

The Performance of The Lee-Carter Model on Heterogeneous Adult Mortality Data in a Limited Data Situation

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Abstract

Adult mortality in developing Countries remains one of the greatest challenges for monitoring, analyzing and projecting the health situation of a large proportion of the world's population. Because of limited resources, many developing Countries, especially in sub-Saharan Africa lack vital registration systems that could reliably and continuously collect information on adult mortality. However, periodic reviews of mortality rate and pattern using mortality models could show new trends and may provide information for the planning of a Country's health care delivery services and a host of other programs.

Previous studies have found that the Lee-Carter model works well with homogenous mortality data. In this current study, we want to see the performance of the model on heterogenous adult mortality data in a limited data situation. A modified version of the Lee-Carter method is used to model sex-combined adult mortality data of Nigerians aged 15-84 years for the time periods 1990, 2000 and 2012. The model's parameters are estimated using the approach proposed by Lee and Carter (1992) based on the singular value decomposition technique, while the mortality index is predicted using the approach developed by Li *et al.* (2002). Our results show that on the overall, the model follows the mortality pattern very well for most of the ages despite the heterogenous nature of the data used. Forecast values of the mortality index show a gradual decline in mortality from 2013-2025 in Nigeria.

KEYWORDS: Adult mortality; Age; Lee-Carter model; Nigeria.

1. Introduction

Whether it is the *eye* or the *toe*, every part of the human body is designed to play a particular function or the other. Oftentimes, we do not realize the importance of the '*less significant part*' until it brings a form of discomfort we can't bear. The workforce of any nation forms an integral part of the adult population. Therefore, the latter should not be overlooked or treated as though they do not '*really*' matter. For several years, emphasis has been placed on infant and child mortality while adult mortality has been given little prominence especially in a developing country like Nigeria.

The level of adult mortality varies markedly across the world, both between countries and between regions and social groups within particular countries. At the global level, adult mortality is said to have reduced especially in low income countries (WHS 2014). However, estimates of life expectancy show that Nigeria is one of the nine Countries in Africa with life expectancy below 55 years. Adult mortality is higher in Nigeria compared to her neighbors such as Benin, Ghana and Cameroon (WHS 2014).

With a wide availability of mortality history data, the Lee-carter model has been applied and has given successful results for various Countries. For instance the U.S. (Lee and Carter 1992); (Preston and Wang 2009), Canada: (Lee and Nault 1993); Li and Chan (2007), Chile (Lee and Rofman 1994), Japan (Wilmoth 1998), Taiwan (Wang & Liu 2010), Nordic countries (Koissi, Shapiro & Högnäs 2004), Belgium (Brouhns *et al.* 2002), Australia (Booth *et al.* 2002), the U.K. (Renshaw and Haberman 2003), Sweden: (Lundström & JanQvist 2004); Wang (2007). Bâte and Scherp (2007) and Chukwu & Oladipupo (2012) discovered that the Lee-carter model works well on homogenous mortality data.

1.1 Study Objectives

The main objective of this study is to see the performance of the model on heterogenous adult mortality data. Heterogeneity in this study implies that the data used for the study is sex-combined and the age groups have varying mortality trends. Others objectives are:

1. To determine past time trends in the general pattern of adult mortality across the age groups
2. To assess the sensitivity of the hazard rate at age x to the time trend.
3. To forecast for future expectation of the mortality trend

2. Data and Methods

The data used for this study was extracted from the global health observatory of the World Health Organization. It covers the age-specific mortality rate records of persons aged 15 to 84 years for the time periods 1990, 2000 and 2012 in Nigeria. Methods used for analysis include the Singular value decomposition technique using the principal component analysis option. Forecast of the mortality index was done by applying the random walk with drift model. The analysis for this study was done using the *biplot* ad-in in Microsoft Excel 10.0.

2.1 The Lee-Carter Model

The Lee-Carter model is a simple bilinear model in the variables x (age) and t (calendar year) defined as;

$$\ln m_{xt} = \hat{a}_x + \hat{b}_x \hat{k}_t + \varepsilon_{xt}$$

Where;

m_{xt} : is the matrix of the observed age-specific death rate at age x during year t . It is obtained from observed deaths divided by population exposed to risk.

\hat{a}_x : represents the logarithm of the geometric mean of the empirical mortality rates, averaged over the historical data. It describes the average shape of the age profile.

\hat{k}_t : represents the underlying time trend for the general mortality. It captures the main time trend on the logarithmic scale in mortality rates at all ages. It is also referred to as the mortality index.

\hat{b}_x : represents the sensitivity of the hazard rate at age x to the time trend. It indicates the relative pace of change in mortality by age as \hat{k}_t varies. It modifies the main time trend according to whether change at a particular age is faster or slower than the main trend.

ε_{xt} : is the residual term at age x and time t . It reflects the age specific influences not captured by the model. Each of ε_{xt} is independent and identically distributed $\varepsilon_{xt} \sim N(0, \sigma^2)$.

2.1.1 Merits of the Model

- It produces an excellent fit to mortality trends;
- It is parsimonious in the number of parameters used;
- It linearizes mortality trends and thereby adds confidence to extrapolations;
- It produces sensible estimates of forecast of uncertainty and
- The model outperforms other models with respect to its prediction errors.

Despite its advantages over other extrapolative methods, the Lee-Carter model also shares the fundamental weaknesses of extrapolation: historical patterns may not hold for the future, and structural changes may therefore be missed.

2.1.2 Assumptions of the Model

- The model assumes that b_x is invariant (remains constant) over time for all x .
- It assumes that k_t is fixed over age-groups for all t .
- The practical use of the model assumes that the disturbances ε_{xt} are normally distributed i.e. $\varepsilon_{xt} \sim N(0, \sigma^2)$

2.2 Estimating the Parameters

The Lee-Carter model unlike most regression formulations is not a common model structure with observed variables on the right hand side and thus it cannot be fit with simple regression formulations. Hence, the estimation of \hat{b}_x and \hat{k}_t cannot be solved explicitly. The *singular value decomposition* (SVD) method was used to find a least squares solution in this study

The Singular Value Decomposition (SVD) is a technique used to decompose a matrix into several component matrices, exposing many of the useful and interesting properties of the original matrix. Ideally, the matrix is decomposed into a set of factors (often orthogonal or independent) that are optimal based on some criterion. In other words, any real $m \times n$ matrix A can be decomposed uniquely as:

$$A = UDV^T$$

U is $m \times n$ and orthogonal (its columns are eigenvectors of AA^T)

V is $n \times n$ and orthogonal (its columns are eigenvectors of $A^T A$)

D is $n \times n$ diagonal (non-negative real values called *singular values*)

$D = \text{diag}(\rho_1, \rho_2, \dots, \rho_n)$ ordered so that $\rho_1 \geq \rho_2 \geq \dots \geq \rho_n$

(If ρ is a singular value of A , its square is an eigenvalue of $A^T A$)

The application of the SVD approach follows about 6 steps (see: *Chukwu & Oladipupo 2012*)

The purpose of using Singular value decomposition is to transfer the task of forecasting an age-specific vector $\ln m_{xt}$ into forecasting a scalar κ_t , with small error. As earlier stated, the model is given by: $\ln m_{x(t)} = \hat{a}_x + \hat{b}_x \hat{\kappa}(t) + \varepsilon_{x,t}$ and we need to estimate a_x , b_x and κ_t .

In order to achieve a unique solution the following restrictions are used;

$$\sum_{x \min}^{x=m} b_x^2 = 1 \quad \dots \dots \dots (1)$$

$$\sum_{t \min}^{t=n} \kappa_t = 0 \quad \dots \dots \dots (2)$$

Where $x = 1 \dots m$ age groups and $t = 1 \dots n$ calendar years.

To estimate the parameters we choose the values that minimize Q ;

$$Q = \sum_{x,t} (a_x + b_x \kappa_t - q_{xt})^2, \quad \dots \dots \dots (3)$$

Where $\ln m_{xt}$ and Q are subject to the constraints in (3.1) and (3.2) above.

To find values that minimize Q we introduce Lagrange's multipliers; α and β

$$R = Q - \alpha \sum_t \kappa_t - \beta \sum_x b_x^2. \quad \dots \dots \dots (4)$$

We substitute equation (3) into (4) and then minimize;

$$R = \sum_{x,t} (a_x + b_x \kappa_t - q_{xt})^2 - \alpha \sum_t \kappa_t - \beta \sum_x b_x^2$$

Thus we first take the derivative of R in respect of a_x , b_x and κ_t respectively;

$$\frac{dR}{da_x} = 2 \sum_t (a_x + b_x \kappa_t - q_{xt}) \quad \forall x \quad \dots \dots \dots (5)$$

$$\frac{dR}{db_x} = 2 \sum_t \kappa_t (a_x + b_x \kappa_t - q_{xt}) - 2\beta b_x \quad \forall x \quad \dots \dots \dots (6)$$

$$\frac{dR}{d\kappa_t} = 2 \sum_x b_x (a_x + b_x \kappa_t - q_{xt}) - \alpha \quad \forall t \quad \dots \dots \dots (7)$$

The derivatives are set equal to

$$\frac{dR}{da_x} = 0 \quad , \quad \frac{dR}{db_x} = 0 \quad \text{and} \quad \frac{dR}{dk_t} = 0$$

We solve equation (3.5);

$$2 \sum_t (a_x + b_x \kappa_t - q_{xt}) = 0 \quad \Rightarrow$$

$$na_x + b_x \sum_t \kappa_t - \sum_t q_{xt} = 0 \quad \left(\sum_{t \min}^{t=n} \kappa_t = 0 \right) \quad \Rightarrow$$

$$na_x - \sum_t q_{xt} = 0 \quad \Rightarrow$$

$$\hat{a}_x = \frac{1}{n} \sum_t q_{xt}$$

$$\hat{a}_x = \frac{1}{n} \sum_{t=1}^n \ln m_{xt}$$

The estimate for a_x is thus computed as the average over time of the logarithm of the central death, which corresponds to the definition of a_x in section 3.0.

We now define $z_{xt} = q_{xt} - a_x \Rightarrow \sum_t z_{xt} = 0$ for every x .

Equation (3.7) can now be rewritten as:

$$2 \sum_x b_x (a_x + b_x \kappa_t - q_{xt}) - \alpha = 2 \sum_x b_x (b_x \kappa_t - z_{xt}) - \alpha$$

If we set the new expression of the derivative with regard to κ_t equal to zero we have;

$$2 \sum_x b_x (b_x \kappa_t - z_{xt}) - \alpha = 0 \quad \Rightarrow$$

$$\sum_x b_x (b_x \kappa_t - z_{xt}) = \frac{\alpha}{2} \quad \Rightarrow$$

$$\kappa_t \sum_x b_x^2 - \sum_x b_x z_{xt} = \frac{\alpha}{2} \quad \left(\sum_{x \min}^{x=m} b_x^2 = 1 \right)$$

$$\kappa_t - \sum_x b_x z_{xt} = \frac{\alpha}{2} \quad \dots \dots \dots (8)$$

Taking the sum over t in the equation (8) above we get,

$$\sum_t (\kappa_t - \sum_x b_x z_{xt}) = \sum_t \frac{\alpha}{2} \quad \Rightarrow$$

$$\sum_t \kappa_t - \sum_t \sum_x b_x z_{xt} = \sum_t \frac{\alpha}{2} \quad \left(\sum_{t \min}^{t=n} \kappa_t = 0 \right) \quad \Rightarrow$$

$$m \sum_x b_x \sum_t z_{xt} = \frac{m\alpha}{2} \quad \left(\sum_t z_{xt} = 0 \right) \Rightarrow \alpha = 0$$

We now have an expression for κ_t by putting $\alpha = 0$ in equation (3.8).

$$\text{Hence, } \kappa_t = \sum_x b_x z_{xt} \quad \text{for every } t \quad \dots\dots\dots (9)$$

The constraint for κ_t is now fulfilled.

If we substitute the expression $z_{xt} = q_{xt} - a_x$ into equation (3.6), we obtain;

$$2 \sum_t \kappa_t (a_x + b_x \kappa_t - q_{xt}) - 2\beta b_x = 2 \sum_t \kappa_t (\kappa_t b_x - z_{xt}) - 2\beta b_x$$

Setting this equal to zero we have,

$$2 \sum_t \kappa_t (\kappa_t b_x - z_{xt}) - 2\beta b_x = 0$$

$$b_x (\sum_t \kappa_t^2 - \beta) = \sum_t \kappa_t z_{xt} \quad \forall x \quad \dots\dots\dots (10)$$

We take the square of both sides and summarise over x and get

$$\sum_x (b_x (\sum_t \kappa_t^2 - \beta))^2 = \sum_x (\sum_t \kappa_t z_{xt})^2 \Rightarrow$$

$$(\sum_t \kappa_t^2 - \beta)^2 \sum_x (b_x)^2 = \sum_x (\sum_t \kappa_t z_{xt})^2 \quad \left(\sum_{x \text{ min}}^{x \text{ max}} b_x^2 = 1 \right) \Rightarrow$$

$$(\sum_t \kappa_t^2 - \beta)^2 = \sum_x (\sum_t \kappa_t z_{xt})^2 \quad (\text{Taking the square root of both sides}) \Rightarrow$$

$$\sum_t \kappa_t^2 - \beta = \sqrt{\sum_x (\sum_t \kappa_t z_{xt})^2} \Rightarrow$$

$$\beta = \sum_t \kappa_t^2 - \sqrt{\sum_x (\sum_t \kappa_t z_{xt})^2} \quad \dots\dots\dots (11)$$

We are able to get an expression for b_x by inserting the equation for β in (11) into equation (10):

$$b_x (\sum_t \kappa_t^2 - \beta) = \sum_t \kappa_t z_{xt}$$

$$\left(\beta = \sum_t \kappa_t^2 - \sqrt{\sum_x (\sum_t \kappa_t z_{xt})^2} \right) \Rightarrow$$

$$b_x (\sum_t \kappa_t^2 - \sum_t \kappa_t^2 + \sqrt{\sum_x (\sum_t \kappa_t z_{xt})^2}) = \sum_t \kappa_t z_{xt} \Rightarrow$$

$$\hat{b}_x = \frac{\sum_t \kappa_t z_{xt}}{\sqrt{\sum_x (\sum_t \kappa_t z_{xt})^2}} \quad \forall x.$$

Hence, $\hat{a}_x = \frac{1}{n} \sum_{t=1}^n \ln m_{xt}$, $\hat{b}_x = \frac{\sum_t \kappa_t z_{xt}}{\sqrt{\sum_x (\sum_t \kappa_t z_{xt})^2}}$ $\hat{\kappa}_t = \sum_x b_x z_{xt}$

2.3 Forecasting $\hat{\kappa}_{u(t)}$ Using Data at Unequal Intervals

After fitting the mortality data to the model and the values of the vectors \hat{a}_x , \hat{b}_x and $\hat{\kappa}_{u(t)}$ are found, only the mortality index $\hat{\kappa}_{u(t)}$ needs to be predicted. Suppose mortality data is collected at times $u(0)$, $u(1) \dots u(T)$. The random walk with drift equation becomes

$$\hat{\kappa}_{u(t)} = \hat{\kappa}_{u(t-1)} + \hat{\theta}[u(t) - u(t-1)] + [(\varepsilon_{u(t-1)} + 1) + \dots + \varepsilon_{u(t)}]$$

$$\text{So } \hat{\kappa}_{u(t)} - \hat{\kappa}_{u(t-1)} = \hat{\theta}[u(t) - u(t-1)] + [(\varepsilon_{u(t-1)} + 1) + \dots + \varepsilon_{u(t)}] \dots \dots \dots (12)$$

Because the means of the second term in the right-hand side of equation (12) are zero, the unbiased estimate of $\hat{\theta}$ is obtained as:

$$\hat{\theta} = \frac{\sum_{t=1}^T \hat{\kappa}_{u(t)} - \hat{\kappa}_{u(t-1)}}{\sum_{t=1}^T [u(t) - u(t-1)]} = \frac{\hat{\kappa}_{u(T)} - \hat{\kappa}_{u(0)}}{u(T) - u(0)}$$

$$\text{Var}(\varepsilon_{u(t)}) = \frac{\sum_{t=1}^T [\hat{\kappa}_{u(t)} - \hat{\kappa}_{u(t-1)} - \hat{\theta}[u(t) - u(t-1)]]^2}{[u(T) - u(0) - \frac{\sum_{t=1}^T [u(t) - u(t-1)]^2}{u(T) - u(0)}]}$$

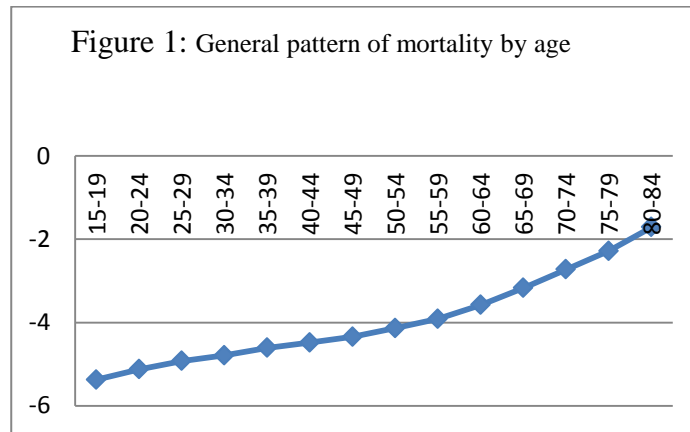
$$\begin{aligned} \text{Var}(\hat{\theta}) &= \frac{\text{var}\{\sum_{t=1}^T [(\varepsilon_{u(t-1)} + 1) + \dots + \varepsilon_{u(t)}]\}}{[u(T) - u(0)]^2} \\ &= \frac{\sigma^2}{u(T) - u(0)} \approx \frac{\text{Var}(\varepsilon_{u(t)})}{u(T) - u(0)} \end{aligned}$$

3. Summary of Results

We present below a summary of our findings.

$$\hat{a}_x =$$

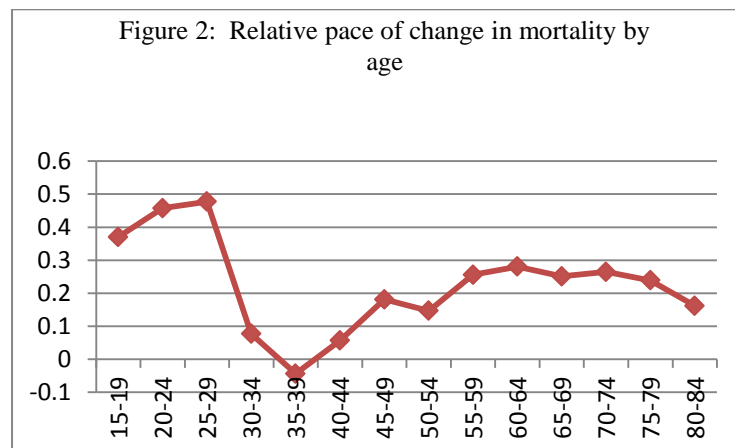
15-19	-5.3727
20-24	-5.12539
25-29	-4.92421
30-34	-4.78905
35-39	-4.60852
40-44	-4.48086
45-49	-4.34478
50-54	-4.13647
55-59	-3.91462
60-64	-3.57863
65-69	-3.17261
70-74	-2.72591
75-79	-2.28179
80-84	-1.70845



Parameter \hat{a}_x represents the general pattern of mortality by age. Our results show that \hat{a}_x values are increasing with age. This implies an upward trend in mortality with respect to the age-groups. As shown in figure 1 above, the younger ages have a lower mortality rate than the older ages. The reason for this disparity is probably due to differences in exposure to risk such as family responsibilities, stress, health issues so on.

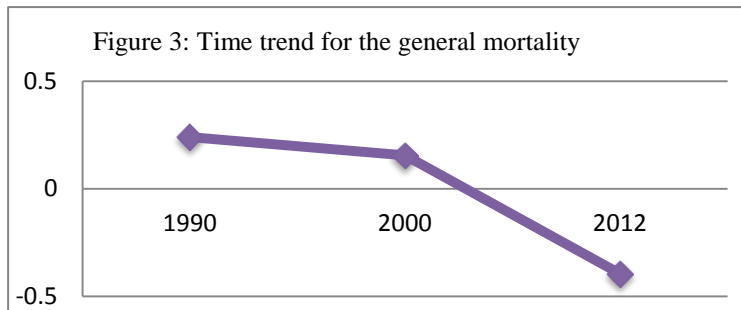
$$\hat{b}_x =$$

15-19	0.370253
20-24	0.457515
25-29	0.477339
30-34	0.077003
35-39	-0.04363
40-44	0.056886
45-49	0.181261
50-54	0.146721
55-59	0.255776
60-64	0.280355
65-69	0.250819
70-74	0.264712
75-79	0.238962
80-84	0.161811

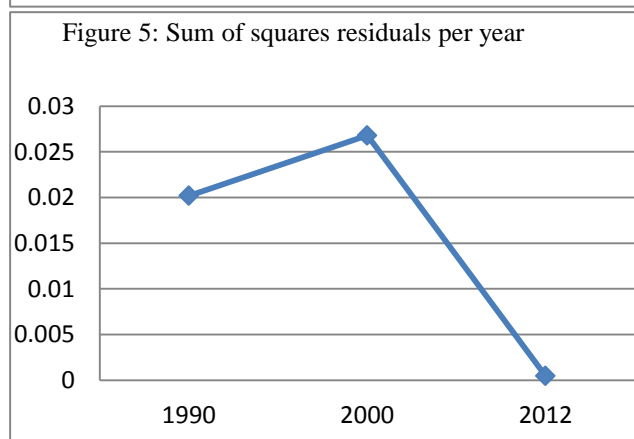
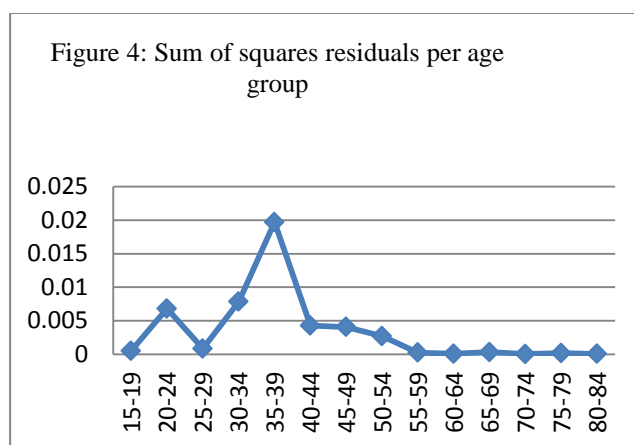


Parameter \hat{b}_x describes the tendency of mortality at age x to change as the general level of mortality changes. The larger the value of \hat{b}_x at a particular age-group, the more fluctuant the mortality rate at that age-group as compared to the general level of mortality change. The results obtained show that persons aged 20-29 have a more fluctuant mortality pattern than other age groups with age group 25-29 having the highest fluctuant mortality pattern. The relative pace of change is lowest in age group 35-39 as illustrated in figure 2 above.

1990 2000 2012
 [0.23988 0.15597 -0.39586]



Parameter $\hat{k}_{u(t)}$ is the mortality index and it captures the main time trend on the logarithmic scale in death rates at all ages. When $\hat{k}_{u(t)}$ values decrease overtime, it signifies a decline in mortality trend as the years increase. Our findings show a downward trend from 1990 to 2012 as shown in figure 3. However, it is important to note that the straight line obtained in figure 3 is as a result of the 3-time point nature of our data. *It does not necessarily imply that the mortality pattern between 1990-2000 and 2000-2009 was not a fluctuating one.* Furthermore, the significant decline in mortality in between the years can be attributed to social-economic factors such as improved Government policies on health, for instance the National health insurance scheme (NHIS).



Sum of squares residuals per age group & per year: The residuals indicate the age specific influences not captured by the model. As shown in figure 6 above, residuals were lowest at 2012.

4. Conclusion

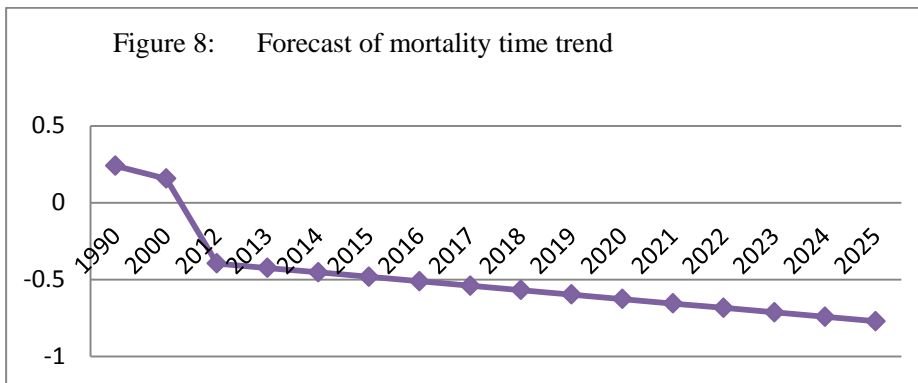
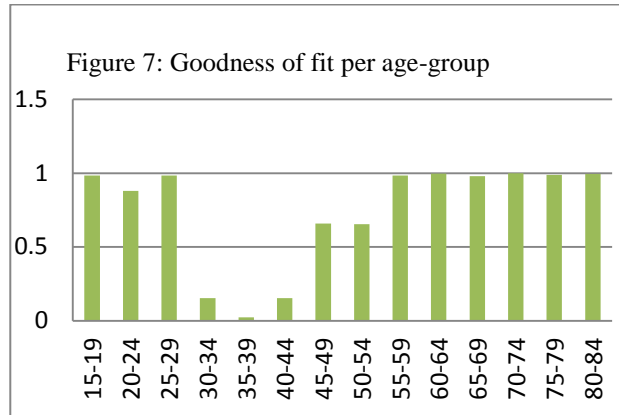
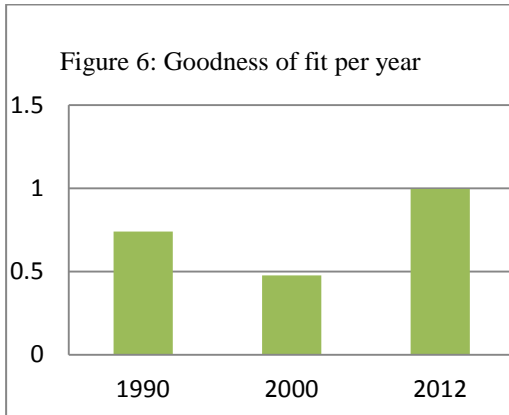
The goodness of fit as shown in figure 7 of the appendix shows that the Lee-Carter model follows the mortality pattern very well for most of the ages despite the heterogenous nature of the data. On the overall, an R^2 of 0.8341 was obtained as the goodness of fit of the model. The forecast of the mortality time trend show a gradual decline in mortality from 2013-2025 as expressed in figure 8 while the general pattern of mortality shows an upward trend from age 15-84 years all things being equal. We can therefore say that the lee-carter model can be used to model the Nigeria mortality data even when the data available is heterogenous in nature. However, the level of data heterogeneity for which the lee-carter model will give successful result is another issue for discussion.

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Appendix



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