The Probability of the Truth on a Truth Table

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Abstract

In symbolic (or mathematical) logic the truth table is used to establish the truth or falsity (falsehood) of both simple and compound statements (or arguments). However, the use of the term “truth table” falsely suggests that all arguments through the table are true. It is known that the table contains both true and false arguments. So the use of “truth table” is indiscriminate. Consequently, this study was focused on solving this problem by finding out the probability of having a true argument associated with every one of the four binary proposition connectives- “and”, “double implication” “inclusive v”, single implication” used in the arguments. The obtained probabilities are ordered as 1/4, ½ and ¾ respectfully for “and”, “double implication”, and (“inclusive v” and “single implication”). So the “truth tables” are discriminately decomposed into “falsehood tables”, “neutral tables” and “truth tables” at probabilities of ¼, ½ and ¾ respectively. These probabilities are independent of the number of statements, n, greater than unity.

1.0 Introduction

The objective of this study is embedded in the title, “The Probability of the Truth on a Truth Table” as the phrase *a truth table* gives an impression that all the table gives is the truth which is not the case because a truth table in its present form contains both the truth (T) and the falsehood or falsity(F) of a statement. Symbolic logic is the use of symbols to represent and connect mathematical statements and to determine the truth or the falsity of such compound statements. The connections of such arguments with truth tables are used in mathematics, law, business and philosophy. The word *logic* comes from the word *logos* in the Greek Language where it means word, reason, science, study of, knowledge, etc. and the word *theo* means *God*. Hence, “theo” and “logos” combined becomes “theology” , the study of or knowledge of God. Generally, logic is the study of the methods and laws used to distinguish correct reasoning or argument from the incorrect or faulty one. It is the science of thoughts. Thinking is haphazard and, hence, may not lead to a definite conclusion. However, reasoning is an ordered thought that actually centres on a specific problem at a particular time. Studies have shown that Aristotle (384-322B.C.) was the first philosopher that separated logic from other parts of philosophy and that the title, “logic”, as a field of study was given by Zeno in about 300 B.C. Further, Alfred North Whitehead (1872-1970 A. D.) wrote a book titled “Principia Mathematica”, a landmark in the history of symbolic logic.

A statement that is not a command, a question or an exclamation is an assertion that can be determined to be either true or false. Such a statement is said to be mathematical. A mathematical statement is either true or false by definition. The truth value of the statement is T if it is true and F if it is false. Two mathematical statements are connected using one of the propositional connectives such as *not* or *negation* (˜), *and* (. or hat ˄ or vector (vec)), *or* (v or v), *implication* (→), *double implication* or *equivalence* (↔ or ≡). the T or F of the combination can also be established using a truth table (which is actually a table containing both T and F) of the compound statement or argument of the connection made.Types of argument include inductive reasoning which takes off from particular examples and finally generalizes. A deductive argument, on the other hand, begins with the general and finally comes down to particulars. A syllogism is an argument involving a major premise, a minor premise and a conclusion. Again an argument consists of premises and a conclusion. One or more or all the premises or statements may be either true (T) or false (F). Further, the validity or invalidity of such arguments are also determined using various methods such as Venn diagrams in set theory and truth tables. A contradiction is a statement form on a truth table with only false conclusions. In that case, the use of the *truth table* is not appropriate. A tautology is an argument with only true conclusions on a truth table where the use of the *truth table* is correct.

Table1 is a list of the details of the five binary propositional connectives.

The truth tables

Here are truth tables for an n-statement argument.

Letters such as *p*, *q* and *r* are usually used to represent statements. Binary propositional connectives are used to join two simple statements to form a compound statement (or an argument from a legal point of view).

Consider the following statements and their symbolic or literal representations

All professors are research-oriented ….s_1=p

Abala is a professor ………………….. s_2=q

Hence, Abala is research-oriented…..….s_3 = r

Abala is a mathematician……………….s_4=s

Majoram (1975), Mendelson (1979) and Copi (1979) give compound statements or arguments presented in a truth table such as Table 2A.

Zameeruddin, Khanna and Bhambri(2004) gave an eight-row truth table. Some authors of symbolic logic assume the knowledge of truth tables. So they do not mention them. For example, see Antonelli and Thomason (2002), Goguadze, Piazza and Venema (2003), Nwagbogwu and Akinfenwa (2006).

Alabi-Labaika (2008) modelled the alternating appearances of batches of T's and F's in an n-statement argument as 2^{n-c} where n is the number of statements in question, c is the column number, and the number of rows of T's and F's is 2^n .

On table 2 the number of statements is 2 and so 2^n becomes 2^2 = 4 rows. Similarly a three-statement argument has $2^n = 2^3 = 8$ rows. A four-statement argument has 2^4 = 16 rows which form the scope of this study but true for all positive integers.

Using 2^{n-c}

when c=1, 2^{n-1} T's are followed by 2^{n-1}F's and the alternation continues until there are 2^n rows.

It is to be noted that 2^{n-c} can be used symmetrically. That is, the column number can start from the first column (c=1) or from the nth column(c=n).

Probability Concepts

What is probability?

Frank and Althoen(2002) define probability as

"If an experiment can produce m different and mutually exclusive results, all of which are equally likely, and if f of theses are favourable, the probability of a favourable result is f/m."

Gupta (2011) explains probability in the following way. Suppose that an event A occurs m times in N repetitions of a random experiment. The ratio, m/N, gives the relative frequency of the event A and it will not vary from one trial to another. In the limiting case when N becomes sufficiently large, it tends to settle to a number which is called the probability of A.

Symbolically,

P(A) = lim m/N as N →∞

In all it is pertinent to ask, "What is the probability of the truth on a truth table?"

2.0 Methodology

Here is a proposed step to find out "the probability of the truth on a truth table".

Let an order of ^, ↔, v, → be followed in the truth table for n-statement arguments represented as $s_1, s_2, s_3, \ldots, s_n$.

Also let ordered data be applied to the resulting probabilities.

Then, the conditional probability of the truth (T) on the truth table for an n-statement argument is proposed to be:

(1) P(T)/$s_1, s_2, s_3, \ldots, s_n$ for s_i^s_j, i≠j, (i, j) =1,2,3,…,n is ¼
(2) P(T)/ $s_1, s_2, s_3, \ldots, s_n$ for s_i↔s_j, i≠j, (i, j) =1,2,3,…,n is 1/2
(3) P(T)/ $s_1, s_2, s_3, \ldots, s_n$ for s_ivs_j, i≠j, (i, j) =1,2,3,…,n is 3/4
(4) P(T)/ $s_1, s_2, s_3, \ldots, s_n$ for s_i→s_j, i≠j, (i, j) =1,2,3,…,n is 3/4

Proof : constructing the truth table for n= 2, 3, 4 statements using ^, ↔, v, → result in truth Tables 2A-4:

3.0 Results and Discussion

On Table 2 the truth T appears only once out of 4 occasions for ^ giving P(T) =1/4

On Table 3 the truth T appears 2 times out of 8 occasions for ^ giving P(T) =2/8 =1/4

On Table 4 the truth T appears 4 times out of 16 occasions for ^ giving P(T) =4/16=1/4

The probabilities for the other symbols (↔, v, and →) are similarly obtainable. The results and comments or conclusions are summarized on Table 5 where f is the frequency of T for a given value of n.

Summary: The phrase "truth table" is viewed to be too gross for describing the contents of the analysis of mathematical logic because both truth and falsehood pervade the analysis and not just truth. So for every binary propositional connective, the probability of appearance of truth (T) has been found and shown on Table5.

Conclusion: The table of symbolic or mathematical logic is to be named in accordance with the magnitude of the probability of truth (T). For "and" it is a Falsehood Table. For "…if and only if …" it is a Neutral Table. For "or" and "…if then…" it is a Truth Table.

Recommendation: The indiscriminate use of the general term, "truth table" for all the binary propositional connectives should be discouraged.

Application to other areas of study where integrity is a component: What is the probability of auditing in auditing? When an account is said to have been audited, auditing of an audited account is finding out the probability of true auditing, that is, whether or not the auditing has been truthfully carried out with equity and without any bias or its possible cause. Next is the accountability. Inflated payment vouchers and receipts for purposes of accountability raise the question of true probability of true accountability in accountability.

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Table 1: Five Binary Propositional Connectives

Connective	Symbol	Pronounciation	Technical(or functional) name
Not	˜	Tilde or curl	Negation
And	. or ^	Dot, conjunction	Conjunction
Or	V (inclusive) and v (exclusive)	V	Disjunction
if …, then,	→	Arrow	Forward implication, conditional
…if and only if….	↔ **or** ≡	Two-edged arrow or triple parallel dash	Opposite-directions or double implication, equivalence, bi-conditional

Table 2A: A 2-Statement Truth Table

S1	**S2**	**S1 ^S2**	**S1↔S2**	**S1vS2**	**S1→S2**
T	**T**	**T**	**T**	**T**	**T**
T	**F**	**F**	**F**	**T**	**F**
F	**T**	**F**	**F**	**T**	**T**
F	**F**	**F**	**T**	**F**	**T**

Table 2B: A 2-statement truth table

S1	S2	S1 ^S2	S1↔S2	S1vS2	S1→S2
T	T	T	T	T	T
T	F	F	F	T	F
F	T	F	F	T	T*
F	F	F	T	F	T
Argument number		1	2	3	4
Probability of T		¼	2/4	¾	¾

Table 3: A 3-statement truth table

	S1	S2	S3	S_1 ^ S_2	S1 ^ S3	S2 ^ S3	S1 ↔ S2	S1 ↔ S3	S2 ↔ S3	S1 V S2	S1 V S3	S2 V S3	S1 → S2	S1 → S3	S2 → S3
1	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
2	T	T	F	T	F	F	T	F	F	T	T	T	T	F	F
3	T	F	T	F	T	F	F	T	F	T	T	T	F	T	T
4	T	F	F	F	F	F	F	F	T	T	T	F	F	F	T
5	F	T	T	F	F	T	F	F	T	T	T	T	T	T	T
6	F	T	F	F	F	F	F	T	F	T	F	T	T	T	F
7	F	F	T	F	F	F	T	F	F	F	T	T	T	T	T
8	F	F	F	F	F	F	T	T	T	F	F	F	T	T	T
Argument number				1	2	3	4	5	6	7	8	9	10	11	12
Probability of T				2/8 1/4	2/8 1/4	2/8 1/4	4/8 1/2	4/8 ½	4/8 ½	6/8 3/4	6/8 ¾	6/8 ¾	6/8 3/4	6/8 3/4	6/8 ¾

To minimise space used let s_1=p, s_2=q, s_3=r and s_4=s. Consequently, a 4-statement truth table (table 4)based on the new notations is obtained.

Table 4: A 4-Statement Truth Table

	P	Q	R	S	p ^ q	p ^ r	p ^ s	q ^ r	q ^ s	r ^ s	p ↔ q	p ↔ r	p ↔ s	q ↔ r	q ↔ s	r ↔ s	p v q	P v r	P v s	Q v r	Q v s	r v s	p → q	p → r	p → s	Q → R	q → s	r → s
1	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
2	T	T	T	F	T	T	F	T	F	F	T	T	F	T	F	F	T	T	T	T	T	T	T	T	F	T	F	F
3	T	T	F	T	T	F	T	F	T	F	T	F	T	F	T	F	T	T	T	T	T	T	T	F	T	F	T	T
4	T	T	F	F	T	F	F	F	F	F	T	F	F	F	F	T	T	T	T	T	T	F	T	F	F	F	F	T
5	T	T	T	T	F	T	T	F	F	T	F	T	T	F	F	T	T	T	T	T	T	T	F	T	T	T	T	T
6	T	T	T	F	F	T	F	F	F	F	F	T	F	F	T	F	T	T	T	T	F	T	F	T	F	T	T	F
7	T	F	F	T	F	F	T	F	F	F	F	F	T	T	F	F	T	T	T	F	T	T	F	F	T	T	T	T
8	T	F	F	F	F	F	F	F	F	F	F	F	F	T	T	T	T	T	T	F	F	F	F	F	F	T	T	T
9	T	T	T	T	F	F	F	T	T	T	F	F	F	T	T	T	T	T	T	T	T	T	T	T	T	T	T	T
10	F	T	T	F	F	F	F	T	F	F	F	F	T	T	F	F	T	T	F	T	T	T	T	T	T	T	F	F
11	F	T	F	T	F	F	F	F	T	F	F	T	F	F	T	F	T	F	T	T	T	T	T	T	T	F	T	T
12	F	T	F	F	F	F	F	F	F	F	F	T	T	F	F	T	T	F	F	T	T	F	T	T	T	F	F	T
13	F	F	T	T	F	F	F	F	F	T	T	F	F	F	F	T	F	T	T	T	T	T	T	T	T	T	T	T
14	F	F	T	F	F	F	F	F	F	F	T	F	T	F	T	F	F	T	F	T	F	T	T	T	T	T	T	F
15	F	F	F	T	F	F	F	F	F	F	T	T	F	T	F	F	F	F	T	F	T	T	T	T	T	T	T	T
16	F	F	F	F	F	F	F	F	F	F	T	T	T	T	T	T	F	F	F	F	F	F	T	T	T	T	T	T
Argument number					1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	2
Probability of T					4/16 ¼						8/16 ½						12/16 3/4						12/16 3/4					

Table 5: Probability of the Truth (T) on a Truth Table

Symbol	n	2^n	f	$P(T)=f/2^n$	P(F)=1-P(T)	Comment (Conclusion)
^	2	4	1	¼	¾	This is a "falsehood table" as P(F)=3/4>P(T)=1/4
	3	8	2	2/8=1/4	¾	"
	4	16	4	4/16=1/4	¾	"
↔	2	4	2	2/4 =1/2	½	This is a "neutral table" as P(F)=P(T)=1/2
	3	8	4	4/8=1/2	½	"
	4	16	8	8/16=1/2	½	"
						It is not surprising that ↔ has a neutral value of ½: it is a two-way connective.
V	2	4	3	¾	¼	This is a "truth table" as P(T)=3/4 > P(F)=1/4
	3	8	6	6/8=3/4	¼	"
	4	16	12	12/16=3/4	¼	"
→	2	4	3	¾	¼	This is a truth table as P(T)=3/4 > P(F)=1/4
	3	8	6	6/8=3/4	¼	"
	4	16	12	12/16=3/4	¼	"