Modeling of Piecewise functions via Microsoft Mathematics: Toward a computerized approach for fixed point theorem

Dr. Faisal Abdulateef Shaghati

Open Educational College, Mathematics Department, Iraq – Baghdad, Al adhamiya - near the judicial institute

E-mail : fisalshagati@yahoo.com

Abstract

This paper basically relies on topology, math-analysis & instructional programming, it details within a mathematical modeling of piecewise defined functions through strict computer programming instructions, It is also going on math-analysis of those functions & other multi-functions as well as it offers many illustrations for their impact while doing mathematics or models programming, it implicitly appears when we deal within analysis of different function topics & especially meanwhile a computerized approach for fixed-point theorem.

Keywords: Smooth piecewise , continuous piecewise , Fixed point theorem , Microsoft Mathematics program

Introduction

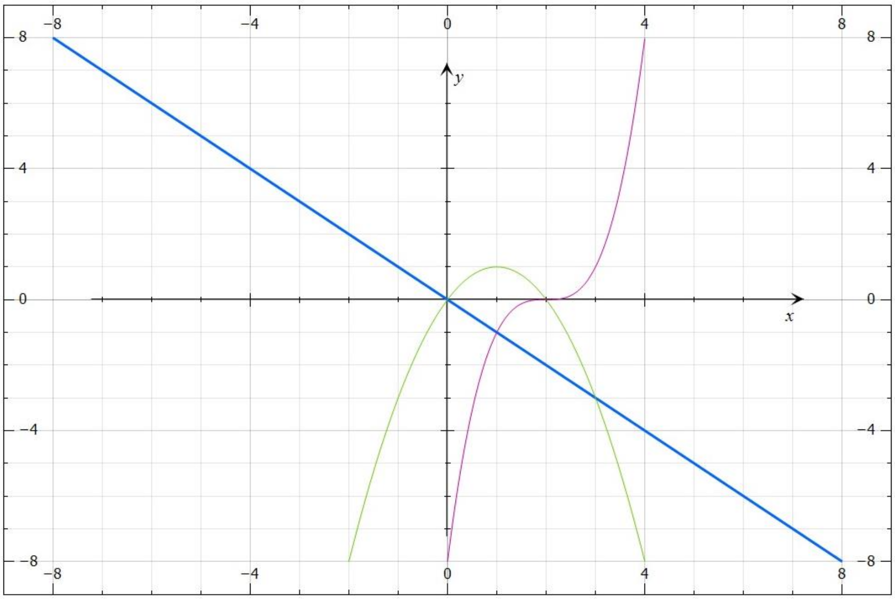
It is relevant to assign that a modeling process had been applied on a spectrum of piecewise defined functions, continuous, smooth as well as chaotic ones, the apparatus used for that application were me-PC and Microsoft-Math program ,a part of this work went in a transparent discussion about the piecewise functions ,its analysis as well as their construction & models, those models presented at this paper are constructed, programmed, elaborated & analyzed orderly to be intact within the main concept going to construct specific models for T(x) & $T_2(x)$ functions, these models were granted to enhance an elegance approach for fixed-point theorem. Microsoft-Math Program instructions that used for models construction are usually written at its worksheet, for symmetry & fitness purpose we preferred to let them appear at an index attached to this work. Also it seems worthwhile to remind that Microsoft Mathematics instructions are compatible and/or easily adaptable across it's well developed versions.

1. Mathematical Processing

We would firstly detail within Piecewise-defined functions, consider the one below:

$$F(x) = \left\{ \begin{array}{ll} -x & -\infty<x\leq 0 \\ -x^2+2x & 0\leq x\leq 2 \\ (x-2)^3 & 2\leq x <\infty \end{array} \right\}$$

Then, we have to graph a function, factually they are three functions defined over three sub-domains, plot instructions of Math-program Plot (1), plot (2), plot (3) produce the following output:



Model (1)

Although one aspect of the picture assign to the most significance feature of the function sample, that is constructed from three parts where every part of them is a function, we may ask our self, Dose it the real graph we sake for it ? Mathematical analysis gives a negative answer, therefore why?

The first reason is obvious, that we have to consider it as a function constructed of strict but respectively connected functions within each other as one function, although it is not necessary for every piecewise defined function to be in continuous fashion, but the consequent domains (-∞, 0], [0,2], [2, ∞) clearly refer to this formal continuity, paradoxically the function target is also piecewise continuous but in view which isn't only contradicted with the intuitive definition of continuity but it relatively differs within the continuity concept in calculus as it will be discussed in details.

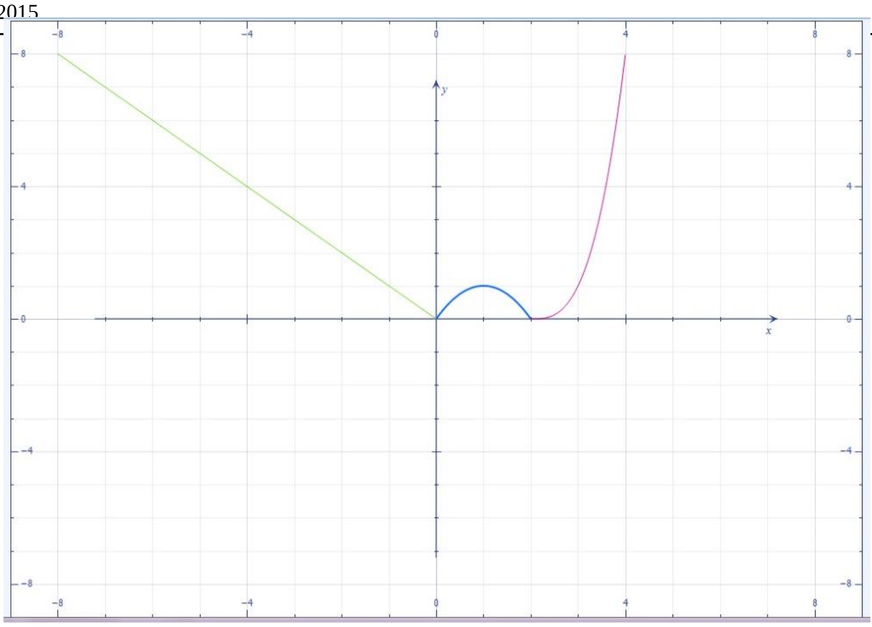
The second reason involved in the first, but it related within comparison between these piecewise functions domains and the domains of their images at the graph-board, whereby the domains of these pieces are (-∞, 0], [0, 2], (2, ∞) respectively, the graph tells the same domain, i.e. (-∞, ∞) for every one of them.

Therefore, what could we do to obtain the reasonable & readable graph of such function as well as those which look like them?

From the very beginning, we already recognize that the graph of a given function is just its image on the coordinate axes , it`s only a cut of its real path for limited domain and in turn for limited range, thus we mostly can`t draw the whole picture of a function, it most specially touches the sample case under our investigation, hereafter the best technique is to construct a short program in order to strictly plot every piece of the sample function taking in account its suitable sub-domain and consequently its sub-range.

One thing is remained, the plot instructions usually used to create graphs for those pieces are written at the worksheet of Math program, wherein it couldn't be well controlled through the INPUT panes of the graphing tab, despite that some functions specially those of two pieces within different formulas can be easily fitted there with an appropriate arrangement of their ranges.

Therefore, just assigning redraw-icon at the worksheet of Math-program within soft click, we ad hoc obtain the graph correspondence within our mathematical imagination as well as obviously readable in accordance with its entire domain (-∞,∞), as it shown at the Model (2).

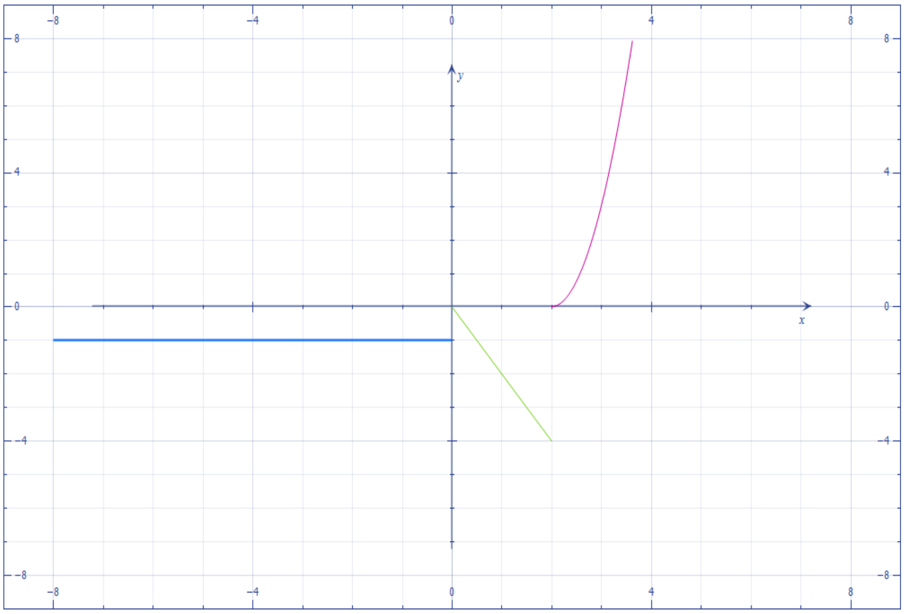


Model (2)

The model above is an instance for piecewise defined function, due to that its pieces are differentiable then continuous and separated by finite points ,then it is piecewise continuous [1] , more details will be at the ascending paragraph. Just now, we would like to investigate another class of piecewise functions, then let us consider the 1st derivative of the preceding function:

$$D\,f(x) = \begin{cases} -1 & -\infty < x \leq 0 \\ -2x & 0 \leq x \leq 2 \\ 3(x-2)^2 & 2 \leq x < \infty \end{cases}$$

The graph below represents its model:

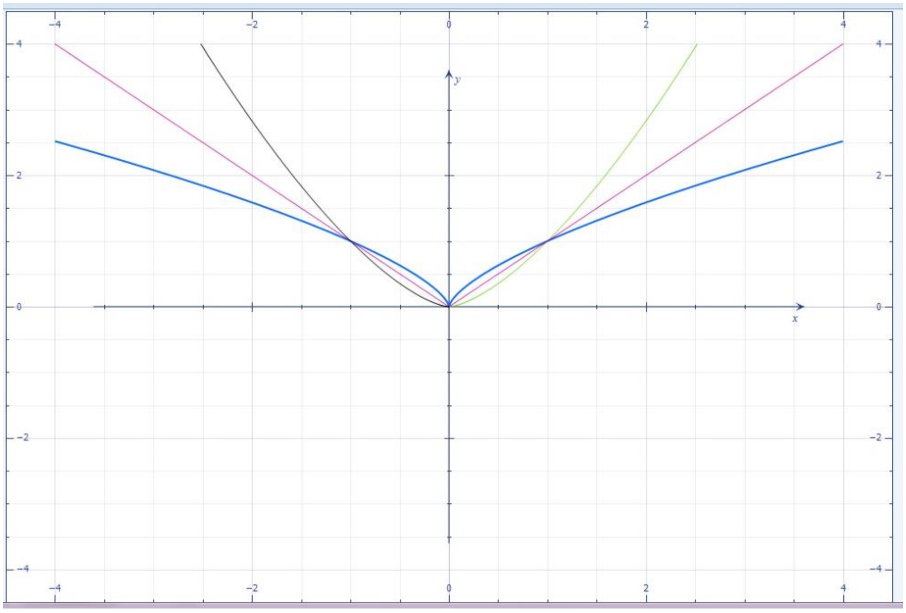


Model (3)

Obviously it is piecewise continuous, also its pieces are separated by only many isolated points; therefore it verifies the conditions of smooth piecewise function [4],[8].

Another type is a multifunction but not every one of its function can be considered as piecewise-defined one

$$F(x) = \left\{ \begin{array}{ll} \sqrt[3]{x^2} & -\infty < x < \infty \\ \sqrt{x^3} & 0 < x < \infty \\ \sqrt{-x^3} & -\infty < x < 0 \\ |x| & -\infty < x < \infty \end{array} \right\}$$

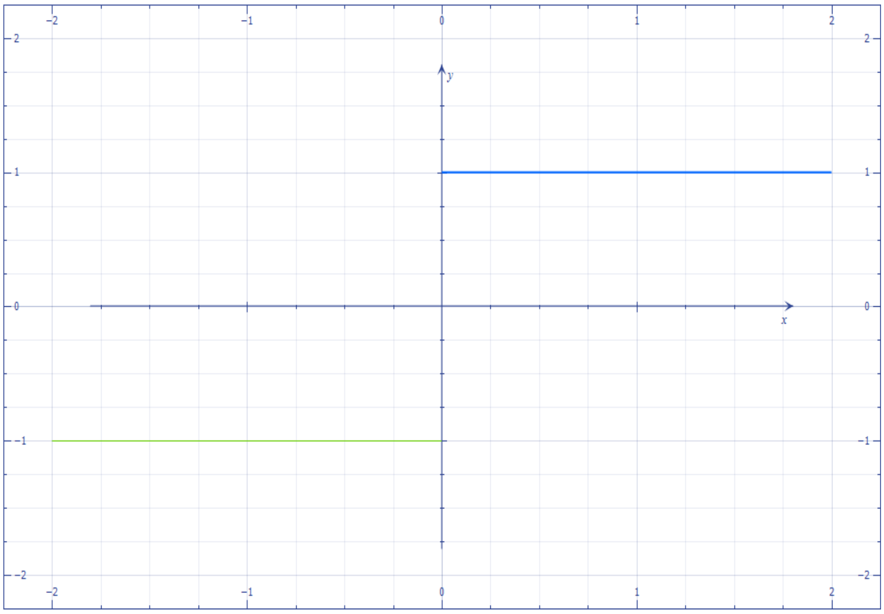


Model (4)

Due to that every function of this collection seems to be constructed by two pieces, it intuitively seems as a collection of piecewise functions, But factually there are only two imbedded piecewise defined functions at the coordinate axes, aren`t they f(x) =|x|, f(x) =$\sqrt{x^3}$ & f(x) =$\sqrt{-x^3}$ ?, although f(x) = $\sqrt[3]{x^2}$ can be defined as two functions over the domains (-∞,0)&(0,∞), but it`s not piece-wisely defined function, for these two functions have the same formula in spite of their different domains i.e. $f(x) = \sqrt[3]{x^2}$ over (-∞, 0) & $f(x) = \sqrt[3]{x^2}$ over (0,∞).

Furthermore, with respect to what seems a certain contradiction within the math interpretation of the continuity, we can emphasis it is "linguistic" & not mathematical. The usual intuition or/and tests of the continuity investigate the path or behavior of a function as one distinct function over its entire domain, while at the case of piecewise defined functions class we have to investigate a function constructed of many distinct functions over different domains, then we assign our investigation to each of its pieces orderly every piece (function) over its own domain. Once more a contradiction appears while analysis of the absolute value function f(x)=|x|, of course it is discontinuous function , while if we investigate it as piecewise defined function, we consider it as continuous one, for that each piece of it is differentiable then continuous over its domain, (differentiability implies continuity)[10]. Therefore, which analysis is the right one? Really both of them are alright. Moreover, f(x) = |x| is considered as smooth piecewise for that its derivative is also piecewise continuous.

Let us consider the model below:

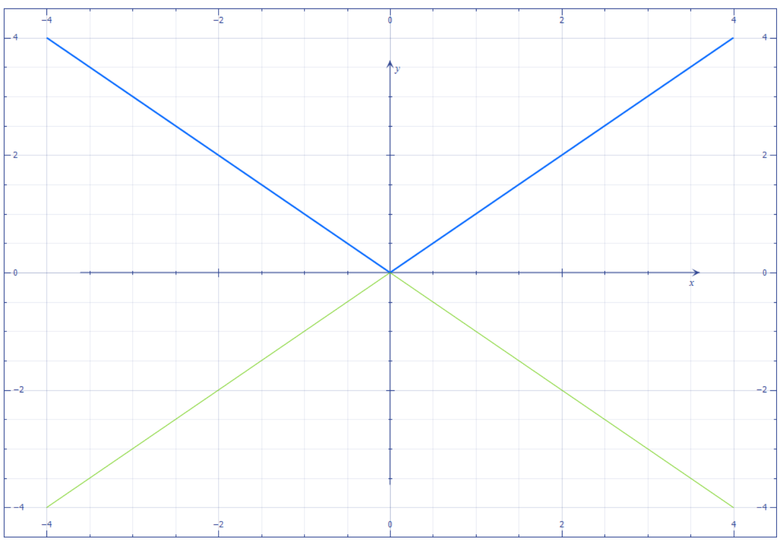


Model (5)

Furthermore, let's investigate the simplest two functions y = x & y = -x, what's new? It is not unexpectedly we would recognize two piecewise-defined functions transparently hidden at the board, aren't they?

$$f(x)= \left\{ \begin{array}{ll} x & x \geq 0 \\ -x & x < 0 \end{array} \right\}$$

$$f(x) = \left\{ \begin{array}{ll} -x & x \geq 0 \\ x & x < 0 \end{array} \right\}$$



Model (6)

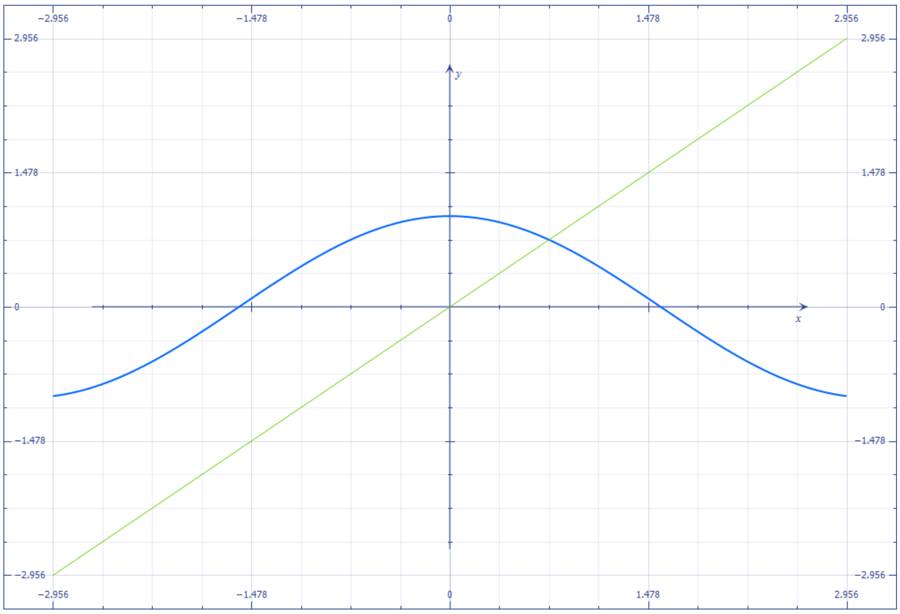
In accordance with in ascending analysis they are smooth piecewise.

Therefore, we can construct other several linearly piecewise defined functions in accordance within the model above, to remain Some of them, consider the specific ones like Heaviside function [3], Step function[6] & the most significance one so called T-function [2].

Also interestingly that the equation y = x play a unique role to illustrate the theorem below :

Fixed point theorem : if f is a continuous function from [0 , 1] to [0 , 1] then there exist z ∈ [0 , 1] such that f(z) = z .[5]

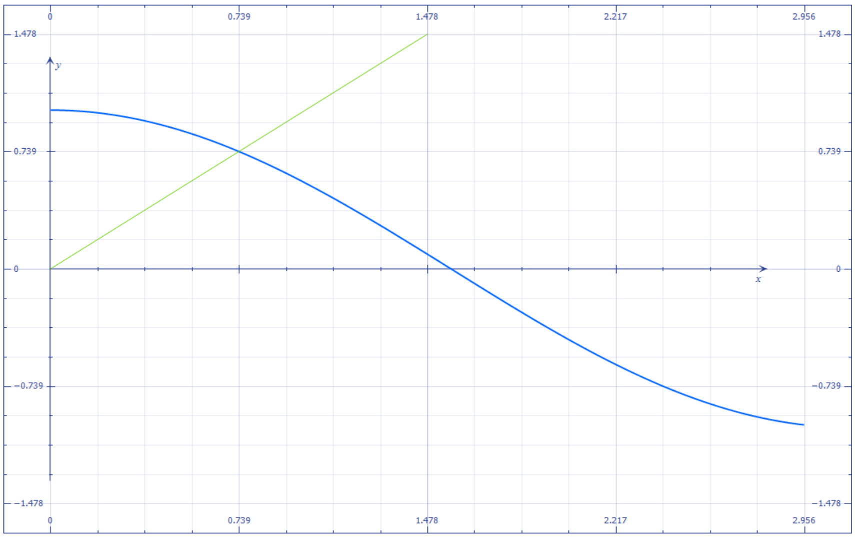
Then let's look how can we determine the fixed point of a continuous function in [0, 1]? Immediately, it's by graphing two functions as the function target f(x) =$\cos(x)$ & the function f(x) = x, consider the model below:



Model (7)

This model offers an illustrative instance for S.A.MORRIS approach [7], as well as accurate numerical value to the fixed point, giving into consideration that $\cos(x)$ = x, f(x) = x.

The graph pointer (trace) of Math program assigns the point values within decimal difference interval [0.001 , 0.1] , to avoid such nuances it seemed reasonable to construct a specific scales for the coordinate axes in away it can be obviously readable. Hereafter, for verification we constructed a separate graph to clearly determine the fixed point graphically as well as numerically:



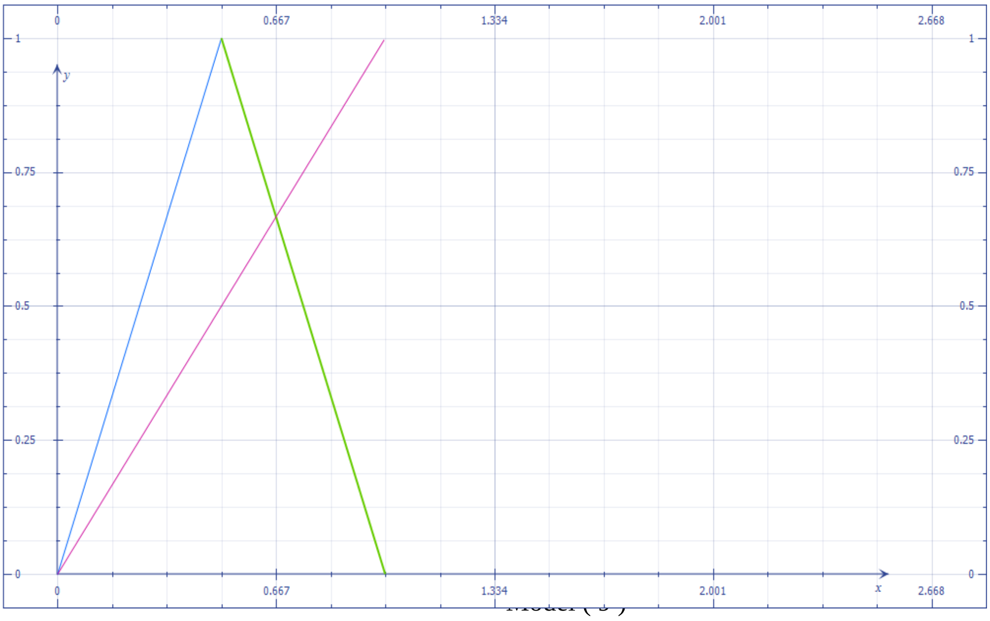
Model (8)

3. An approach for FPT :

Aiming at a computerized approach for fixed point theorem, let us state the formula of a piecewise defined function T(x) as below:

$$T(x) = \begin{cases} 2x & 0 \le x \le 1/2 \\ 2-2x & 1/2 \le x \le 1 \end{cases}$$

Then we can a transparently observe its computer-graph :



Model (9)

Just a glance at it explains two fixed points such that $f_1(x_1) = f_2(x_1) = 0$ & $f_1(x_2) = f_2(x_2) = 0.667$.

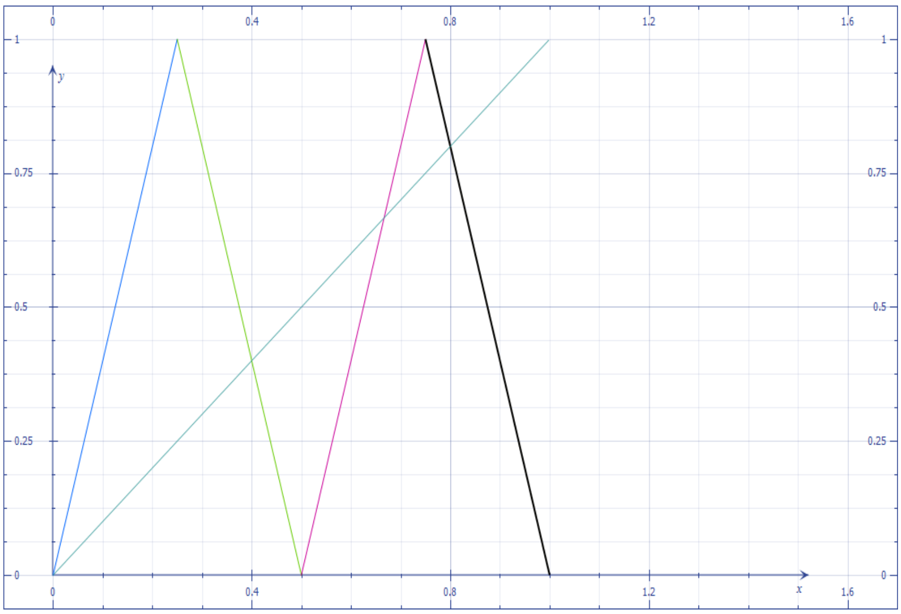
Moreover, we would like to formulate $T^2(x)$ as below:

$$T^2(x) = \begin{cases} 4x & 0 \le x \le 0.25 \\ 2-4x & 0.25 \le x \le 0.5 \\ 4x-2 & 0.5 \le x \le 0.75 \\ 4-4x & 0.75 \le x \le 1 \end{cases}$$

$$T^2(x) = \begin{cases} 4x & 0 \le x \le 1/4 \\ 2-4x & 1/4 \le x \le 1/2 \\ 4x-2 & 1/2 \le x \le 3/4 \\ 4-4x & 3/4 \le x \le 1 \end{cases}$$

$T^2(x)$ has be written in two styles, the first one appears more suitable in accordance within scales of the coordinate plane, while the second in order to be more elegance & comparable within the form of T(x).

Hereafter, let us produce it's relatively a unique model:



Model (10)

This model illustrates four fixed points for $T^2(x)$ whereas $T^2(x_1) = f(x_1) = 0$, $T^2(x_2) = f(x_2) = 2/5$, $T^2(x_3) = f(x_3) = 2/3$ & $T^2(x_4) = f(x_4) = 4/5$. Therefore It is a convenient moment to formulate the following,

Theorem: If f is a continuous function from [0 , 1] to [0 , 1] , then there exist at least one fixed point $z_1 \in [0,1]$ such that $f(z_1) = z_1$.

References:

1- Bill Cox, Understanding Engineering Mathematics, 2002.
2- Colin Adams & Robert Franz's, Introduction to topology, 2008.
3- David McMahon, MATLAB 2009.
4- Greg Fasshauer, Fourier series and boundary value problems, 2011.
5- Microsoft Mathematics Program2006.
6- Michael krihel, chaos theory & dynamical systems, 2012.
7- Murray Spiegel, "Probability & statistics", 2001.
8- R. T. Rockafellar[1], some properties of piecewise smooth functions, 2011.
9- Sidney. A. Morris, Topology without tears, 2007.
10- Thomas,Weir & Hass, Thomas' Calculus, 2014.

Model No.	Microsoft Mathematics Program Instructions
1	$\text{show}(\text{plot}(-x,\{x,-8,8\},\{y,-8,8\}),\text{plot}(-\text{x}^2+2\text{x}),\{\text{x},-8,8\},\{\text{y},-8,8\},\text{plot}((x-2)^3,\{x,-8,8\},\{y,-8,8\}),\{\text{ShowGrid},true\})$
2	$\text{show}(\text{plot}(-x,\{x,-8,0\},\{y,-8,8\}),\text{plot}(-x^2+2,\{x,\{x,0,2\},\{y,-8,8\}),\text{plot}((x-2)^3,\{x,-8,8\},\{y,0,8\}),\{\text{ShowGrid},true\},\{x,-8,8\},\{y,-8,8\})$
3	$\text{show}\big(\text{plot}(-1,\{x,-8,0\},\{y,-8,8\}),\text{plot}\big((-2),\{x,\{x,0,2\},\{y,-8,8\}\big),\text{plot}(\ 3(x-2)^2,\{x,2,8\},\{y,-8,8\}),\{\text{ShowGrid},true\},\{x,-8,8\}\big)$
4	$\text{show}\left(\text{plot}\left(\sqrt[3]{x^2},\{x,-4,4\},\{y,-4,4\}\right),\text{plot}\left(\sqrt{x^3},\{x,-4,4\},\{y,-4,4\}\right),\text{plot}\left(\sqrt{-x^3},\{x,-4,4\},\{y,-4,4\}\right),\text{plot}(\text{abs}(x),\{x,-4,4\},\{y,-4,4\}),\{\text{ShowGrid},true\}\right)$
5	$\text{show}(\text{plot}(1,\{x,0,2\},\{y,-2,2\}),\text{plot}(-1,\{x,-2,-0.001\},\{y,-2,2\}),\{\text{ShowGrid},true\},\{x,-2,2\})$
6	$\text{show}(\text{plot}(\text{abs}(x),\{x,-4,4\},\{y,-4,4\}),\text{plot}(-\text{abs}(x),\{x,-4,4\},\{y,-4,4\}),\{\text{ShowGrid},true\})$
7	$\text{show}(\text{plot}(\cos(x),\{x,-2.956,2.956\},\{y,-2.956,2.956\}),\text{plot}(x,\{x,-2.956,2.956\},\{y,-2.956,2.956\}),\{\text{ShowGrid},true\})$
8	$\text{show}(\text{plot}(\cos(x),\{x,0,2.956\},\{y,-1.478,1.478\}),\text{plot}(x,\{x,0,2.956\},\{y,-1.478,1.478\}),\{\text{ShowGrid},true\})$
9	$\text{show}(\text{plot}(2\ x,\{x,0,0.5\},\{y,0,1\}),\text{plot}(2-2\ x,\{x,0.5,1\},\{y,0,1\}),\text{plot}(x,\{x,0,2.668\},\{y,0,1\}),\{\text{ShowGrid},true\},\{x,0,2.668\})$
10	$\text{show}(\text{plot}(4\ x,\{x,0,2\},\{y,0,1\}),\text{plot}(2-4\ x,\{x,0,2\},\{y,0,1\}),\text{plot}(4\ x-2,\{x,0,2\},\{y,0,1\}),\text{plot}(4-4\ x,\{x,0,2\},\{y,0,1\}),\text{plot}(x,\{x,0,1\},\{y,0,1\},\{\text{Selected},true\}),\{\text{ShowGrid},true\},\{x,0,1.2\})$

The IISTE is a pioneer in the Open-Access hosting service and academic event management. The aim of the firm is Accelerating Global Knowledge Sharing.

More information about the firm can be found on the homepage:
http://www.iiste.org

CALL FOR JOURNAL PAPERS

There are more than 30 peer-reviewed academic journals hosted under the hosting platform.

Prospective authors of journals can find the submission instruction on the following page: http://www.iiste.org/journals/ All the journals articles are available online to the readers all over the world without financial, legal, or technical barriers other than those inseparable from gaining access to the internet itself. Paper version of the journals is also available upon request of readers and authors.

MORE RESOURCES

Book publication information: http://www.iiste.org/book/

Academic conference: http://www.iiste.org/conference/upcoming-conferences-call-for-paper/

IISTE Knowledge Sharing Partners

EBSCO, Index Copernicus, Ulrich's Periodicals Directory, JournalTOCS, PKP Open Archives Harvester, Bielefeld Academic Search Engine, Elektronische Zeitschriftenbibliothek EZB, Open J-Gate, OCLC WorldCat, Universe Digtial Library , NewJour, Google Scholar