimal fractions. For instance, to multiply 5.8 by 2.13, a line would be drawn straight down from the decimal in 5.8, and a line straight out from the decimal in 2.13. The lines are extended until they reach each other, at which point they merge and follow the diagonal. The positioning of this diagonal line in the final result is the location of the decimal point.

Lattice multiplication can easily be extended to multiply decimal fractions. To multiply 2.314 by 1.57, we draw lines from the decimal points down and to the left until they meet, then follow the diagonal to the left or bottom of the grid. The point where this diagonal emerges from the grid is the position of the decimal point in the answer.



Finally, the result of the multiplication $2.314 \times 1.57 = 3.63298$.

4. Data Analysis

The data obtained from the pre-test and post-test were recorded and analyzed using basic descriptive statistics such as percentages, mean, maximum and minimum values and bar charts.

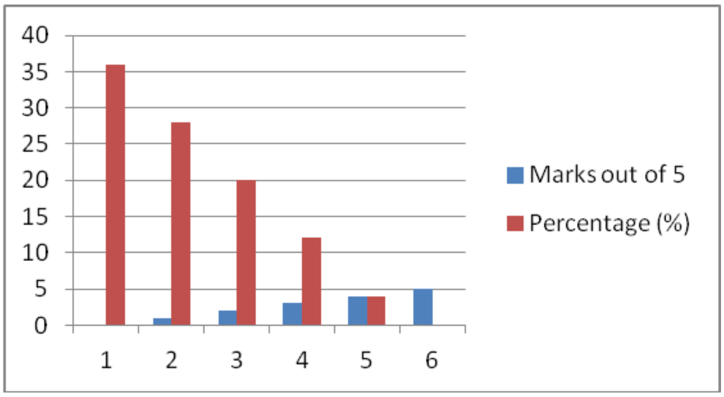
4.1 Analysis and Interpretation of Results

The results obtained were analyzed in both the pre-test and post-test respectively as follows

Analysis of pre-test

Table 1

Marks out of 5	Number of students	Percentage (%)
0	9	36
1	7	28
2	5	20
3	3	12
4	1	4
5	0	0

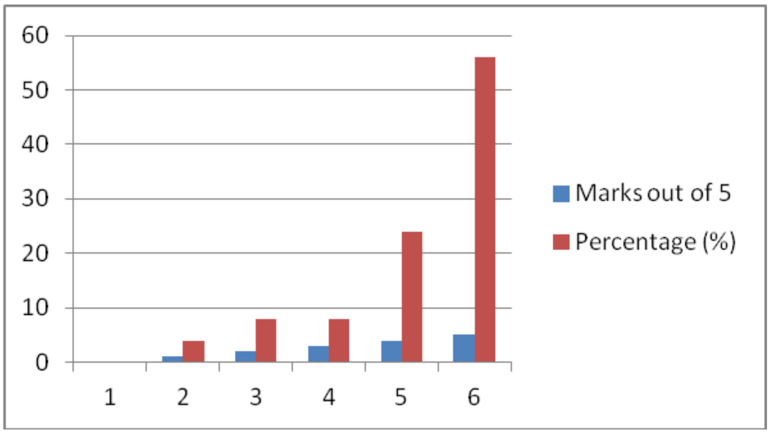


From the pre-test which was marked over 5, a mean of 1.2 was obtained. 36% of the students scored a minimum mark of 0 while 4% had a maximum score of 4 marks. Based on the pre-test data, it is obvious that 84% of the students failed the pre-test with 16% making the pass mark.

Analysis of post-test

Table 2

Marks out of 5	Number of students	Percentage (%)
0	0	0
1	1	4
2	2	8
3	2	8
4	6	24
5	14	56



The result of the post-test indicates a mean mark of 4.2, a maximum of 5 marks was scored by students representing 56%. A minimum mark of 1 representing 4% was also scored by students. Hence the post-test results indicate that 12% of the students failed while 88% passed the test.

Interpretation of Results

The researchers having done a good analysis of the pre-test and post-test results were of the view that pupil's inability to find the product of two multi-digit multiplicands which was discovered through observation can be minimized. The frequent use of teaching and learning materials in the classroom will give a good bearing on pupil's performance.

Pupils' difficulties in solving problems involving multi-digit multiplicands were also noticed from the pre-test that was conducted. This was checked using the lattice. Enough evidence of this was gotten from the pre-test which produced 16% pass as compared to the post-test which gave an 88% pass mark.

The post-test data however is an indication that pupil's ability to multiply two multi-digit numbers can be done using discovery activities.

5. Conclusion

In conclusion, the twenty-five pupils took part in the study. Two test were employed. These were the pre-test and the post-test. A cautious study at the analyzed data of the pre-test and post-test results points out that the mean mark of 4.2 for the post-test is an evidence of improvement in students' performance. From a 16% pass mark of the pre-test and an 88% pass from the post-test, the researchers are convinced that the guided discovery teaching method is a prudent intervention measure.

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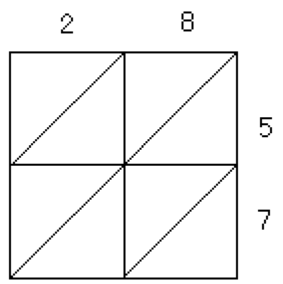
APPENDIX A

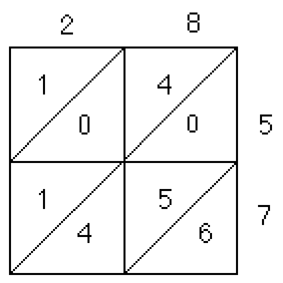
Find the product of 28 $\times$ 57

Solution

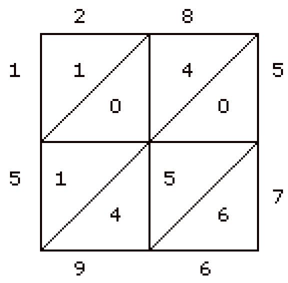
As 28 and 57 have two digits each, a lattice is set out with two columns and two rows. The diagonals are drawn in each cell as shown below. 28 is written above the lattice with 2 above the first column and 8 above the second. 57 is written to the right of the lattice with 5 along the first row and 7 along the second.



The partial products of these digits taken two at a time is set out in the corresponding cells with the tens above the diagonal and ones below. For example, the partial products in this case are 5 $\times$ 8 (= 40), 5 $\times$ 2 (= 10), 7 $\times$ 8 (= 56) and 7 $\times$ 2 (= 14).These are set out as shown below.



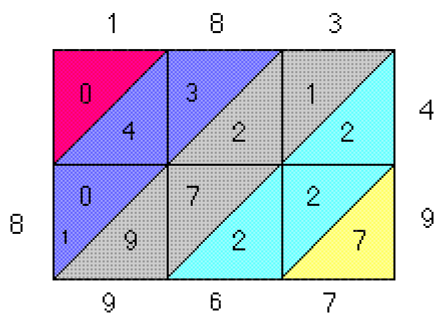
The sum along each diagonal is then recorded as shown below and these digits 1, 5, 9 and 6 form the answer to the multiplication. As usual, start adding at the ones (in this case '6' which comes from multiplying 8 ones by 7 ones), proceeding from right to left around the lattice.



Thus 28 × 57 = 1596

Find the product of 183 × 49 = 8967

The lattice set out for this multiplication will have 3 columns and two rows as 183 has 3 digits (it could also be done as 2 columns and 3 rows as 49 × 183). As before the numbers are set out as shown below and the partial products are written down in their respective positions. The numbers along the diagonals are added to give the answer.



Note that in this example adding along the third diagonal gives 19 which needs 1 to be carried to the diagonal to the left, in other words, 19 hundreds is 10 hundreds + 9 hundreds, then the 10 hundreds is renamed as 1 thousand and the 1 is then written in the thousands column. Therefore the addition should begin with the lowest diagonal on the right hand side (the product of the ones from the two numbers).

183 × 49 = 8967

APPENDIX B

A Sample of the Post – Test Questions

Use the lattice to find the product of

1. 18 × 409

2. 218 × 570

A Sample of the Post – Test Questions

Use the lattice to find the product of

1. 583 × 49

2. 28 × 57