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A Mathematical model for the effect of Gastrin in humans using Fuzzy Rayleigh distribution

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Abstract

Gastrin is a potent stimulant of lower esophageal sphincter pressure in human. Several observations suggest that lower esophageal sphincter pressure is under the control of gastrin concentration. Injections of gastrin produce a marked increase in lower esophageal sphincter pressure. Various doses of synthetic gastrin-I significantly increased lower esophageal sphincter pressure. In this paper, we developed a mathematical model using the fuzzy Rayleigh distribution for the effects of gastrin on lower esophageal sphincter pressure in humans.

Key Words: Cumulative damage model, Rayleigh distribution, Gastrin.

1. Introduction

All real world systems are deteriorating in nature and the progressive system degradation is often reflected in higher production cost, lower product quality. The optimum control-limit policy where a unit is replaced at a threshold level was derived, when it fails with a known probability that is a function of the total damage. More discussions on such replacement policies were carried out [5], [7], [8]. Among the many approaches to modelling deteriorating systems, shock models have found favour with reliability analysis. [10],[11]Shocks are events which cause perturbation to the system, leading to its deterioration and consequent failure. The effect of these shocks to the system is measured by a process called Wear process of damage process. In a typical cumulative damage model, a system suffers damage due to shock and failure occurs when the total damage exceeds a pre specified level. [1]Such a model generates a cumulative process. [4]Esary et al., discussed that any component or device when exposed to shocks which cause damage to the system is likely to fail when the total accumulated damage exceeds a level called the threshold. The rate of accumulation of damage determines the lifetime of the system.

In 1905, Edkins discovered a potent gastric acid Secretagogue in extracts of antral mucosa and named it gastrin. The existence of gastrin was firmly established in the early 1960s. Gastrin is the best-studied gastrointestinal hormone. There are presently three known biologically active forms of the gastrin molecule. Gastrin is a potent stimulant of lower esophageal sphincter pressure in human[2]. Gastrin has a physiological role in the control of sphincter strength[3]. Intravenous injections of synthetic gastrin-I caused lower esophageal sphincter pressure to rise promptly[6].

Usual cumulative damage process has crisp parameters, however the parameter used in cumulative damage process are often not known precisely, while there also some uncertainty exists in the parameters. The theory of fuzzy set is used in solving such problems. The objective of this paper is to find the expected life time and variance using crisp and fuzzy parameters by assuming that the threshold distribution is continuous and the amount of damage occurs in each shock are independent.

2. Notations

W_i	–	Random variable denoting the shock value in the i^{th} shock
g(.), G(.), $\overline{G}(.)$	–	Probability density, cumulative density and survivor functions of W_i
Z	–	Random variable denoting the inter arrival time between two shocks.
f(.), F(.)	–	Probability density and cumulative density function of Z.
D	–	Random variable denoting the threshold value which follows Rayleigh distribution.
h(.), H(.), $\overline{H}(.)$	–	Probability density, cumulative density and survivor functions of D.
$g^*(.)$	–	Laplace transform of g(.).

$g_k(.)$	–	K-fold convolution of g(.).
$F_k(.)$	–	K-fold convolution of F(.).
σ	–	scale parameter of Rayleigh distribution.
μ	–	scale parameter of exponential distribution.
$\sigma[\alpha]$	–	Alpha cut of scale parameter in Rayleigh distribution.
$\mu[\alpha]$	–	Alpha cut of scale parameter in exponential distribution.
$E(T)$	–	expected life time in crisp parameter.
$\overline{E}(T)$	–	expected life time in fuzzy parameter.

3. Fuzzy Mathematical model

The random variable D denoting the threshold value follows generalized Rayleigh distribution with shape parameter one has the cumulative distribution function defined as $H(x) = 1 - e^{-(\sigma x^2)}$

The corresponding survival function is

$$\overline{H}(x) = 1 - \left(1 - e^{-(\sigma x^2)}\right) = e^{-\sigma x^2}$$

The shock survival probability are given by

$$P(W_i < D) = \int_0^\infty g_K(x)\overline{H}(x)dx$$

$$= \int_0^\infty g_k(x)\, e^{-\sigma x^2} dx$$

$$= [g^*(\sigma^2)]^k$$

The survival function which gives the probability that the cumulative threshold will fail only after time t.

$$S(t) = P(T > t)$$

$$= \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\ [g^*(\sigma^2)]^k$$

Now the life time is given by

$$L(t) = 1 - S(t$$

$$= 1 - \sum_{k=0}^{\infty}[F_k(t) - F_{k+1}(t)]\ [g^*(\sigma^2)]^k$$

$$= [1 - g^*(\sigma^2)] \sum_{k=1}^{\infty} F_k(t)\, [g^*(\sigma^2)]^{k-1}$$

Taking Laplace Stieltjes transformation, we get

$$L^*(s) = \frac{[1 - g^*(\sigma)^2] f^*(s)}{[1 - g^*(\sigma)^2\, f^*(s)]}$$

Let the random variable Z denoting the inter arrival time which follows exponential distribution with parameter c

Now $f^*(s) = \frac{c}{c+s}$

$$L^*(s) = \frac{[1-g^*(\sigma)^2](\frac{c}{c+s})}{[1-g^*(\sigma)^2\,(\frac{c}{c+s})}$$

$$L^*(s) = \frac{[1-g^*(\sigma)^2]c}{[c+s-g^*(\sigma)^2]\,c}$$

$$E(T) = -\frac{d}{ds}L^*(s)\ at\ s=0$$

$$=\frac{[1-g^*(\sigma)^2]c}{c^2[1-g^*(\sigma)^2]}$$

$$=\frac{1}{c[1-g^*(\sigma)^2]}$$

$$E(T^2) = \frac{d^2}{ds^2}L^*(s)\ at\ s=0$$

$$=\frac{2}{c^2[1-g^*(\sigma)^2]^2}$$

$$V(T) = E(T^2) - E(T)^2$$

$$=\frac{1}{c^2[1-g^*(\sigma)^2]^2}$$

If the shock value follows exponential distribution with parameter μ then

$$E(T) = \frac{1}{c\left[1-\frac{\mu}{\mu+\sigma^2}\right]}$$

Therefore

$$E(T) = \frac{\mu+\sigma^2}{c\sigma^2} \qquad \text{-------------}\rightarrow (1)$$

and $$V(T) = \frac{(\mu+\sigma^2)^2}{c^2\sigma^4} \qquad \text{--------------}\rightarrow (2)$$

In most of the times, the parameter used in the cumulative damage model is not known precisely. The theory of fuzzy set can be used in solving such problems. [9]The uncertainty existing in the parameter is resolved by assuming that the parameter is a triangular fuzzy number.

The alpha cut of fuzzy expected value is

$$\overline{E}(T) = [\overline{E}_l(T), \quad \overline{E}_u(T)]$$

Where

$$\overline{E}_l(T) = \min \{\frac{\overline{\mu} + \overline{\sigma}^2}{c\overline{\sigma}^2}, \overline{\mu} \in \overline{\mu}[\alpha], \overline{\sigma} \in \overline{\sigma}[\alpha]\}$$

$$\overline{E}_u(T) = \max \{\frac{\overline{\mu} + \overline{\sigma}^2}{c\overline{\sigma}^2}, \overline{\mu} \in \overline{\mu}[\alpha], \overline{\sigma} \in \overline{\sigma}[\alpha]\}$$

The alpha cut of fuzzy variance is $\overline{V}(T) = [\overline{V}_l(T), \overline{V}_u(T)]$

Where $\overline{V}_l(T) = \min \{\left(\frac{\overline{\mu}+\overline{\sigma}^2}{c\overline{\sigma}^2}\right)^2, \overline{\mu} \in \overline{\mu}[\alpha], \overline{\sigma} \in \overline{\sigma}[\alpha]\}$

$$\overline{V}_u(T) = \max \{\left(\frac{\overline{\mu} + \overline{\sigma}^2}{c\overline{\sigma}^2}\right)^2, \overline{\mu} \in \overline{\mu}[\alpha], \overline{\sigma} \in \overline{\sigma}[\alpha]\}$$

4. Application

Let us consider an example of the effect of lower esophageal sphincter pressure on synthetic gastrin in human. Increases in lower esophageal sphincter pressure were prompt in onset. Fig.4.1 shown lower esophageal sphincter pressure recorded from a single normal patient for three days during intravenous three 30 sec injections of synthetic gastrin-I. Each dose of gastrin-I caused sphincter pressure to rise promptly. The response peaked at 3 min and returned to normal within 10 minutes. The magnitude of the peak response at 3 minute is directly proportional to the amount of gastrin-I.

From the Fig.1, the scale parameter of the Rayleigh distribution for the three different synthetic gastrin-I namely 0.5, 0.25 and 0.1 $\mu g/hg$ respectively are 25, 18.13 and 13.78. Also from the Fig.1, the scale parameter of exponential distribution for the three different synthetic gastrin-I respectively are 0.03, 0.04 and 0.06.

Under the alpha cut zero, the fuzzy expected value for the three different synthetic gastrin-I are calculated from $\overline{E}(T) = [\overline{E}_l(T), \overline{E}_u(T)]$ and shown in Table-1. Also the fuzzy variances are calculated from $\overline{V}(T) = [\overline{V}_l(T), \overline{V}_u(T)]$ and shown in Table-2. The mean and variance for the three different synthetic gastrin-I injections are calculated from equation (1) and (2),which is shown in Fig.2, Fig.3 and Fig.4 respectively.

5. Conclusion

In this paper we study the effect of lower esophageal sphincter pressure of normal subjects after the administration of three different doses of synthetic gastrin-I. The expected value and variance are calculated by using both crisp and fuzzy parameter. The expected value to cross the threshold value is decreasing when the inter arrival time c, which follows exponential distribution, is increasing. The same case is found at variance which is observed in Fig.2, Fig. 3 and Fig. 4.

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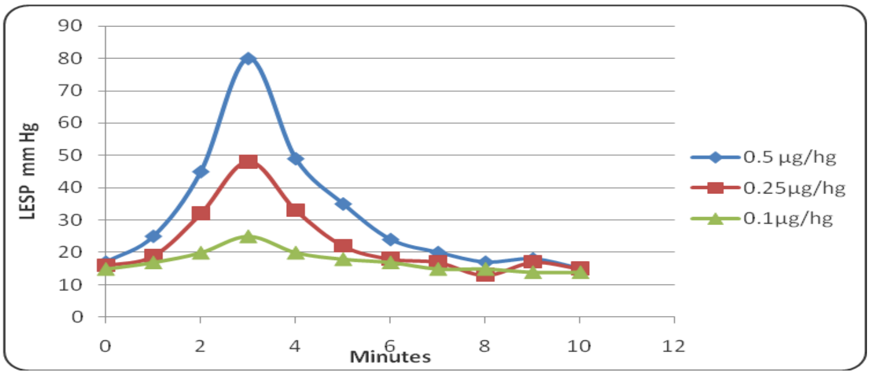

Fig.1: lower esophageal sphincter response to three 30 sec intravenous injections of synthetic gastrin-I

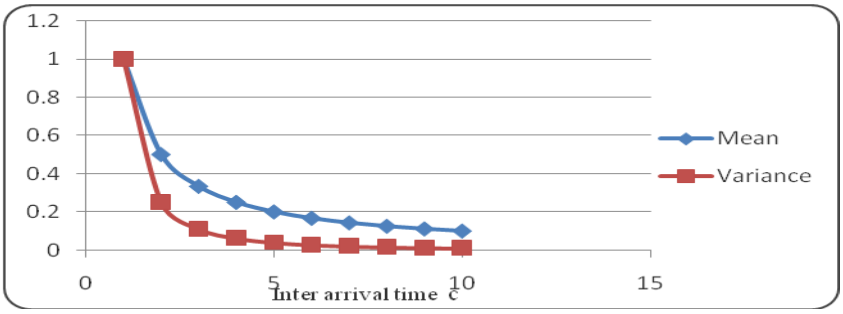

Fig. 2: Mean and variance of 0.5 $\mu g/hg$ synthetic gastrin-I injections

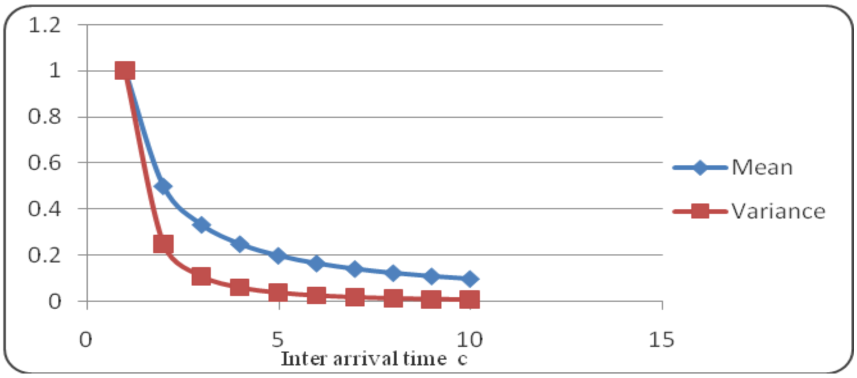

Fig. 3: Mean and variance of 0.25 $\mu g/hg$ synthetic gastrin-I injections

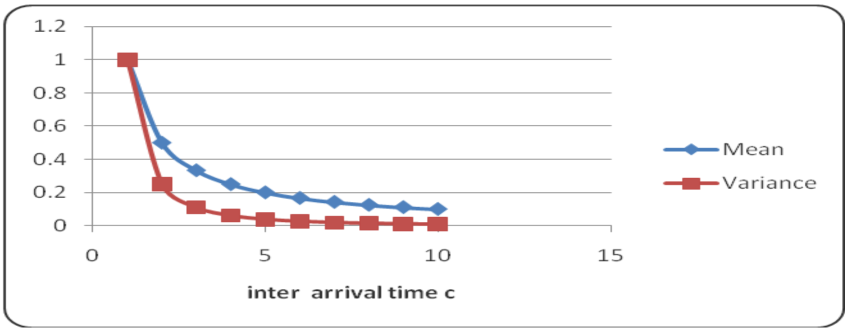

Fig. 4 Mean and variance of 0.1 $\mu g/hg$ synthetic gastrin-I injections

Table 1: Mean for three different doses of synthetic gastrin-I injections

C	0.5		0.25		0.1	
	$\overline{E}_l(T)$	$\overline{E}_u(T)$	$\overline{E}_l(T)$	$\overline{E}_u(T)$	$\overline{E}_l(T)$	$\overline{E}_u(T)$
1	1.00002	1.000066	1.000095	1.00013	1.000321	1.000344
2	0.50001	0.500033	0.500043	0.500095	0.500161	0.500172
3	0.33334	0.333355	0.333362	0.333397	0.33344	0.333448
4	0.250005	0.250017	0.250022	0.250048	0.25008	0.250086
5	0.200004	0.200013	0.200017	0.200038	0.200064	0.200069
6	0.16667	0.166678	0.166681	0.166698	0.16672	0.166724
7	0.14286	0.142867	0.14287	0.142884	0.142903	0.142906
8	0.125002	0.125008	0.125011	0.125024	0.12504	0.125043
9	0.111113	0.111118	0.111121	0.111132	0.111147	0.111149
10	0.100002	0.100007	0.100009	0.100019	0.100032	0.100034

Table 4.2: Variance for three different doses synthetic gastrin-I injections

C	0.5		0.25		0.1	
	$\overline{V}_l(T)$	$\overline{V}_u(T)$	$\overline{V}_l(T)$	$\overline{V}_u(T)$	$\overline{V}_l(T)$	$\overline{V}_u(T)$
1	1.00004	1.000132	1.00019	1.00026	1.000642	1.000688
2	0.25001	0.250033	0.250043	0.250095	0.250161	0.250172
3	0.111116	0.111126	0.11113	0.111154	0.111182	0.111188
4	0.062503	0.062509	0.062511	0.062524	0.06254	0.062543
5	0.040002	0.040005	0.040007	0.040015	0.040026	0.040028
6	0.027779	0.027782	0.027783	0.027788	0.027796	0.027797
7	0.020409	0.020411	0.020412	0.020416	0.020421	0.020422
8	0.015626	0.015627	0.015628	0.015631	0.015635	0.015636
9	0.012346	0.012347	0.012348	0.01235	0.012354	0.012354
10	0.01	0.010001	0.010002	0.010004	0.010006	0.010007