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Modelling an Infectious Disease Prediction and control using S-I-R Model

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Abstract

This paper is concerned with the SIR model of Infectious diseases. Mathematical models based on the demographical factors, which provide a conceptual framework for understanding the process of transmission and control of infectious diseases at a basic level is proposed. The proposed model has advantages of translating theoretical and experimental work into information that can be used in clinical setting. Discussion on various measures for judging effectiveness of policies and control of infectious diseases was presented. Illustrative example based on this model was considered, in particular the infectious disease Rubeolla(measles) was used to validate this model. Mathematical problem was formulated and solved using a numerical technique.

Keywords and Phrases: SIR model, Infectious disease, Rubeolla, Basic Reproductive Ratio.

1 Introduction

A disease is an abnormal condition that affects the body of an organism. It is often construed as a medical condition associated with specific symptoms and signs[7].In humans,"disease" is often used more broadly to refer to any condition that causes pain, dysfunction, distress,social problems or death to the afflicted person or for those in contact with the person.
It is classified as infectious or non-infectious if it is caused by factors originating from an external source.

The micro-organism that cause these diseases are known as pathogens and includes varieties of bacteria, viruses, protozoa and fungi. Infectious diseases can be transmitted e.g by hand-to-mouth contact with infectious materials on surfaces, by bites of insects or other carriers of the diseases and from contaminated water or food [3].
An infectious disease is contagious if it is easily transmitted from person to person. These infectious diseases can be endemic or emerging; it is endemic if it is maintained in a population without the need for external inputs. This work is concerned with the SIR model of infectious diseases and various measures for judging effectiveness of policies and control methods. The measures discussed are: the basic reproductive ratio, Herd Immunity threshold, Effective Reproductive number and control vaccination number. These measures were discussed for our Rubeola example.

2 S.I.R Model

A model is a representation of reality. Models are also used to study and modify existing systems and processes, and as part of control systems.
The first account of mathematical modelling of spread of infectious disease was carried out by Daniel Bernoulli in 1766, which was recently republished (Bernoulli and Blower)[4]. Also, W.O. Kermack and A.G. McKendrick [11] also developed models of epidermic which was later expanded by Anderson and May [1].
in this work, we make use of compartmental model, in which the population was divided into three different groups.

2.1 Basic Assumptions

Before going into details analysis, we shall make some necessary assumptions that will govern our derived equations. However, some of these assumptions could be relax later to better reflect advances in diseases control measures.

- The Population is fixed but considered demography factor i.e birth and death
- The same number of births and deaths per unit time

- The member of the population mix homogeneously or uniformly
- Social status ,sex, Age, and colour dos not affect the probability of being infected

2.2 Model Analysis

We start with the following basic notations
N is the total number of populations
S(t) is the number of susceptible individuals at time t
I(t) is the number of infected individuals at time t
R(t) is the number of recovered individuals at time t

The assumptions lead us to a set of differential equations

$$\frac{dS}{dt} = \mu - \beta S(t)I(t) - \mu S(t) \tag{1}$$

$$\frac{dI}{dt} = \beta S(t) - \gamma I(t) - \mu I(t) \tag{2}$$

$$\frac{dR}{dt} = \gamma I(t) - \mu R(t) \tag{3}$$

where γ is the removal or recovery rate,
α is the probability of becoming infected,
β is the transmission rate. μ is the per capita death rate, and the population level birth rate.

$$S(t) + I(t) + R(t) = N \tag{4}$$

By applying Euler's method, we have the following schemes;

$$S_{n+1} = \mu + [1 - (\beta I_n - \mu)]S_n \Delta t \tag{5}$$

$$I_{n+1} = \beta S_n + [1 - (\gamma + \mu)]I_n \Delta t \tag{6}$$

$$R_{n+1} = \gamma I_n + [1 - \mu]R_n \Delta t \tag{7}$$

where $S_{n+1}, I_{n+1}, R_{n+1}$ are the number of susceptible, infected, and recovered people at time (n+1). Δt is a small change in time, and will be equal to one from now on [8]. The equations above are primarily used to calculate β, γ and μ.

2.3 Basic Reproduction Ratio

An important quantity in epidemiology is the Basic Reproductive Ratio denoted as R_o . R_o is defined as the mean number of secondary infections produced when one infected individual is introduced into a host population where everyone is susceptible[9]. It is affected by the infection, removal and birth or death rates, i.e β , γ , μ and is obtained by

$$R_o = \frac{\beta}{(\gamma + \mu)} \tag{8}$$

When $R_o > 1$, the occurrence of the disease will increase i.e Epidemic , When$R_o < 1$, the disease occurrence will decrease and the disease will eventually be eliminated i.e Eradication . When $R_o = 1$, the disease occurrence will be constant i.e Endemic [2]. The Basic Reproduction ratio also tells if a population is at risk from a disease and helps to predict who will not become infected at all.

2.4 Herd Immunity Threshold

Herd immunity is defined as the process where " for each person that is vaccinated the risk of infection for the rest of the community decreases" [9,10]. The main purpose of vaccination is to create herd immunity while having the amount of infected people to be very small [9,10]. The herd immunity Threshold (H_I) is the percentage of the population that needs to be immune to control transmission of a disease, i.e . R_o equal to one. The equation given by [6] for figuring out the Herd Immunity Threshold is

$$H_t = \frac{R_o - 1}{R_o} = 1 - \frac{1}{R_o} \tag{9}$$

As the amount of vaccinations increase, the herd immunity threshold also increase. By decreasing the amount of susceptible people, the herd immunity threshold decreases.

2.5 Effective Reproductive Number

Effective Reproductive Number, denoted E_R is the average number of secondary cases generated by an infectious case during the epidemic. To calculate this number , we multiply the basic reproductive ratio by how many

people are susceptible at time t, i.e

$$E_R = R_o \frac{S_t}{N} \tag{10}$$

Effective Reproductive Number is important since it helps researchers and health officials determine how effective their policies are [9].

2.6 Control Vaccination Number

The Control Vaccination Number , denoted C_v , is the average number of secondary cases generated by an infectious case during epidemic with control measures, i.e vaccinations. The formula for calculating vaccination control is given as

$$C_v = R_o[1 - hf] \tag{11}$$

where h is the vaccine efficacy and f is vaccination coverage. The main goal of researchers and health officials is to have $C_v < 1$.

3 Case Study: Measles

3.1 Measles background

Measles, also known as morbilli or rubeolla is an infection of the respiratory system caused by a virus . Measles is spread through respiration or contact with fluids from an infected person's nose and mouth, either directly or through transmission and is highly contagious. According to [5], [8], [12], and [13] the incubation period of measles is approximately seven to twelve days while the infectious period is between six to seven days. The probability of becoming infected is 75% [11] and 90% when in close contact [13] .Before the vaccine , there was a high mortality rate. Now that the vaccine is available , the probability of dying from rubeola is < 0.0001 [8]

3.2 SIR Model and Rubeolla

Rubeolla is a disease that we can model using the SIR model. The assumptions in section 2 are all satisfied . For our example, population is made up of 100 people. Since the disease is highly contagious or infectious, everyone

will eventually become infected. In tables 1 and 2 we see the number of people in each state at a given period of time. We started with everyone being susceptible to the disease, and one person suddenly becomes infected. It is important to note that equations (5), (6) and (7), were not used to make these cases, as these are just formulated scenarios.

Table 1: The period for $\alpha = 0.75$

	State		
period	S	I	R
0	100	0	0
1	99	1	0
2	25	74	1
3	6	19	75
4	1	5	94
5	0	1	99
6	0	0	100

Table 2: The period for $\alpha = 0.85$

	State		
period	S	I	R
0	100	0	0
1	99	1	0
2	15	84	1
3	2	13	85
4	0	2	98
5	0	0	100

From tables I and II , we can calculate β . To do this we manipulate equation(8) using $\Delta t = 1$ to get the following

$$\beta = \frac{2S_n - S_{n+1}}{S_n I_n} \tag{12}$$

using this equation we can get the β for each period. These β are shown in the following tables:

3.3 The Effects of Infectious rate and the amount of Initial Infectious persons

One of the most important part of disease modeling is the infectious rate. Rubeola's infetion rate is 75-90% as calculated by [11]. This number affects

Table 3: Different $\beta's$ for $\alpha = 0.75$

period	Beta
0	0
1	1.7475
2	0.0238
3	0.0965
4	0.4
5	0
6	0
Average	0.3239

Table 4: Different $\beta's$ for $\alpha = 0.85$

period	Beta
0	0
1	1.8485
2	0.0222
3	0.1538
4	0
5	0
Average	0.3374

the amount of people in the susceptible , infected, and recovered groups, and how long it takes until everyone that will get the disease, recovers from it . The next set of graphs shows how the infectious rates affects the amount of people in the susceptible , infected, and recovered groups.
draw graph.

3.4 Rubeola's Basic Reproductive Ratio

Since we have calculated β for our outbreak of rubeola, we can calculate the basic Reproductive Ratio.
When the probability of becoming infected is 75%,

$$R_o = \frac{\beta}{(\gamma + \mu)} S_o = 16.19$$

When the probability of becoming infected is 85 % ,

$$R_o = \frac{\beta}{(\gamma + \mu)} S_o = 16.8$$

In general for Rubeola ,R_o is usually between sixteen and eighteen [5,12] and our result equals the actual R_o. From our result we see R_o (and from the actual R_o's), that $R_o > 1$. Thus the disease cannot be eliminated(though no more people can become infected on our case).

3.5 Rubeola's Herd Immunity Threshold

Since we now know the Basic Reproductive Ratio , we can calculate the Herd Immunity Threshold,H_I [9]. From our cases,when the probability of becoming infected is 75%,

$$H_I = \frac{R_o - 1}{R_o} = 0.9382.$$

When the probability of becoming infected is 85%,

$$H_I = \frac{R_o - 1}{R_o} = 0.9405$$

3.6 Rubeola's Effective Reproductive Number

We can calculate the Effective Reproductive number , E_R, for our two cases [9]. We get the following tables (Tables 5 and 6) that shows the Effective Reproductive Number at each period

Table 5: E_R for $\alpha = 0.75$

period	S	I
0	100	16.19
1	99	16.0281
2	25	4.2275
3	6	0.9714
4	1	0.1619
5	0	0
6	0	0

Table 6: E_R for $\alpha = 0.85$

period	S	I
0	100	16.8
1	99	16.632
2	15	2.52
3	2	0.336
4	0	0
5	0	0

We notice that $E_R < 1$, at period 3 when the probability of becoming infected is 0.75 and 0.85. This means that any policies that were implemented were effective.

3.7 Rubeola's Control Vaccination Number

The African Regional recommended schedule for measles vaccination contains one measles vaccine given at or shortly after the 9th month of age. However, until May 2008, the African region accepted policies for second measles doses in routine immunization.Research has shown that the vaccine has $95-98\%$ effectiveness [14].The vaccination coverage estimated by WHO and UNICEF for Africa Region in 2007 is 74% [14]. We can calculate the control Vaccination Number , C_V, for Rubeola for our data using the 2007 vaccine coverage for Africa region percentage.

R_o	C_v
16.19	4.7922
16.8	4.9728

Table VII: C_v **for different** R_o **when** $h = 0.95$

From table VII we discovered that the control vaccination number is not less than 1 which is the aim of researchers. However if the vaccine coverage is increased from 74% to 97% , the control vaccination number will be less than one i.e $C_v < 1$

4 Conclusion

In this paper, we have developed a simple SIR model of an infectious diseases under the assumption of flat birth and death rates. The resulting mathematical problem was solved using an explicit numerical scheme. Our work could be viewed as a substantial progress made in developing a more realistic SIR model for an infectious disease (Rubeolla).

As well as analysing various theoretically measures for judging effectiveness of policies and control of Rubeolla, our numerical results are valuable to health practitioners. We discovered that researchers and health officials can check if their policies are effective and if the occurrence of the disease is increasing, decreasing or stable. Our results reveal that when $R_o > 1$,the disease cannot be eliminated(though no more people can become infected in our case). Moreover, if Herd immunity threshold is less than one in both cases, it implies that the policy is effective since main purpose of vaccina-

tion is to create Herd immunity. However, in order to have a $C_v < 1$ with 100% vaccination effectiveness, we would only need to vaccinate 95-98% of the population.

However, the SIR model has some disadvantages; The population has to be fixed and the population needs to mix homogeneously, and the model does not take into account different sexes, races, or ages.

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