Fully Fuzzy Time-Cost Trade-Off in a Project Network - A New Approach

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Abstract

In this paper, we propose a new approach to Fuzzy network crashing in a project network whose activity times are uncertain. The uncertain parameters in the project network are represented by triangular fuzzy numbers. By using a new type of fuzzy arithmetic and a fuzzy ranking method we propose a method for finding an optimal duration by crashing the fuzzy activities of a project network without converting the fuzzy activity times to classical numbers. A numerical example is provided to illustrate the proposed method.

Keywords: Triangular fuzzy number, fuzzy ranking, fuzzy project network, Critical path, crashing.

1. Introduction

Project management involves project scheduling, planning, evaluating, and controlling. Activity networks have proven very useful for the performance evaluation of certain types of projects. The critical path method is an important tool to identify the critical activities in the critical path of an activity network for project management. Project crashing is a technique used to shorten the project duration by accelerating some of its activities at an additional expense. The objective of crashing is to reduce project duration while minimizing the cost, which is also known as time-cost trade off problem. Reducing project durations can be done by providing additional resources to those activities in the critical path. Project managers are intended to spend the minimum possible amount of money and achieve the maximum crashing time, as a result both direct and indirect costs will be influenced in the project. While crashing any critical activity by providing extra resources the direct cost will increase whereas the indirect cost will reduce and vice versa. Therefore there is some compromise in project duration – a balance between excessive direct costs for shortening the project duration and excessive indirect costs for lengthening the project duration which we call as optimum project duration.

The successful implementation of CPM requires the exact values on activity durations, resources, money etc. However in reality, due to the non availability and uncertainty of information as well as the variation of management scenario, it is often difficult to obtain the exact values of the decision parameters of the problems. Thus, the conventional approaches, both deterministic and random process, tend to be less effective in conveying the imprecision or vagueness nature of the linguistic assessment. Consequently, the fuzzy set theory can play a significant role in this kind of decision making environment to tackle the unknown or the vagueness about the decision parameters in a project network.

The fuzzy PERT was originated by Chanas and Kamburowski [3]. Motivated by this article several authors have studied the fuzzy crashing of fuzzy PERT problem. Steve and Dessouky**Error! Reference source not found.** described a procedure for solving the project time/cost tradeoff problem of reducing project duration at a minimum cost. Rehab and Carr **Error! Reference source not found.** described the typical approach that construction planners take in performing time-Cost Trade-off (TCT). Pulat and Horn **Error! Reference source not found.** described a project network with a set of tasks to be completed according to some precedence relationship; the objective is to determine efficient project schedules for a range of project realization times and resource cost per time unit for each resource. Van Slyke**Error! Reference source not found.** demonstrated several advantages of applying simulation techniques to PERT, including more accurate estimates of the true project length, flexibility in selecting any distribution for activity times and the ability to calculate "criticality indexes", which are the probability of various activities being on the critical path. Van Slyke was the first to apply Monte Carlo simulations to PERT. Coskun[3]formulated the problem as a Chance Constrained Linear Programming. Johnson and Schon [7] used simulation to compare three rules for crashing stochastic networks. Badiru[1]reported development of another simulation program for project management called STARC. Feng *et al* [5] presented a hybrid approach that combines simulation techniques with a genetic algorithm to solve the time-cost trade-off problem under uncertainty.

In this paper, we use triangular fuzzy numbers to effectively deal with the ambiguities involved in the process of linguistic values. Applying a new type of fuzzy arithmetic and a fuzzy ranking method [8,9], we propose a method for finding an optimal duration by crashing the fuzzy activities of a project network without converting the fuzzy activity times to classical numbers. A numerical example is provided to illustrate the proposed method. The rest of this paper is organized as follows. In section 2 we recall the basic concepts of fuzzy numbers, fuzzy arithmetic and their rankings. In section 3, we introduce the concept of fuzzy network crashing. In section 4, a numerical example is given to illustrate the method proposed in this paper.

2. Preliminaries

The aim of this section is to present some notations, notions and results which are of useful in our further study.

Definition 2.1

A fuzzy set $\tilde{a}$ defined on the set of real numbers R is said to be a fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following study.

(a) $\tilde{a}$ is convex,

$$\tilde{a}\{\lambda x_1+(1\text{-}t)x_2\}=\min\{\tilde{a}(x_1),\tilde{a}(x_2)\},\ \text{for all }\ x_1, x_2 \in R\ \text{ and } \lambda \in [0,1]$$

(b) $\tilde{a}$ is normal i.e. there exists an $x \in R$ such that $\tilde{a}(x) = 1$

(c) $\tilde{a}$ is Piecewise continuous.

Definition 2.2

A fuzzy number $\tilde{a}$ on R is said to be a triangular fuzzy number (TFN) or linear fuzzy number if its membership function $\tilde{a}: R \to [0,1]$ has the following characteristics:

$$\tilde{a}(x) = \begin{cases} \dfrac{x - a_1}{a_2 - a_1}, & \text{for } a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & \text{for } a_2 \le x \le a_3 \\ 0, & \text{elsewhere} \end{cases}$$

We denote this triangular fuzzy number by $\tilde{a} = (a_1, a_2, a_3)$. We use $F(R)$ to denote the set of all triangular fuzzy numbers.

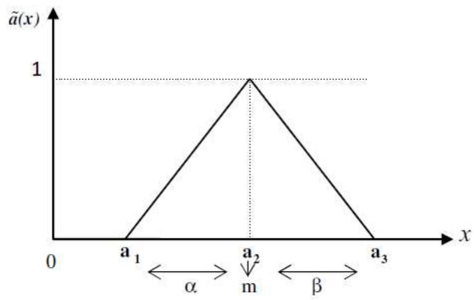


Fig 1. Triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

Also if $m = a_2$ represents the modal value or midpoint, $\alpha = (a_2 - a_1)$ represents the left spread and $\beta = (a_3 - a_2)$ represents the right spread of the triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$, then the triangular fuzzy number $\tilde{a}$ can be represented by the triplet $\tilde{a} = (\alpha, m, \beta)$. i.e. $\tilde{a} = (a_1, a_2, a_3) = (\alpha, m, \beta)$.

Definition 2.3 A triangular fuzzy number $\tilde{a} \in F(R)$ can also be represented as a pair $\tilde{a} = (\underline{a}, \overline{a})$ of functions $\underline{a}(r)$ and $\overline{a}(r)$ for $0 \le r \le 1$ which satisfies the following requirements:

(i). $\underline{a}(r)$ is a bounded monotonic increasing left continuous function.

(ii). $\overline{a}(r)$ is a bounded monotonic decreasing left continuous function.

(iii). $\underline{a}(r) \le \overline{a}(r)$, $0 \le r \le 1$

Definition 2.4 For an arbitrary triangular fuzzy number $\tilde{a} = (\underline{a}, \overline{a})$, the number $a_0 = \left(\frac{\underline{a}(1) + \overline{a}(1)}{2} \right)$ is said to be a location index number of $\tilde{a}$. The two non-decreasing left continuous functions $a_* = (a_0 - \underline{a})$, $a^* = (\overline{a} - a_0)$ are called the left fuzziness index function and the right fuzziness index function respectively. Hence every triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3)$ can also be represented by $\tilde{a} = (a_0, a_*, a^*)$

2.1 Ranking of triangular Fuzzy Numbers

Many different approaches for the ranking of fuzzy numbers have been proposed in the literature. Abbasbandy and Hajjari [1] proposed a new ranking method based on the left and the right spreads at some α -levels of fuzzy numbers.

For an arbitrary triangular fuzzy number $\tilde{a} = (a_1, a_2, a_3) = (a_0, a_*, a^*)$ with parametric form $\tilde{a} = (\underline{a}(r), \overline{a}(r))$, we define the magnitude of the triangular fuzzy number $\tilde{a}$ by

$$\mathrm{Mag}(\tilde{a}) = \frac{1}{2}\left(\int_0^1 (\underline{a} + \overline{a} + a_0)\ f(r)\ dr\right)$$
$$= \frac{1}{2}\left(\int_0^1 (a^* + 4a_0 - a_*)\ f(r)\ dr\right).$$

where the function $f(r)$ is a non-negative and increasing function on $[0,1]$ with $f(0)=0$, $f(1)=1$ and $\int_0^1 f(r)\,dr = \frac{1}{2}$. The function $f(r)$ can be considered as a weighting function. In real life applications, $f(r)$ can be chosen by the decision maker according to the situation. In this paper, for convenience we use $f(r)=r$.

Hence $\mathrm{Mag}(\tilde{a}) = \left(\frac{a^* + 4a_0 - a_*}{4}\right) = \left(\frac{\underline{a} + \overline{a} + a_0}{4}\right).$

The magnitude of a triangular fuzzy number $\tilde{a}$ synthetically reflects the information on every membership degree, and meaning of this magnitude is visual and natural. Mag ($\tilde{a}$) is used to rank fuzzy numbers. The larger Mag($\tilde{a}$), the larger fuzzy number.

For any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R), we define the ranking of $\tilde{a}$ and $\tilde{b}$ by comparing the Mag ($\tilde{a}$) and Mag ($\tilde{b}$) on R as follows:

(i). $\tilde{a} \succeq \tilde{b}$ if and only if Mag($\tilde{a}$) $\geq$ Mag($\tilde{b}$)

(ii). $\tilde{a} \preceq \tilde{b}$ if and only if Mag($\tilde{a}$) $\leq$ Mag($\tilde{b}$)

(iii). $\tilde{a} \approx \tilde{b}$ if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$)

Definition 2.5

A triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*)$ is said to be symmetric if and only if $a_* = a^*$.

Definition 2.6

A triangular fuzzy number $\tilde{a} = (a_0, a_*, a^*)$ is said to be non-negative if and only if Mag ($\tilde{a}$) ≥ 0 and is denoted by $\tilde{a} \succeq \tilde{0}$. Further if Mag($\tilde{a}$) > 0, then $\tilde{a} = (a_0, a_*, a^*)$ is said to be a positive fuzzy number and is denoted by $\tilde{a} \succ \tilde{0}$.

Definition 2.7

Two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R) are said to be equivalent if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$).That is $\tilde{a} \approx \tilde{b}$ if and only if Mag($\tilde{a}$) = Mag($\tilde{b}$).Two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ in F(R) are said to be equal if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$. That is

$\tilde{a} = \tilde{b}$ if and only if $a_0 = b_0, a_* = b_*, a^* = b^*$.

2.2 *Arithmetic operations on triangular Fuzzy Numbers*

Ming Ma et al. [9] have proposed a new fuzzy arithmetic based upon both location index and fuzziness index functions. The location index number is taken in the ordinary arithmetic, whereas the fuzziness index functions are considered to follow the lattice rule which is least upper bound in the lattice L. That is for a, b∈L we define $a \vee b = \max\{a, b\}$ and $a \wedge b = \min\{a, b\}$.

For arbitrary triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$ and $* = \{+, -, \times, \div\}$, the arithmetic operations on the triangular fuzzy numbers are defined by $\tilde{a} * \tilde{b} = (a_0 * b_0, a_* \vee b_*, a^* \vee b^*)$. In particular for any two triangular fuzzy numbers $\tilde{a} = (a_0, a_*, a^*)$ and $\tilde{b} = (b_0, b_*, b^*)$, we define

(i) Addition: $\tilde{a} + \tilde{b} = (a_0, a_*, a^*) + (b_0, b_*, b^*) = (a_0 + b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$

(ii) Subtraction: $\tilde{a} - \tilde{b} = (a_0, a_*, a^*) - (b_0, b_*, b^*) = (a_0 - b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$.

(iii) Multiplication: $\tilde{a} \times \tilde{b} = (a_0, a_*, a^*) \times (b_0, b_*, b^*) = (a_0 \times b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$.

(iv) Division: $\tilde{a} \div \tilde{b} = (a_0, a_*, a^*) \div (b_0, b_*, b^*) = (a_0 \div b_0, \max\{a_*, b_*\}, \max\{a^*, b^*\})$.

3. Fuzzy Crashing

Fuzzy project network is an acyclic digraph, where the vertices represent events, and the directed edges represent the activities, to be performed in a project. We denote this fuzzy project network by $\tilde{N} = (\tilde{V}, \tilde{A}, \tilde{T})$. Let $\tilde{V} = \{\tilde{v}_1, \tilde{v}_2, \tilde{v}_3, ..., \tilde{v}_n\}$ be the set of fuzzy vertices (events), where $\tilde{v}_1$ and $\tilde{v}_n$ are the tail and head events of the project, and each $\tilde{v}_1$ belongs to some path from $\tilde{v}_1$ to $\tilde{v}_n$. Let $\tilde{A} \in (\tilde{V}$ 鬃 $\tilde{V})$ be the set of directed edges $\tilde{A} = \{\tilde{a}_{ij} = (\tilde{v}_i, \tilde{v}_j) / \text{ for } \tilde{v}_i, \tilde{v}_j \in \tilde{V}\}$, that represents the activities to be performed in the project. Activity $\tilde{a}_{ij}$ is then represented by one, and only one, arrow with a tail event $\tilde{v}_i$ and a head event $\tilde{v}_j$. For each activity $\tilde{a}_{ij}$, a fuzzy number $\tilde{t}_{ij} \in \tilde{T}$ is defined as the fuzzy time required for the completion of $\tilde{a}_{ij}$. A critical path is a longest path from the initial event $\tilde{v}_1$ to the terminal event $\tilde{v}_n$ of the project network, and an activity $\tilde{a}_{ij}$ on a critical path is called a critical activity.

Most of the existing time-cost trade-off models rely on conditions where the activity time-cost relationships are assumed to be deterministic. A typical activity time - cost relationship is shown in Fig. 1. However, during project implementation, many uncertain factors, such as weather, productivity level, human factors etc., dynamically affect activity durations and costs. The activity time-cost relationship is no longer a monotonously decreasing curve as shown in Fig. 1. In this paper durations and costs are both characterized by triangular fuzzy numbers and the fuzzy relationship between time and cost is shown in Fig. 2.

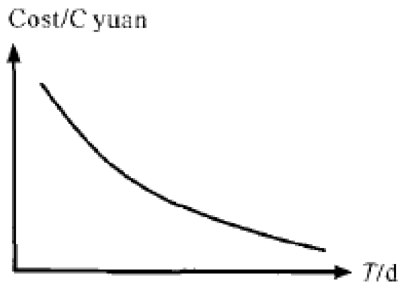
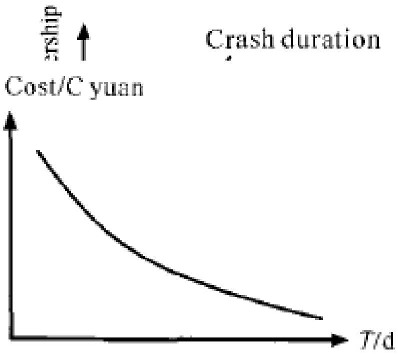
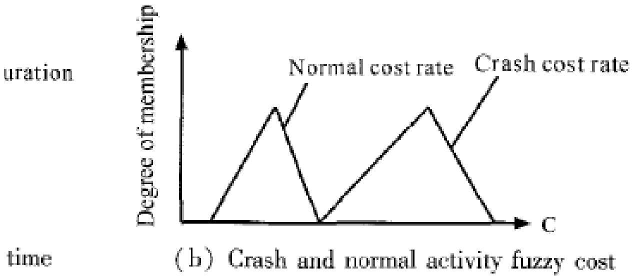


Figure 1 Deterministic relationship between time and cost of an active





ionship between time and cost of an activity

3.1 Notations

$\tilde{a}_{ij}$:	Project's activities, where, i = (1, 2, 3,…, n)
$\tilde{B}$:	Maximum available budget
$\tilde{T}$:	Shortest possible duration to complete the project at least cost within the maximum available budget
$\tilde{F}$:	The desired project completion time at least cost
$\tilde{T}_{N, I;}$:	Normal time for activity i
$\tilde{T}_{C, I;}$	Crash time for activity i
$\tilde{C}_{N, I;}$	Normal cost for activity i
$\tilde{C}_{C, I:}$	Crash cost for activity i
$\tilde{U}_{i:}$	Cost slope for activity i
$\tilde{C}$ P:	Critical path (longest path in the project network)
$\tilde{D}_{N, q:}$	Normal time for critical activity q, where q = (1, 2, 3,…, L)
$\tilde{D}_{C, q:}$	Crash time for critical activity q
$\tilde{C}_{N, j:}$	Normal cost for noncritical activity j, where j = (1, 2, 3,…, m)
$\tilde{C}_{C, q:}$	Crash cost for critical activity q

$\tilde{U}_{S}$:	Cost slope for critical activity S, where S begin with critical activity that has the smallest cost slope = (1, 2, 3,…, Y)
$\tilde{U}_{x}$:	Cost slope for noncritical activity x, where x begin with noncritical activity that has the biggest cost slope = (1, 2, 3,…, z)
$\tilde{D}_{r,S}$:	Max reduction in duration for critical activity S
$\tilde{T}C_N$:	Total cost to complete the project in normal condition
$\tilde{T}C_a$:	Total cost to complete the project by crashing all activities (crash condition)
$\tilde{T}C_C$:	Total cost to complete the project by CCA
$\tilde{T}E_C$:	Extra cost that adding to crash critical activities
$\theta_{CrashForT}$:	Number of steps to meet F (considering F>T) or obtain T (considering F = T) by CCA
CCA:	Approach of crashing critical activities

3.2 General descriptions and formulations

- Critical path of the project network in normal condition:

$$\tilde{C}P_k = \sum_{q=1}^{L} \tilde{D}_{N,q} \quad (1)$$

- Total cost of the project in normal condition:

$$\tilde{T}C_N = \sum_{i=1}^{n} \tilde{C}_{N,2} \quad (2)$$

- Total cost of the project by crashing all activities:

$$\tilde{T}C_a = \sum_{i=1}^{n} \tilde{C}_{c,i} \quad (3)$$

- Critical path of the project network by crashing all activities:

$$\tilde{C}P_C = \sum_{q=1}^{L} \tilde{D}_{c,q} \quad (4)$$

- Total extra cost that adding to crash critical activities:

$$\tilde{T}E_t = \sum_{S=1}^{Y} \tilde{D}_{r,s} \ \tilde{U}_s \quad (5)$$

- Total cost of the project by CCA:

$$\tilde{T}C_C = \tilde{T}C_N + \tilde{T}C_E \quad (6)$$

- Number of steps to meet F (considering F>T) or complete the project within T (considering F = T) by CCA:

$$\theta_{Crash\ Fort} = \sum_{s=1}^{Y} \tilde{D}_{r,s} \quad (7)$$

3.3 Algorithm

Step1: Draw the project network

Step2: Determine the normal time and normal cost for each activity to determine the critical and noncritical activities

Step3: Compute the normal total cost and normal duration of the project completion. If F equals the normal duration of completion then we stop the procedure

Step 4 : Find the project cost by the formula
Project cost = (Direct cost + (Indirect cost*project duration))

Step 5 : Find the minimum cost slope by the formula
Cost slope = (Crash cost - Normal cost)/(Normal time - Crash time)

Step 6 : Determine the crash time and crash cost for each activity to compute the cost slope

Step 7 : Identify the activity with the minimum cost slope and crash that activity. Identify the new critical path and find the cost of the project by formula
Project Cost= ((Project Direct Cost + Crashing cost of crashed activity)
+ Indirect Cost*project duration))

Step 8 : Crash all activities in the project simultaneously

Step 9 : Draw the project network after crashing all activities

Step10 : Determine the critical path and noncritical paths Also, identify the critical activities

Step11: In the In the new Critical path select the activity with the next minimum cost slope, and repeat this step until all the activities along the critical path are crashed upto desired time.

Step12: At this point all the activities are crashed and further crashing is not possible. The crashing of non critical activities does not alter the project duration time and is of no use.

4. Numerical Example

A precedence relationships network of a project with nine activities is depicted in the diagram. The crash time and normal time of an activity are triangular fuzzy numbers and crash cost rate and normal cost rate also are triangular fuzzy numbers (as shown in Table 1).

Table 1 Fuzzy durations and fuzzy cost rate of shipbuilding scheduling $\tilde{a} = (a_1, a_2, a_3)$

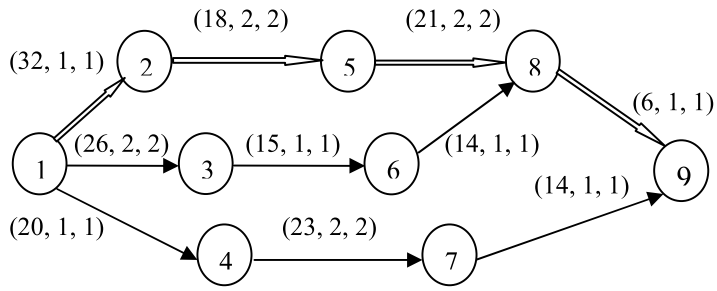
Activity index	Activity name	Fuzzy duration/d		Fuzzy cost rate/10^4 yuan d^{-1}		
		Min. crash Duration	Min. normal Duration	Crash cost rate	Normal cost Rate	Indirect cost rate
1-2	Material preparing	(28,30,32)	(31,32,33)	(1.7,1.9,2. 1)	(0.3,0.35,0.4)	(0.1, 0.12, 0. 14)
1-3	JP3Self-making omponents	(20,23,26)	(24,26,28)	(1.9,2.0,2.1)	(0.6,0.65,0.7)	(0.18,0.20,0.22)
1-4	The cell manufacture	(15,19,23)	(19,20,21)	(1.7,1.8,1.9)	(0.45,0.55,0.65)	(0.14,0.17,0.2)
2-5	Pipi-parts manufacture	(10,14,18)	(16,18,20)	(1.5,1.55,1.6)	(0.5,0.55,0.6)	(0.15,0.17,0.19)
5-8	Parts assembly	(17,20,23)	(19,21,23)	(1.8,1.85,1.9)	(0.4,0.5,0.6)	(0. 14,0. 17,0.2)
3-6	Block building	(10,13,16)	(14,15,16)	(2.0,2. 1,2.2)	(0.55,0.6,0.65)	(0.15,0. 17,0. 19)
6-8	Pallet assembly	(8 ,11,14)	(13,14,15)	(1.4,1.45,1.5)	(0.4,0.45,0.55)	(0. 13,0. 15,0. 17)
4-7	Sections building	(16,19,22)	(21,23,25)	(1.9,2.0,2.1)	(0.6,0.7,0.8)	(0.18,0.20,0.22)
7-9	Zone construction	(10,12,14)	(13,14,15)	(2.3,2.4,2.5)	(0.7,0.75,0.8)	(0.19,0.21,0.23)
8-9	Building berth healing	(4,5,6)	(5,6,7)	(2.4,2.5,2.6)	(0.8,0.85,0.9)	(0.2,0.23,0.26)

Here all the decision parameters are triangular fuzzy numbers represented in the form of $\tilde{a} = (a_1, a_2, a_3)$.

According to Definition 2.4 these triangular fuzzy numbers can also be represented in a convenient form $\tilde{a} = (a_1, a_2, a_3) = \left(a_0, a_*, a^*\right)$, where a_0 - location index number, $a_* = (a_0 - \underline{a})$ - left fuzziness index function and $a^* = (\overline{a} - a_0)$ - right fuzziness index function of $\tilde{a}$ respectively. Hence the given data in the form $\tilde{a} = \left(a_0, a_*, a^*\right)$ is

Table 2 Fuzzy durations and fuzzy cost rate of shipbuilding scheduling $\tilde{a} = (a_0, a_*, a^*)$

Activity index	Activity name	Fuzzy duration/d		Fuzzy cost rate/10^4 yuan d^{-1}		
		Min. crash Duration	Min. normal Duration	Crash cost rate	Normal cost Rate	Indirect cost rate
1-2	Material preparing	(30,2,2)	(32,1,1)	(1.9,.0.2,0.2)	(0.35,0.05,0.05)	(0.12,0.02,0.02)
1-3	JP3Self-making components	(23,3,3)	(26,2,2)	(2,0.1,0.1)	(0.65,0.05,0.05)	(0.2,0.02,0.02)
1-4	The cell manufacture	(19,4,4)	(20,1,1)	(1.8,0.1,0.1)	(0.55,0.1,0.1)	(0.17,0.03,0.03)
2-5	Pipi-parts manufacture	(14,4,4)	(18,2,2)	(1.55,0.05,0.05)	(0.55,0.05,0.05)	(0.17,0.02,0.02)
5-8	Parts assembly	(20,3,3)	(21,2,2)	(1.85,0.05,0.05)	(0.5,0.1,0.1)	(0.17,0.03,0.03)
3-6	Block building	(13,3,3)	(15,1,1)	(2.1,0.1,0.1)	(0.6,0.05,0.05)	(0.17,0.02,0.02)
6-8	Pallet assembly	(11,3,3)	(14,1,1)	(1.45,0.05,0.05)	(0.45,0.05,0.05)	(0.15,0. 02,0.02)
4-7	Sections building	(19,3,3)	(23,2,2)	(2,0.1,0.1)	(0.7,0.1,0.1)	(0.20,0.02,0.02)
7-9	Zone construction	(12,2,2)	(14,1,1)	(2.4,0.1,0.1)	(0.75,0.05,0.05)	(0.21,0.02,0.02)
8-9	Building berth healing	(5,1,1)	(6,1,1)	(2.5,0.1,0.1)	(0.85,0.05,0.05)	(0.23,0.03,0.03)

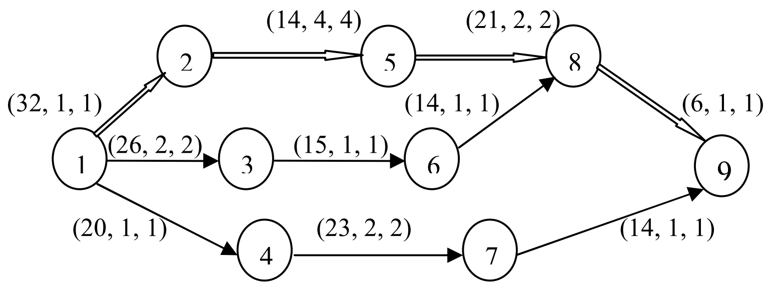


1 – 2 – 5 – 8 – 9 is the critical path and 1 – 2, 2 – 5, 5 – 8, 8 – 9 are the critical activities

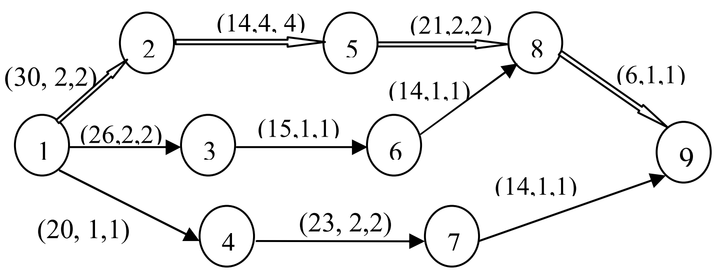
Table 3 Slope of Fuzzy durations and fuzzy cost rate of shipbuilding scheduling $\tilde{a} = \left(a_0, a_*, a^*\right)$

	Normal time $\tilde{NT}$	Normal cost $\tilde{NC}$	Crash time $\tilde{CT}$	Crash cost $\tilde{CC}$	$\tilde{\Delta t} = \tilde{NT} - \tilde{CT}$	$\tilde{\Delta C} = \tilde{CC} - \tilde{NC}$	$\frac{\tilde{\Delta C}}{\tilde{\Delta t}}$
1-2	(32,1,1)	(0.35,0.05,0.05)	(30,2,2)	(1.9,0.2,0.2)	**(2,2,2)**	(1.55,0.2,0.2)	**(0.78,2,2)-II**
1-3	(26,2,2)	(0.65,0.05,0.05)	(23,3,3)	(2,0.1,0.1)	(3,3,3)	(1.35,0.1,0.1)	(0.45,3,3)
1-4	(20,1,1)	(0.55,0.1,0.1)	(19,4,4)	(1.8,0.1,0.1)	(1,4,4)	(1.25,0.1,0.1)	(1.25,4,4)
2-5	(18,2,2)	(0.55,0.05,0.05)	(14,4,4)	(1.55,0.05,0.05)	**(4,4,4)**	(1.00,0.05,0.05)	**(0.25,4,4)-I**
5-8	(21,2,2)	(0.5,0.1,0.1)	(20,3,3)	(1.85,0.05,0.05)	**(1,3,3)**	(1.35,0.1,0.1)	**(1.35,3,3)-III**
3-6	(15,1,1)	(0.6,0.05,0.05)	(13,3,3)	(2.1,0.1,0.1)	(2,3,3)	(1.5,0.1,0.1)	(0.75,3,3)
6-8	(14,1,1)	(0.45,0.05,0.05)	(11,3,3)	(1.45,0.05,0.05)	(3,3,3)	(1,0.05,0.05)	(0.33,3,3)
4-7	(23,2,2)	(0.7,0.1,0.1)	(19,3,3)	(2,0.1,0.1)	(4,3,3)	(1.3,0.1,0.1)	(0.33,3,3)
7-9	(14,1,1)	(0.75,0.05,0.05)	(12,2,2)	(2.4,0.1,0.1)	(2,2,2)	(1.65,0.1,0.1)	(0.83,2,2)
8-9	(6,1,1)	(0.85,0.05,0.05)	(5,1,1)	(2.5,0.1,0.1)	**(1,1,1)**	(1.65,0.1,0.1)	**(1.65,1,1)-IV**
Direct cost		(5.95,1,1)					

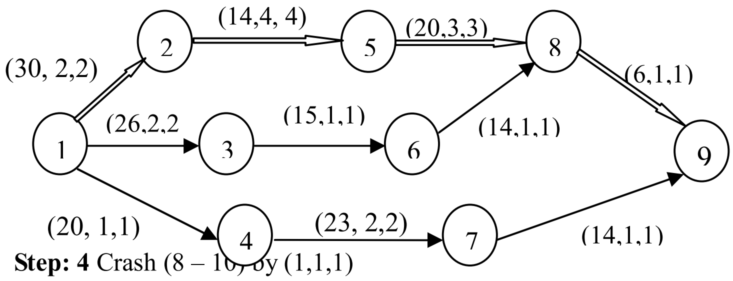
Step: 1 Since (2-5) has least crash slope, crash (2-5) by (4 , 4, 4)



Step: 2 Crash (1 – 2) by (2, 2, 2)



Step: 3 Crash (5 – 8) by (1, 3, 3)



Step: 4 Crash (8 – 10) by (1,1,1)

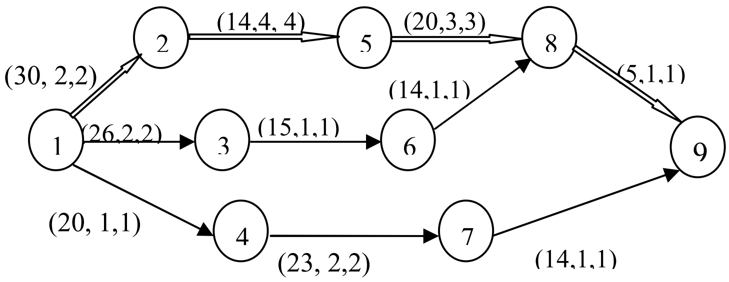


Table-4

Stage	**Crash**	**Current duration**	**Direct cost = (Normal cost + Crash cost)**	**Indirect cost**	**Total cost**
(1)	(2-5) by (4,4,4)	(73,4,4,)	(12.15,4,4)	(11.17,4,4)	(23.52,4,4)
(2)	(1-2) by (2,2,2)	(71,4,4)	(9.75,4,4)	(10.93,4,4)	(20.68,4,4)
(3)	**(5-8) by (1,3,3)**	**(70,4,4)**	**(7.8,4,4)**	**(10.76,4,4)**	**(18.56,4,4)**
(4)	(8-10) by (1,1,1)	(69,4,4)	(8.45,4,4)	(10.53,4,4)	(18.98,4,4)

From the above table we can see that after stage 3 the total cost starts increasing. Hence the optimal duration is (70,4,4) and the corresponding optimal cost is (18.56,4,4) 10^4 yuan.

5. Conclusion

This paper investigated the fuzzy network crashing problem in the project network with several fuzzy parameters. The validity of the proposed method is examined with numerical example. The applicability of the above procedure is useful for various types of problems in which uncertain results are compassed. We can consider this approach faster and easier to complete the project in shortest possible duration.

References

[1]. Abbasbandy.S and Hajjari. T, (2009). "A new approach for ranking of trapezoidal fuzzy numbers", Computers and Mathematics with Applications, 57, 413–419.

[2]. Badiru, A.B., (1991) "A simulation approach to Network analysis". Simulation, 57: 245-255.

[3]. Chanas S, Kamburowski J (1981) The use of fuzzy variables in PERT. Fuzzy Sets Syst 5:11–19

[4]. Coskun, O., (1984) "Optimal probabilistic compression of PERT networks", J. Construct. Eng. Manage.,

110: 437-446.

[5]. Feng C.W., L. Liu and S.A. Burns, (2000) "Stochastic construction time-cost tradeoff analysis". J. Comput. Civil Eng., 14: 117-126.

[6]. Jin Chao-guang, Jl Zhuo-shang I , Lin Yan'- Zhao Yuan-min, and Huang Zhen-dong (2005) "Research on the fully fuzzy time-cost trade-off based on genetic algorithms", Journal of Marine Science and Application, Vol. 4, No. 3

[7]. Johnson, G.A. and C.D. Schou, (1990) "Expediting projects in PERT with stochastic time estimates", Project Manager. J., 21: 29-32.

[8]. Liou.T.S and Wang.M.J, (1992) "Ranking fuzzy numbers with integral value", Fuzzy sets and systems,50,3,247-255

[9]. Ming Ma, Menahem Friedman, Abraham Kandel, (1999) "A new fuzzy arithmetic, Fuzzy sets and systems",108,83-90

[10]. Mohanaselvi S. et al. (2012) "Fuzzy Optimal Solution to Fuzzy Transportation Problem: A New Approach" / International Journal on Computer Science and Engineering (IJCSE) ISBN: 0975-3397 Vol. 4 No. 03 367-375.

[11]. Pulat, P.S. and S.J. Horn, (1996) "Time-resource tradeoff problem [project scheduling]". IEEE Trans. Eng. Manage., 43: 411-417.

[12]. Rehab, R. and R.I. Carr, (1989) "Time-cost trade-off among related activities", J. Construct. Eng. Manage., 115: 475-486.

[13]. Shakeela Sathish, K. Ganesan (2011) "A simple approach to fuzzy critical path analysis in project networks" International Journal of Scientific & Engineering Research Volume 2, Issue 12, ISSN 2229-5518.

[14]. Steve, P. Jr. and M.I. Dessouky, (1977) "Solving the project time/cost tradeoff problem using the minimal cut concept", Manage. Sci., 24: 393-400.

[15]. Van Slyke, R.M., (1963) "Monte carlo methods and the PERT problem", Operat. Res., 33: 141-143.

[16]. Wakas S. Khalaf, Leong Wah June, Mohd Rizam B. Abu Bakar and Lee Lai Soon (2011) "A Comparative Study on Time-cost Trade-off Approaches within Critical Path Method, Journal of Applied Sciences 11 (6): 920-931, ISSN 1812-5654, DOI: 10.3923/jas.2011.920.931.

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