Cost Analysis of a Compound System with the Concept of Waiting

Sonendra Kumar Gupta
Department of Mathematics
Research Scholar, Shri Venkateshwara, University (Gajraula), UP, India
Email- sonendrag@gmail.com

Dr. Rekha Choudhary
Department of Engineering Mathematics
Govt. Engineering College, Bharatpur, Rajasthan, India

Abstract- In this research paper the authors, Suppose a compound system having two sub-systems namely '*P*' and '*Q*' connected in chain. Sub-system '*P*' consists of *n* unlike units in chain, while sub-system '*Q*' consists of *m* like units chain. The compound system is subjected to minor failure, major failure critical human failure and common cause failure.

Keywords – Asymptotic behavior, Availability of function, Abel lemma, Cost function and Supplementary variable technique.

1. Introduction

A compound system having two sub-systems namely '*P*' and '*Q*' connected in chain. Sub-system '*P*' consists of *n* unlike units connected in chain, while sub-system '*Q*' consists of *m* like units in chain. The compound system is subjected to minor failure, major failure, critical human failure and common cause failure. Failures follow exponential time distribution where as repairs follow general time distribution. The system is repaired immediately when it is in the state $A_1 ... A_6 ... A_7 ... A_8$. However in the states A_2 & A_4 the system has to wait with a constant rate u_i till the adequate facilities are made available to repair the system. System goes to reduced efficiency state if i^{th} unit of sub-system '*P*' is failed while system goes to complete breakdown if more than one unit of subsystem '*Q*' failed or j^{th} unit of sub-system '*Q*' failed.

Earlier research [1, 2, 3, 4], different techniques have been applied to evaluate the reliability of distribution system, including distributed generation such as an analytical technique using the load duration curve, distributed processing technique, Characteristic function based approach for computing the probability distributers of reliability indices, probabilistic method for assessing the reliability and quantity of power supply to a customer, composite load point model, practical reliability assessment algorithm, validation method and impact of substation on distribution reliability respectively.

2. ASSUMPTION

1. Initially at time $t = 0$, the system is in good state.
2. The system consists of two sub-systems namely; *P* and *Q* connecting in chain**.**
3. Sub-system *P* consists of *n* –unlike units in chain. While subsystem '*Q*' consists of *m* like in chain.
4. The system has three states as good, degraded and failed.
5. Each unit of the system has a constant failure rate.
6. All the failures are statistically independent.
7. Common cause failure and human error rates are constant.
8. Repair is giving only when the system is in either degraded or in failed state.
9. After repair, the system works like new one and never damages anything.

3. Notations:

The following notations have been used in this paper:

$\overline{f}(s)$	Laplace transform of function $f(t)$
$\int$	Integration in the range 0 to ∞
$\alpha_1 / \alpha_2 / \alpha_3 / \alpha_B$	Constant failure rates of state A_0 to A_1 / A_0 to A_2 / A_0 to A_4 / A_0 to A_8
$\alpha_C / \alpha_{C_i} / \alpha_h / \alpha_{1\eta}$	Constant failure rate of state A_0 to A_6 / A_1 to A_6 / A_0 to A_7 / A_1 to A_7

$\beta_i(x)\,\Delta / \eta_i(\mathrm{y})\,\Delta$	The first order probability that the system will be repaired in the time interval $(x,\, x+\Delta)/(\mathrm{y},\, \mathrm{y}+\Delta)$ conditioned that it was not repaired up the time x/y
$\lambda_j(u)$	General repair rates for sub-system 'B' from state A_8 to A_0
$r(w)/\mu(z)$	General repair rates for common cause failure and critical human error
$i,\ j$	Subscript denotes the serial number of *P*-unit and *Q*-unit $[i=1,2\,.....\,n],[\,j=1,2\,.....\,m]$
$P_0(t)$	The probability that at time '*t*' the system is in good state
$P_1(x,t)\,\Delta$	The probability that at time '*t*' the system is in degraded state due to the failure of i^{th} unit of sub-system '*P*'. The elapsed repair time lies in the interval $(x,\, x+\Delta)$
$P_2(t)$	The probability that at time '*t*' the system is in failed state A_2.
$P_3(\mathrm{y},t)\,\Delta$	The probability that at time '*t*' the system is in failed state A_3 and the elapsed repair time lies in the interval $(\mathrm{y},\, \mathrm{y}+\Delta)$
$P_4(t)$	The probability that at time '*t*' the system is in failed state A_4
$P_5(\mathrm{y},t)\,\Delta$	The probability that at time '*t*' the system is in failed state A_5 and the elapsed repair time lies in the interval $(\mathrm{y},\, \mathrm{y}+\Delta)$

Transition State Diagram

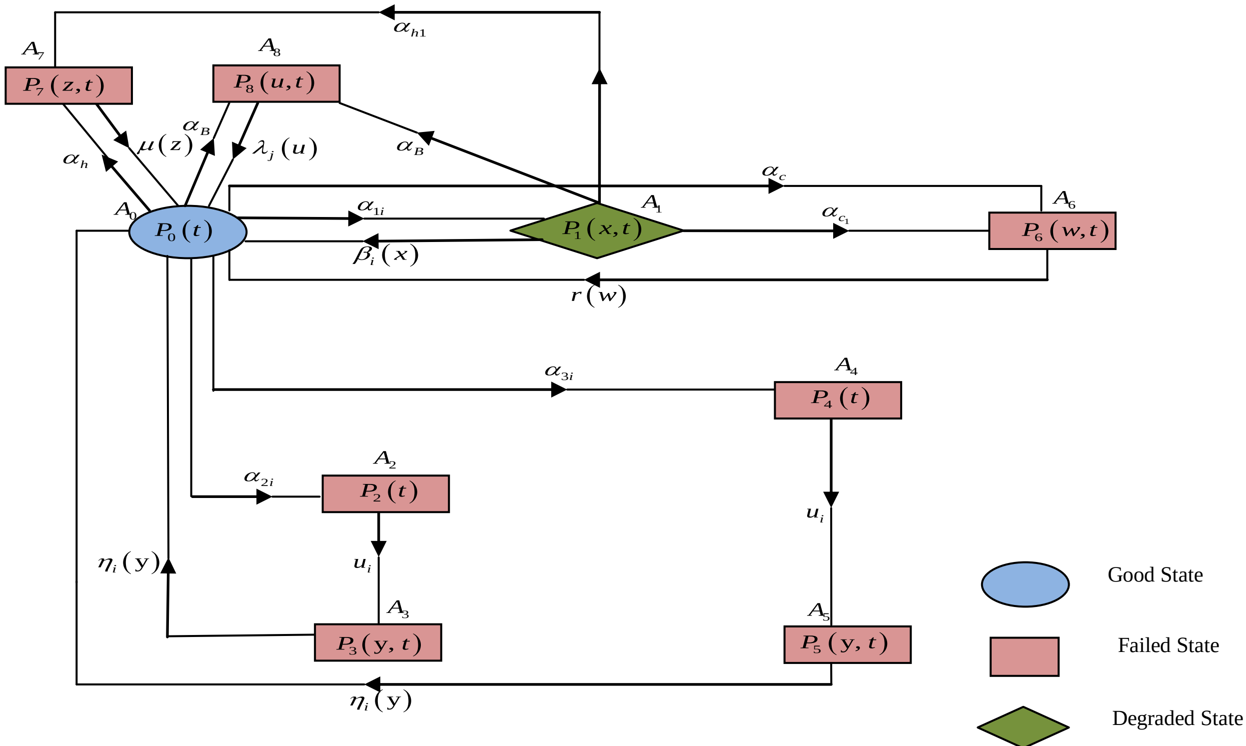

Figure 1 represents the state transition diagram of the system.

4. FORMULATION OF THE MATHEMATICAL MODEL

The analysis crucially depends on the method of supplementary variables technique and the supplementary variable *x* denotes the time that a unit has been elapsed undergoing repair. Viewing the nature of the problem, we obtain the following set of difference-differential equations:

$$\left[\frac{\partial}{\partial t}+\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B\right]P_0(t)=\sum_i\int P_1(x,t)\,\beta_i(x)\,dx+\sum_i\int P_3(\mathrm{y},t)\,\eta_i(\mathrm{y})\,d\mathrm{y}$$

$$+\sum_i\int P_5(\mathrm{y},t)\,\eta_i(\mathrm{y})\,d\mathrm{y}+\int P_6(w,t)\,r(w)\,dw+\int P_7(z,t)\,\mu(z)\,dz$$

$$+\sum_j\int P_8(u,t)\,\psi_j(u)\,du \qquad \text{…. (1.1)}$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\alpha_{c_1}+\alpha_{i1_\eta}+\alpha_B+\beta_i(x)\right]P_1(x,t)=0 \qquad \text{…. (1.2)}$$

$$\left[\frac{\partial}{\partial t}+u_i\right]P_2(t)=\alpha_{2i}\,P_0(t) \qquad \text{…. (1.3)}$$

$$\left[\frac{\partial}{\partial \mathrm{y}}+\frac{\partial}{\partial t}+\eta_i(\mathrm{y})\right]P_3(\mathrm{y},t)=0 \qquad \text{…. (1.4)}$$

$$\left[\frac{\partial}{\partial t}+u_i\right]P_4(t)=\alpha_{3i}\,P_0(t) \qquad \text{…. (1.5)}$$

4.1 Boundary Conditions

$$P_1(0,t)=\alpha_{1i}\,P_0(t) \qquad \text{…. (1.6)}$$

$$P_3(0,t)=u_i\,P_2(t) \qquad \text{…. (1.7)}$$

$$P_5(0,t)=u_i\,P_4(t) \qquad \text{…. (1.8)}$$

$$P_6(0,t)=\alpha_c\,P_0(t)+\alpha_{c_1}\,P_1(t) \qquad \text{…. (1.9)}$$

$$P_7(0,t)=\alpha_h\,P_0(t)+\alpha_{1_\eta}\,P_1(t) \qquad \text{…. (2.0)}$$

$$P_8(0,t)=\alpha_j\left[P_0(t)+P_1(t)\right] \qquad \text{…. (2.1)}$$

4.2 Initial Conditions:

$P_0(0)=1$, otherwise zero …. (2.2)

5. SOLUTION OF THE MODEL

Taking Laplace transforms of equations (1.1) through (2.1) and using initial condition (2.2) one may obtain

$$\left[s+\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B\right]\overline{P}_0(s)=1+\sum_i\int\overline{P_1}(x,s)\,\beta_i(x)\,dx+\sum_i\int\overline{P_3}(\mathrm{y},s)\,\eta_i(\mathrm{y})\,d\mathrm{y}$$

$$+\sum_i\int\overline{P_5}(\mathrm{y},s)\,\eta_i(\mathrm{y})\,d\mathrm{y}+\int\overline{P_6}(w,s)\,r(w)\,dw+\int\overline{P_7}(z,s)\,\mu(z)\,dz$$

$$+\sum_j\int\overline{P_8}(u,s)\,\psi_j(u)\,du \qquad \text{…. (2.3)}$$

$$\overline{P}_1(0,s)=\alpha_{1i}\,\overline{P}_0(s) \qquad \text{…. (2.4)}$$

$$\overline{P}_3(0,s)=\mu_i\,\overline{P}_2(s) \qquad \text{…. (2.5)}$$

$$\overline{P}_5(0,s)=\mu_i\,\overline{P}_4(s) \qquad \text{…. (2.6)}$$

After solving the above equations, we get finally

$$\overline{P}_0(s)=\frac{1}{A(s)} \qquad \text{…. (2.7)} \qquad \overline{P}_1(s)=\frac{\alpha_{1i}}{A(s)}D_{\beta i}\,(s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B) \qquad \text{…. (2.8)}$$

$$\overline{P}_2(s)=\frac{\alpha_{2i}}{(s+u_i)\,A(s)} \qquad \text{…. (2.9)} \qquad \overline{P}_3(s)=\frac{u_i\,\alpha_{2i}}{(s+u_i)\,A(s)}D_{\eta_i}(s) \qquad \text{…. (3.0)}$$

$$\overline{P}_4(s)=\frac{\alpha_{3i}}{(s+u_i)\,A(s)} \quad \ldots.\ (3.1) \qquad \overline{P}_5(s)=\frac{u_i\,\alpha_{3i}}{(s+u_i)\,A(s)}D_{\eta_i}(s) \quad \ldots.\ (3.2)$$

Where,

$$A(s)=s+\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B-\sum_i\alpha_{1i}\,\overline{S}_{\beta i}\,(s+\alpha_{\eta_i}+\alpha_{c_1}+\alpha_B)-\sum_i\frac{u_i}{s+u_i}(\alpha_{2i}+\alpha_{3i})\,\overline{S}_{\eta i}(s)$$
$$-\left[\alpha_c+\alpha_{c_1}\sum_i\alpha_{1i}D_{\beta i}\,(s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B)\right]\overline{S}_r(s)-\left[\alpha_h+\alpha_{1_\eta}\sum_i\alpha_{1i}D_{\beta i}\,(s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B)\right]\overline{S}_\mu(s)$$
$$-\left[1+\sum_i\alpha_{1i}D_{\beta i}\,(s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B)\right]\sum_j\alpha_j\,\overline{S}_{\psi_j}(s) \quad \ldots.\ (3.3)$$

6. ERGODIC BEHAVIOUR OF THE SYSTEM

Using Abel's Lemma $\lim_{s\to 0}s\,\overline{F}(s)=\lim_{t\to\infty}F(t)=F$ (say), provided the limit on the R.H.S. exists, the time independent probabilities are obtained as follows by making use above lemma in the relations (2.3) through (3.2), then we get the following equations,

$$P_0=\frac{1}{A'(0)} \quad \ldots.\ (3.4) \qquad P_1=\frac{\alpha_{1i}}{A'(0)}D_{\beta_i}\left(\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B\right) \quad \ldots.\ (3.5)$$

$$P_2=\frac{\alpha_{2i}}{u_i\,A'(0)} \quad \ldots.\ (3.6) \qquad P_3=\frac{\alpha_{2i}}{A'(0)}M_{\eta_i} \quad \ldots.\ (3.7)$$

$$P_4=\frac{\alpha_{3i}}{u_i\,A'(0)} \quad \ldots.\ (3.8) \qquad P_5=\frac{\alpha_{3i}}{A'(0)}M_{\eta_i} \quad \ldots.\ (3.9)$$

$$P_6=\frac{1}{A'(0)}\left[\alpha_c+\alpha_{c_1}\sum_i\alpha_{1i}D_{\beta i}\left(\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B\right)\right]M_r \quad \ldots.(3.10)$$

$$P_7=\frac{1}{A'(0)}\left[\alpha_h+\alpha_{\eta_1}\sum_i\alpha_{1i}D_{\beta i}\left(\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B\right)\right]M_\mu \quad \ldots.(3.11)$$

$$P_8=\frac{\alpha_j}{A'(0)}\left[1+\sum_i\alpha_{1i}\,D_{\beta i}\left(\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B\right)\right]M_{\psi_j} \quad \ldots.(3.12)$$

Where, $A'(0)=\left[\frac{d}{ds}A(s)\right]_{s=0}$ and M_k = Mean time to repair k^{th} unit $=-\overline{S}'_k(0)$

7. EVALUATION OF UP AND DOWN STATE PROBABILITIES

We have,

$$\overline{P}_{up}=\overline{P}_0(s)+\overline{P}_1(s)=\frac{1}{s+\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B}\left[1+\frac{\alpha_1}{s+\alpha_{\eta_1}+\alpha_{c_1}+\alpha_B}\right]$$

Taking inverse Laplace transform on both sides, we get

$$P_{up}(t)=\left[\frac{\alpha_{c_1}+\alpha_{\eta_1}-\alpha_2-\alpha_3-\alpha_c-\alpha_h}{\alpha_{c_1}+\alpha_{\eta_1}-\alpha_1-\alpha_2-\alpha_3-\alpha_c-\alpha_h}\right]\exp\left[-(\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h+\alpha_B)t\right]$$
$$+\frac{\alpha_1}{\alpha_1+\alpha_2+\alpha_3+\alpha_c+\alpha_h-\alpha_{\eta_1}-\alpha_{c_1}}\exp\left[-(\alpha_{c_1}+\alpha_{1\eta}+\alpha_B)t\right] \quad \ldots.\ (3.13)$$

and, $P_{down}(t)=1-P_{up}(t)$ … . (3.14)

8. COST PROFIT ANALYSIS FUNCTION

The cost function for the considered system is defined as

$$G(t) = C_1 \int_0^t P_{up}(t)\, dt - C_2\, t \qquad \text{…. (3.15)}$$

Where,
$G(t)$ = Expected cost for total time, C_1 = Revenue cost per unit up time and C_2 = Service cost per unit time
Putting the value of $P_{up}(t)$ in equation (3.15), we get

$$G(t) = C_1 \left[\frac{\alpha_{\eta_1} + \alpha_{c_1} - \alpha_2 - \alpha_3 - \alpha_c - \alpha_h}{\alpha_{\eta_1} + \alpha_{c_1} - \alpha_1 - \alpha_2 - \alpha_3 - \alpha_c - \alpha_h} \right] \times \left[\frac{1 - \exp\left\{ -\left(\alpha_1 + \alpha_2 + \alpha_3 + \alpha_h + \alpha_c + \alpha_B \right) t \right\}}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_h + \alpha_c + \alpha_B} \right]$$

$$+ \frac{C_1 \alpha_1}{\alpha_1 + \alpha_2 + \alpha_3 + \alpha_h + \alpha_c - \alpha_{c_1} - \alpha_{\eta_1}} \times \left[\frac{1 - \exp\left\{ -\left(\alpha_{\eta_1} + \alpha_{c_1} + \alpha_B \right) t \right\}}{\alpha_{\eta_1} + \alpha_{c_1} + \alpha_B} \right] - C_2\, t \qquad \text{…. (3.16)}$$

9. NUMERICAL COMPUTATION

Suppose the parameters as

(i). $\alpha_1 = \alpha_h = 0.02,\ \alpha_2 = \alpha_B = 0.03,\ \alpha_3 = 0.04,\ \alpha_c = \alpha_{\eta_i} = \alpha_{1\eta} = 0.05,\ \alpha_{c_1} = 0.03$ and C_1 = Rs. 3.00, C_2 = Rs. 2.00, we get

(ii). $\alpha_1 = \alpha_h = 0.01,\ \alpha_2 = \alpha_B = 0.02,\ \alpha_3 = 0.03,\ \alpha_c = \alpha_{\eta_i} = \alpha_{1\eta} = 0.04,\ \alpha_{c_1} = 0.02$ and C_1 = Rs. 2.00, C_2 = Rs. 1.00, we get

1. *Availability of system:*

Putting the parameters (i) in equation (3.13), then we get the expression,

$$P_{up}(t) = 0.75 \exp(-0.19t) + 0.25 \exp(-0.11t)$$

And putting the parameters (ii) in equation (3.13), then we get the expression,

$$P_{up}(t) = 0.08 \exp(-0.13t) + 0.2 \exp(-0.08t)$$

2. *Cost function of system:*

Putting the parameters (ii) in equation (3.16), then we get the expression,

$$G(t) = 1.6 \left[\frac{1 - e(-0.13t)}{0.13} \right] + 0.4 \left[\frac{1 - e(-0.08t)}{0.08} \right] - t$$

10. EXPERIMENTAL RESULT IN TABULATION AND FIGURE

10.1 Table for $P_{up}(t)$ and Curve:

Table-1

S.No.	t	Pup(t)	Pup(t)
1	0	1	1
2	1	0.8441779	0.8870996
3	2	0.7135258	0.78727
4	3	0.603875	0.6989711
5	4	0.5117589	0.6208462
6	5	0.4342932	0.5517006
7	6	0.3690771	0.4904815
8	7	0.3141112	0.4362612
9	8	0.2677296	0.3882222
10	9	0.2285435	0.345644
11	10	0.1953942	0.3078912
12	11	0.1673147	0.2744037

13	12	0.143497	0.2446874
14	13	0.1232659	0.2183066
15	14	0.1060564	0.1948766
16	15	0.0913957	0.1740581

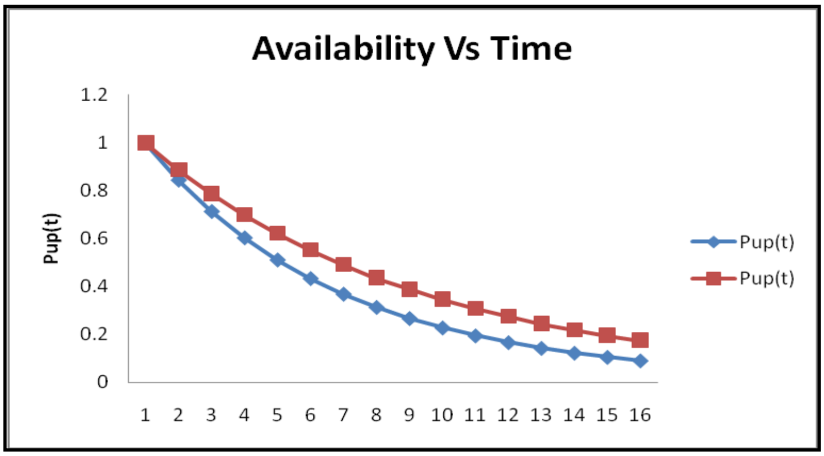

Figure 2 represents the Availability of the system with respect to time.

10.2. Table for G(t) and Curve:

Table-2

S.No.	t	G(t)
1	0	0
2	1	0.884782195
3	2	1.557107691
4	3	2.041545316
5	4	2.359771147
6	5	2.530913287
7	6	2.57185521
8	7	2.497502693
9	8	2.321018717
10	9	2.054030212
11	10	1.706810034
12	11	1.288437168
13	12	0.806937771
14	13	0.269409372
15	14	-0.317869754
16	15	-0.949344248

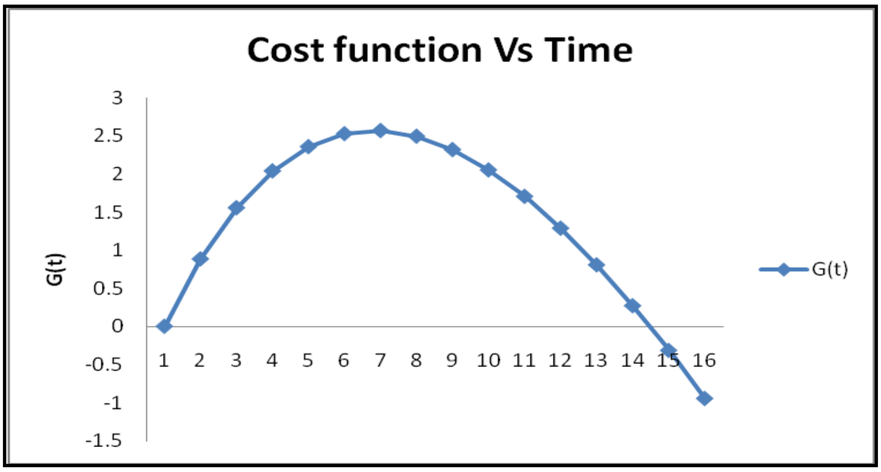

Figure 3 represents the Cost function of the system with respect to time.

11. Conclusion

Table 1 and Figure 2 provide information how availability of the complex engineering repairable system change with respect to time when failure rate increases, then availability of system is decreases.

Table 2 exhibits expected cost function with respect to time and their corresponding Figure 3 shows that when time increase then cost function increase and after some time when time increase, cost function continuously decrease.

The further research area is widely open, where one may think of the application of other members of copula family, MTTF and sensitivity analysis.

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