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Random Fuzzy metric space with cyclic contraction

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Abstract:- In the paper, we define a random fuzzy cyclic contraction and prove the existence and uniqueness of fixed points in a random fuzzy metric space.

Introduction and preliminaries

In 1965, Zadeh [1] introduced the concept of fuzzy sets, as a new way to represent the vagueness in everyday life. Kramosil and Michalek [2] investigated the notion of fuzzy metric space which is closely related to a class of probabilistic metric spaces. In [3,4], George and Veeramani modified the concept of fuzzy metric space of Kramosil and Michalek, and obtained a Hausdorff and first countabletopology on the modified ramdom fuzzy metric space. Grabiec [7] obtained a fuzzy version of the Banach contraction principle in fuzzy metric spaces in Kramosil and Michalek's sense. Many mathematicians proved several fixed point results in fuzzy metric spaces. Gregori and Sapena [19] introduced the concept of fuzzy contractive mapping and proved some fixed point results for such mappings. Kirk et al. [20] introduced the notion of a cyclic representation and characterized the Banach contraction principle in the context of a cyclic mapping. Some interesting fixed point results for cyclic contration in fuzzy metric spaces can be seen in [21-23] In this paper, we generalize and exend the concept of fuzzy contractive mappings to random fuzzy cyclic contraction and prove some fixed point result for taking random fuzzy metric spaces.

For the sake of completeness, we write some definitions and properties of random fuzzy metric spaces.

Definition 1.1 [1]. A fuzzy set A in a nonempty set X is a function with domain X and values in [0,1].

Definition 1.2 [24]. A binary operation $\star\colon [0,1] \times [0,1] \to [0,1]$ is a continuous t-norm if $\{[0,1],\star\}$ is an abelian topological monoid with unit 1 such that $a \star b \leq c \star d$ whenever $a \leq c$ and $b \leq d, a, b, c, d \in [0,1]$. A typical example of t-norm is $a \star b = \min\{a, b\}$ (minimum t-norm)

Definition 1.3 [3]. The triplet $(X, M,\star)$ with measurable function ξ is called a random fuzzy metric space if X is an arbitrary set, $\star$ is a continuous t-nom and M is a fuzzy set in $X^2 \times (0, \infty)$ satisfying the following conditions: for all $x(\xi),y(\xi), z(\xi) \in X$ and $s, t > 0)$

(M1) $M(x(\xi),y(\xi), t) > 0$;

(M2) $M(x(\xi),y(\xi), t) = 1$ if and only if $x(\xi) = y(\xi)$;

(M3) $M(x(\xi),y(\xi), t) = M(y(\xi),x(\xi), t)$;

(M4) $M(x(\xi),y(\xi), t) \star M(y(\xi),z(\xi), s) \leq M(x(\xi), z(\xi), t + s)$;

(M5) $M(x(\xi),y(\xi),.):(0, \infty) \to [0,1]$ is continuous.

Here M with $\star$ is called a random fuzzy metric on X. Note that, $M(x(\xi),y(\xi), t)$

can be thought of as the definition of nearness between $x(\xi)$ and $y(\xi)$ with respect to t. It is known that $M(x(\xi),y(\xi),.)$ is nondecreasing for all $x(\xi),y(\xi) \in X$.

Let $(X, M,\star)$ be a random fuzzy metric spaces with. For $t > 0$, the open ball $B(x(\xi), r, t)$ with center $x(\xi) \in X$ and radius $0 < r < 1$ is defined by

$B(x(\xi), r, t) =\{ y(\xi) \in X: M(x(\xi),y(\xi), t) > 1 - r\}$.

The collection$\{(x(\xi),y(\xi), t): x(\xi) \in X, 0 < r < 1, t > 0\}$ is a neighborhood system for a topology τ on X induced by the random fuzzy metric M. This topology is hausdorff and first countable.

Lemma 1.1 ([4, 26]). Let $(X, M, \star)$ be a random fuzzy metric space. Then M is a continuous function on $X^2 \times (0, \infty)$.

Definition 1.4[3]. A sequence $\{x_n(\xi)\}$ in a random fuzzy metric space $(X, M, \star)$ is said to be convergent to $x(\xi) \in X$ if for each $\varepsilon \in (0,1)$ and each $t > 0$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n(\xi), x(\xi), t) > 1 - \varepsilon$ for all $n > n_0$.

Definition 1.5[3]. A seuence $\{x_n(\xi)\}$ in a random fuzzy metric space $(X, M, \star)$ is said to be a Cauchy sequence if for each $\varepsilon \in (0,1)$ and each $t > 0$ there exists $n_0 \in \mathbb{N}$ such that $M(x_n(\xi), x_m(\xi), t) > 1 - \varepsilon$ for all $n, m > n_0$.

A random fuzzy metric space in which every Cauchy seuence is convergent is called a complete random fuzzy metric space.

Definition 1.6[7]. A sequence $\{x_n(\xi)\}$ in a random fuzzy metric space $(X, M, \star)$ is called G-Cauchy if

$\lim_{n\to\infty} M(x_{n+p}(\xi), x_n(\xi), t) = 1$

For each $t > 0$ and $p > 0$.

Theorem 1.1 [3]. A sequence $\{x_n(\xi)\}$ in a random fuzzy metric space $(X, M, \star)$

Converges to $x(\xi)$ if and only if $M(x_n(\xi), x(\xi), t) \to 1$ as $n \to \infty$.

It is clear that every closed subset of a complete random fuzzy metric space is complete.

Gregori and Sapena [19], motivating by we introduced the concept of random fuzzy contractive mappings as follows:

Let $(X, M, \star)$ be a random fuzzy metric space. We say that the mpping $T: X \to X$ is random fuzzy contractive if there exists $k \in [0,1)$ such that

$$\frac{1}{M(T(\xi,x(\xi)),T(\xi,y(\xi)),t)} - 1 \leq k\left[\frac{1}{M(x(\xi),y(\xi),t)} - 1\right] \qquad (1.1)$$

for all $x(\xi), y(\xi) \in X$ and $t > 0$. Here k is called the random fuzzy contractive constant of T.

2. Main results

In this section we define a class of cyclic operators on random fuzzy metric spaces and investigate the existence and uniqueness of fixed points of these operators.

In [20], the following concept of cyclic representation of a set is defined.

Let X be a nonempty set and $T: X \to X$ be an operator, $A_1, A_2, \ldots, A_m$ be subsets of X. Then $X = \bigcup_{i=1}^m A_i$ is a cyclic representation of X with respect to T if

a) A_i $i = 1,2, \ldots, m$ are nonempty sets;
b) $T(A_1) \subset A_2, \ldots, T(A_{m-1}) \subset A_m, T(A_m) \subset A_1$.

Definition 2.1. Let $(X, M, \star)$ be a random fuzzy meric space, $A_1, A_2, \ldots, A_m$ be subsets of X and $Y = \bigcup_{i=1}^m A_i$. An operator $T: Y \to Y$ is called random fuzzy cyclic contraction if the following conditions hold:

(i) $Y = \bigcup_{i=1}^m A_i$ is a cyclic representation of Y with respect to T;
(ii) There exists $k \in [0,1)$ such that
$$\frac{1}{M(T(\xi,x(\xi)),T(\xi,y(\xi)),t)} - 1 \leq k\left[\frac{1}{M(x(\xi),y(\xi),t)} - 1\right] \qquad (2.1)$$

for all $x(\xi) \in A_i, y(\xi) \in A_{i+1}$($i = 1,2, \ldots, m$, where $A_{m+1=A_1}$ and each $t > 0$.

Definition 2.2 Let $(X, M, \star)$ be a random fuzzy metric space, $A_1, A_2, \ldots, A_m$ be subsets of X and $Y = \bigcup_{i=1}^m A_i$. A self operator T of Y is said to belong to the class $D_M^C(k_1, k_2, k_3)$ if the following concitions hold:

(i) $Y = \bigcup_{i=1}^m A_i$ is a cyclic representation of Y with respect to T;
(ii) There exist $0 \leq k_1, k_2, k_3 < 1$ such that

$$\frac{1}{M(T(\xi,x(\xi)),T(\xi,y(\xi),t))}-1\le k_1\left[\frac{1}{M(x(\xi),y(\xi),t)}-1\right]$$
$$+K_2\left[\frac{1}{M(x(\xi),T(\xi,x(\xi)),t)}-1\right]+K_3\left[\frac{1}{M(y(\xi),T(\xi,y(\xi)),t)}-1\right]$$

for any $x(\xi)\in A_i, y(\xi)\in A_{i+1}(i=1,2,\dots,m,$ where $A_{m+1=A_1}$ and each $t>0$.

It is obvious that if an operator T is in the class $D_M^C(k,0,0)$ with $0\le k<1$, Then T is a random fuzzy cyclic contraction. The first proposition gives uniqueness conditions of the fixed point of an operator provided that the fixed point exists.

Proposition 2.1 Let $(X,M,\star)$ be a random fuzzy metric space. $A_1,A_2,\dots,A_m$ be subsets of X and $Y=\bigcup_{i=1}^m A_i$. Let T be a self operator of Y and belongs to $D_M^C(k_1,k_2,k_3)$ if

$F(T)=\{x(\xi)\in Y: T(\xi,x(\xi))=x(\xi)\}\neq\emptyset$, then $F(T)$ consists of a single point.

Proof. Assume the contrary, that $u(\xi),v(\xi)\in F(T)\subset Y=\bigcup_{i=1}^m A_i$, $u(\xi)\neq v(\xi)$. Note that, $u\in A_i$ for some $1\le i\le m$, so $u=Tu\in A_{i+1}$ and so on. Hence, $u\in\bigcap_{i=1}^m A_i$ and similar result holds for v therefore, it follows from (2.2) that

$$\frac{1}{M(u(\xi),v(\xi),t)}-1=\frac{1}{M(T(\xi,u(\xi)),T(\xi,v(\xi)),t)}-1$$
$$\le k_1\left[\frac{1}{M(u(\xi),v(\xi),t)}-1\right]+k_2\left[\frac{1}{M(u(\xi),T(\xi,u(\xi),t)}-1\right]+k_3\left[\frac{1}{M(v,T(\xi,v(\xi),t)}-1\right]$$
$$\le k_1\left[\frac{1}{M(u(\xi),v(\xi),t)}-1\right]<\frac{1}{M(u(\xi),v(\xi),t)}-1,$$

a contradiction. Therefore we must have $u(\xi)=v(\xi)$.

Definition 2.3. Let $(X,M,\star)$ be any random fuzzy metric space, T be a self mapping of X and $x(\xi)\in X$. The mapping T is said to be asymptotically regular at point $x(\xi)$ if

$\lim_{n\to\infty} M(T^n(\xi,x(\xi)),\ T^{n+1}(\xi,x(\xi)),t)=1$ for all $t>0$.

Proposition 2.2. Let $(X,M,\star)$ be a random fuzzy metric space, $A_1,A_2,\dots,A_m$ be subsets of X and $Y=\bigcup_{i=1}^m A_i$. Let T be a self operator of Y and belongs to $D_M^C(k_1,k_2,k_3)$ with $k_1+k_2+k_3<1$. Then T is asymptotically regular at every point $x(\xi)\in Y$.

Proof. Take an arbitrary point $x_0(\xi)\in Y$ and define the sequence of Picard's iterates $x_n(\xi)=T(\xi,x_{n-1}(\xi)=T^n(\xi,x_0(\xi)$ for all $\in N$. As

$x_0(\xi)\in Y=\bigcup_{i=1}^m A_i$, so for all $n\in\mathbb{N}$ there exists i such that $1\le i\le m$ and $x_n(\xi)\in A_i$ and so $x_{n+1}(\xi)=T(\xi,x_n(\xi))\in A_{i+1.}$ Therefore for all $t>0$, it follows from (2.2) that

$$\frac{1}{M(x_{n+1}(\xi),x_{n+2}(\xi),t)}-1=\frac{1}{M(T(\xi,x_n(\xi)),T(\xi,x_{n+1}(\xi)),t)}-1$$

$$\le k_1\left[\frac{1}{M(x_n(\xi),x_{n+1}(\xi),t)}-1\right]+k_2\left[\frac{1}{M\left(x_n(\xi),T(\xi,x_n(\xi)),t\right)}-1\right]$$
$$+k_3\left[\frac{1}{M(x_{n+1}(\xi),T(\xi,x_{n+1}(\xi)),t)}-1\right]=k_1\left[\frac{1}{M(x_n(\xi),x_{n+1}(\xi),t)}-1\right]$$

$$+k_2\left[\frac{1}{M(x_n(\xi),x_{n+1}(\xi),t)}-1\right]+k_3\left[\frac{1}{M(x_{n+1}(\xi),x_{n+2}(\xi),t)}-1\right],$$

That is,

$$(1-k_3)\ \left[\frac{1}{M(x_{n+1}(\xi),x_{n+2}(\xi),t)}-1\right]\leq(k_1+k_2)\left[\frac{1}{M(x_n(\xi),x_{n+1}(\xi),t)}-1\right]$$

$$\frac{1}{M(x_{n+1}(\xi),x_{n+2}(\xi),t)}-1\leq\frac{k_1+k_2}{1-k_3}\left[\frac{1}{M(x_n(\xi),x_{n+1}(\xi),t)}-1\right]$$

By repetition of this process we obtain

$$\frac{1}{M(x_{n+1}(\xi),x_{n+2}(\xi),t)}-1\leq\left(\frac{k_1+k_2}{1-k_3}\right)^{n+1}\ \left[\frac{1}{M(x_0(\xi),x(\xi),t)}-1\right]\qquad(2.3)$$

Since $k_1+k_2+k_3<1$, so $\frac{k_1+k_2}{1-k_3}<1$ and hence,

$\lim_{n\to\infty} M(x_{n+1}(\xi),x_{n+2}(\xi),t)=1$. Therefore for any arbitrary $x_0(\xi)\in Y$ we have $\lim_{n\to\infty} M(T^{n+1}(\xi,x_0(\xi)),T^{n+2}(\xi,x_0(\xi)),t)=1$ for all $t>0$, and result follows. ■

We now prove the existence of fixed point of operators in the class $D_M^C(k_1,k_2,k_3)$ for random fuzzy metric motivated by [28]

Theorem 2.1. Let $(X,M,\star)$ be a random fuzzy metric space, $A_1,A_2,\ldots,A_m$ be subsets of X and $Y=\bigcup_{i=1}^m A_i$ is such that every G-Cauchy sequence in Y is convergent in Y. Let T be a self operator of Y and belongs to $D_M^C(k_1,k_2,k_3)$ with $k_1+k_2+k_3<1$. Then T has a unique fixed point $u\in Y$ and the sequence of picard's iterates $x_n(\xi)=T(\xi,x_{n-1}(\xi))=T^n(\xi,x_0(\xi))$ for all $n\in\mathbb{N}$, where $x_0(\xi)\in Y$ is arbitrary, converges to the fixed point of T.

Proof. Let $n\in\mathbb{N}$ then for any $p\geq1$ we have

$$M(x_n(\xi),x_{n+p}(\xi),t)\geq M(x_n(\xi),x_{n+1}(\xi),t/2)\star M(x_{n+1}(\xi),x_{n+p}(\xi),t/2)$$

$$\geq M(x_n(\xi),x_{n+1}(\xi),t/2))\star M(x_{n+1}(\xi),x_{n+2}(\xi),t/2^2)$$
$$\star M(x_{n+2}(\xi),x_{n+p}(\xi),t/2^2)$$

$$\geq M(x_n(\xi),x_{n+1}(\xi),t/2)\star M(x_{n+1}(\xi),x_{n+2}(\xi),t/2^2)$$
$$\star M(x_{n+2}(\xi),x_{n+3}(\xi),t/2^3)$$
$$\star\ldots\star\ M\left(x_{n+p-1}(\xi),x_{n+p}(\xi),\frac{t}{2^{p-1}}\right).\qquad(2.4)$$

Setting $\lambda=\frac{k_1+k_2}{1-k_3}$ and $M_n(t)=M(x_n(\xi),x_{n+1}(\xi),t)$ for all $t>0$ and $n\geq0$, it follows from inequality (2.3) of Proposition 2.2 that

$$\frac{1}{M_{n+1}(t)}\leq\frac{\lambda^{n+1}}{M_0(t)}+1-\lambda^{n+1}\leq\frac{\lambda^{n+1}}{M_0(t)}+1,$$

That is,

$$\frac{1}{\frac{\lambda^{n+1}}{M_0(t)}+1}\leq M_{n+1}(t)\text{ for all }t>0\text{ and }n\geq0.$$

Using the above inequality in (2.4) we obtain

$$M(x_n(\xi),x_{n+p}(\xi),t)\geq M_n\left(\frac{t}{2}\right)\star M_n\left(\frac{t}{2^2}\right)\star M_{n+2}\left(\frac{t}{2^3}\right)$$

$$\star\ldots\star\ M_{n+p-1}\left(\frac{t}{2^{p-1}}\right)$$

$$\geq \frac{1}{\frac{\lambda^n}{M_0(t/2)}+1} \star \frac{1}{\frac{\lambda^{n+1}}{M_0(t/2^2)}+1} \star \dots \star \frac{1}{\frac{\lambda^{n+p-1}}{M_0(t/2^{p-1})}+1}$$

$$\geq \frac{1}{\frac{\lambda^n}{M_0(t/2)}+1} \star \frac{1}{\frac{\lambda^n}{M_0(t/2^2)}+1} \star \dots \star \frac{1}{\frac{\lambda^n}{M_0(t/2^{p-1})}+1}.$$

As ,$\lambda < 1$, letting $n \to \infty$, we obtain from the above inequality that

$$\lim_{n\to\infty} M(x_n(\xi), x_{n+p}(\xi), t) = 1 \text{ for all } t > 0, p \geq 1. \qquad (2.5)$$

Thus, $\{x_n(\xi)\}$ is a G-Cauchy sequence in Y therefore by the assumption there exists $u \in Y$ such that $\lim_{n\to\infty} M(x_n(\xi), u, t) = 1$ for all $t > 0$.

We shall show that u is the fixed point point of T.

Note that, As $Y = \bigcup_{i=1}^m A_i$ is a cyclic representation of Y with repect to T, the sequence $\{x_n(\xi)\}$ has infinite terms in each A_i for $i \in \{1,2, \dots, m\}$. Suppose that $u \in A_i$ then we have $Tu \in A_{i+1}$, also take a subsequence $\{x_{n_k}(\xi)\}$ suh that $x_{n_k}(\xi) \in A_{i+1}$. Then, for any $t > 0$. we have

$$\frac{1}{M\left(x_{n_{k+1}}(\xi), T(\xi,u(\xi)), t\right)} - 1 = \frac{1}{M\left(T(\xi, x_{n_k}(\xi)), T(\xi,u(\xi)), t\right)} - 1$$

$$\leq k_1 \left[\frac{1}{M\left(x_{n_k}(\xi), u(\xi), t\right)} - 1\right] + k_2 \left[\frac{1}{M\left(x_{n_k}(\xi), T(\xi, x_{n_k}(\xi), t\right)} - 1\right]$$

$$+ k_3 \left[\frac{1}{M(u(\xi), T(\xi, u(\xi), t)} - 1\right] \leq k_1 \left[\frac{1}{M\left(x_{n_k}(\xi), u(\xi), t\right)} - 1\right]$$

$$+ k_2 \left[\frac{1}{M\left(x_{n_k}(\xi), x_{n_{k+1}}(\xi), t\right)} - 1\right] + k_3 \left[\frac{1}{M(u(\xi), T(\xi, u(\xi)), t)} - 1\right]$$

Letting $k \to \infty$ in the above inequality and using (2.5) and (2.6), we obtain

$$\left[\frac{1}{M(u(\xi), T(\xi, u(\xi), t)} - 1\right] \leq k_3 \left[\frac{1}{M(u(\xi), T(\xi, u(\xi)), t)} - 1\right].$$

As, $0 \leq k_3 < 1$ we must have $\frac{1}{M(u(\xi), T(\xi, u(\xi)), t)} - 1 = 0$. that is, $M(u(\xi), T(\xi, u(\xi)), t) = 1$ for all $t > 0$, hence $T(\xi, u(\xi)) = u(\xi)$. Thus, $u(\xi)$ is a fixed point of T. Therefore,

$F(T) = \{x(\xi) \in Y: T(\xi, x(\xi)) = x(\xi)\} \neq \emptyset$, and then by proposition 2.1, F(T) consists of a single point, that is, the fixed point of T is unique. ■

Following corollaries are immediate consequence of the above theorem.

Corollary 2.1. Let $(X, M, \star)$ be a random fuzzy metric space, $A_1, A_2, \dots, A_m$ be subsets of X and $Y = \bigcup_{i=1}^m A_i$ is such that every G-Cauchy sequence in Y is convergent in Y. Let T be a random fuzzy cyclic contraction on Y. Then T has a unique fixed point $u \in Y$ and the sequence of picard's iterates $x_n(\xi) = T(\xi, x_{n-1}(\xi)) = T^n(\xi, x_0(\xi))$ for all $n \in \mathbb{N}$, where $x_0(\xi) \in Y$ is arbitrary, converges to the fixed point of T.

Corollary 2.2. Let $(X, M, \star)$ be a random fuzzy metric space, $A_1, A_2, \dots, A_m$ be subsets of X and $Y = \bigcup_{i=1}^m A_i$ is such that every G-Cauchy sequence in Y is convergent in Y. Let T be a self operator of Y such that the following conditions hold:

(i) $Y = \bigcup_{i=1}^m A_i$ is a cyclic representation of Y with respect to T;
(ii) There exist $0 \leq k_1, k_2 < 1$ such that $k_1 + k_2 < 1$ and

$$\frac{1}{M(T(\xi,x(\xi)),T(\xi,y(\xi)),t)} - 1 \le k_1\left[\frac{1}{M(x(\xi),T(\xi,x(\xi)),t)} - 1\right] + k_2\left[\frac{1}{M(y(\xi),T(\xi,y(\xi)),t)} - 1\right]$$

for any $x(\xi) \in A_{i+1}(i = 1,2, \ldots\ldots, m$ where $A_{m+1} = A_1)$ and each $t > 0$.

Then T has a unique fixed point $u \in Y$ and the sequence of Picard's iterates

$x_n(\xi) = T(\xi, x_{n-1}(\xi)) = T^n(\xi, x_0(\xi))$ for all $n \in \mathbb{N}$, where $x_0 \in Y$ is arbitrary, converges to the fixed point of T.

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