Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.5, No.5, 2015

An Analysis of Group Theoretic Properties of A Class of (123)-avoiding Pattern of Aunu Numbers Using Thin Cyclic Design

Usman .A* And Magami M.S **

*Department of mathematics and Computer Sciences. Umaru Musa Yar'adua University, P.M.B. 2218 Katsina. Nigeria

**Department of Mathematics, Usmanu Danfodiyo University, P.M.B. 2346 Sokoto. Nigeria

Abstract

This paper looks on some Application of Aunu group using some class of cyclic design. It shows that by using the procedure of thin cyclic designs the element of the generating set can be generated from another, this shows that each of the four element of the generating set has index 5and procedure produce 20 instinct element from Aunu group having only 4 distinct cyclic.

Keywords.Aunu Groups, Aunu Patterns. Cyclic, Permutation, Association scheme, cyclic design.

1 Introduction

Aunu permutation pattern is a possible pairing scheme involving pairs of numbers associated by some precedence relation see (Ibrahim and Audu, 2005). The governing conditions for the generation of these numbers are outlined in (Ibrahim, 2004; 2005). The study of these numbers was conducted based on a precedence parameter governed by some axioms which were centered on probability theory (doubt). The set of numbers was originally called a set of some special numbers with restricted pattern-avoiding words and permutations. Different researchers worked on this set of numbers and named it differently; for instance, Abor (2009) call it Aminu numbers, and later improved in Usman and Ibrahim (2011), on the formulation of a generating Function for Aunu Permutation Pattern s Thus: Where *mp* means taking integer modulo *p* in each of the cycles. The detailed applications of these numbers can be found in Ibrahim (2007). In Association scheme Constructions were undertaken in Magami and Ibrahim (2011) of Association scheme using the (123)-avoiding class of Aunu permutation patterns. Further, Magami et al (2012) constructed some association scheme using some (132)-avoiding class of Aunu permutation. In design theory, theoretic constructions were made in Usman and Ibrahim (2011) of a thin cyclic designs using the (123)-avoiding class of Aunu permutations Patterns. These researches employed the use of the special class of (132) and/or (123)-avoiding permutations pattern referred to as "Aunu pattern" to formulate a generalized generating function for the pattern (section 10). For $n \geq 5$we have the sequence: 2,3,5,6,8,9,11,14

2 Some Basic Definition

2.1 Definition

An association scheme with S associate classes on a finite set Ω is a partition of Ω×Ω in to sets C_o C_1, C_2, ……….$C_{.3}$ (called associate class) such that

(i) $C_0 = Diag(\Omega)$

(ii) C_i is symmetric for i=1,2,………..s

(iii) For all (i, j, k) in{0,1,…………..s} there is an integer p_{ij}^k such that, for all (α, β) in c_k

$$\left|\{\gamma \in \Omega : (\alpha, \gamma) \in C_i \text{ and } (\gamma, \beta) \in C_j\}\right| = P_{ij}^k$$

Note the superscript K in P_{ij}^k does not signify a power.

2.2 Definition

An incomplete-block design with treatment set Z, is a thin cyclic design if there is some subset ϕ of Z_t, such that the blocks are all the distinct translates of ϕ**:** the design is said to generated by ϕ. An incomplete-block design is a cyclic design its block can be partitioned into sets of blocks such that each set is thin cyclic design.

Let $\Theta = Z_t$, for a translate of $\Phi \subset \Theta$ is the set of the form $\Phi + \Theta = \{\phi + \theta : \phi \subset \Phi\}$ for some $\theta \, in \, \Theta$ is a translate of itself

3 Application of Aunu Group in Cyclic Designs

In this section we shall introduce an application of Aunu group using some class of cyclic designs.

Example 3.1. Consider the following generating set of Aunu group

$\{1.\ 2.\ 3.\ 4.\ 0\}\ \{1.\ 3.\ 0.\ 2.\ 4\}\ \{1.\ 4.\ 2.\ 0.\ 3\}\ \{1.\ 0.\ 4.\ 3.\ 2\}$

$\{2.\ 3.\ 4.\ 0.\ 1\}\ \{2.\ 4.\ 1.\ 3.\ 0\}\ \{2.\ 0.\ 3.\ 1.\ 4\}\ \{2.\ 1.\ 0.\ 4.\ 3\}$

$\{3.\ 4.\ 0.\ 1.\ 2\}\ \{3.\ 0.\ 2.\ 4.\ 1\}\ \{.3.\ 1.\ 4.\ 2.\ 0\}\ \{3.\ 2.\ 1.\ 0.\ 4\}$

$\{4.\ 0.\ 1.\ 2.\ 3\}\ \{4.\ 1.\ 3.\ 0.\ 2\}\ \{4.\ 2.\ 0.\ 3.\ 1\}\ \{4.\ 3.\ 2.\ 1.\ 0\}$

$\{0.\ 1.\ 2.\ 3.\ 4\}\ \{0.\ 2.\ 4.\ 1.\ 3\}\ \{0.\ 3.\ 1.\ 4.\ 2\}\ \{0.\ 4.\ 3.\ 2.\ 1\}$

Where 0 denote the number 5 and P = 5.

It follows that using the procedure of thin cyclic designs as (Example 3.1) none of the elements of the generating set can be generated from another. This shows that each of the four elements of the generating set has index 5. This procedure produces 20 distinct element from Aunu group having only 4 distinct cyclic.

Method of Construction

Proposition 3.1.the index of Aunu group generated using thin cyclic design for p>5 is given as

$$\binom{p}{2} \; 20.42.110....p \;\; (p-1) \qquad (2)$$

Proof. For p=5 we have

$$\binom{5}{2} = \frac{5!}{(5-2)!} = \frac{5 \times 4 \times 3!}{3!} = 5 \times 4 = 20$$

For p = 7 we have

$$\binom{7}{2} = \frac{7!}{(7-2)!} = \frac{7 \times 6 \times 5!}{5!} = 7 \times 6 = 42$$

For p = 11 we have

$$\binom{11}{2} = \frac{11!}{(11-2)!} = \frac{11 \times 10 \times 9}{9!} = 11 \times 10 = 110$$

In general for any p we obtain

$$\begin{pmatrix} p \\ 2 \end{pmatrix} = \frac{p!}{(p-2)!} = \frac{p(p-1)(p-2)!}{(p-2)!} = p(p-1)$$

The result follows

Theorem 3.1. using additive modulo p on Aunu permutation patterns and in accidence with the procedure expressed in the order of Aunu groups enlarges respectively from 5.7.11………..to 20.42.110…..

Proof:

It follows from this design that the summation is done using additive notation modulo p on all the cyclic of Aunu group.

From the proposition 2.1 the total sum of all cyclic in Aunu group is given by p (p-1) there for the total number of cyclic obtainable from the design corresponds with the total sum of cyclic in Aunu group that is p(p-1) so that. For p 5.7.11….we obtains 20.42.110……Respectively.

In this section a formulation of the procedure used for identifying the relevant theoretic properties our application is undertaken. Consider the following

The equation $\begin{pmatrix} p \\ 2 \end{pmatrix} = p(p-1)$ for p>2 as follows

2. 6. 20. 42. 110. 156. 272. 342. 506. 812. 930. 1332. 1640. 1806. 2162. 2756. 3422

2660. 4422. 4970. 52567. 6162. 6806. 7832. 9312. 10100. 10506. 11342. 11772.

12656. 16002. 17030. 18632. 19182. 22052. 22650. 24492. 26406. 2772. 39756

31862. 32580. 36290. 37056. 38612. 39402. 44310……..

As an illustrations we construct the concurrencies in thin cyclic using Aunu group for p=5

Application

Let $\Phi \{0.\ 1.\ 2.\ 3.\ 4\} \subset$ .then

$$\Phi \times \Phi \begin{Bmatrix} (0.0).(0.1).(0.2).(0.3).(0.4).(1.0).(1.1).(1.2).(1.3).(1.4). \\ (2.0).(2.1).(2.2).(2.3).(2.4).\ (3.0).(3.1).(3.2).(3.3).(3.4) \\ (4.0).(4.1).(4.2).(4.3).(4.4) \end{Bmatrix}..$$

Also the index of $\{1.\ 2.\ 3.\ 4\}$ is 5and t= 5.so the concurrences in the thin design generated by Φ are:

$$m_0\ (\Phi) = \{(0.0).(1.1).(2.2).(3.3).(4.4)\} = A(0.0)$$

$$m_0\ (\Phi)\ \{(0.1).(1.0).(1.2).(2.1).(2.3).(3.2).(3.4).(4.3)\} = A(0.1)$$

$$m_0\ (\Phi)\ \{(0.2).(2.0).(1.3).(3.1).(2.4).(4.2)\} = A(0.2)$$

$$m_0\ (\Phi)\ \{(0.3).\ (3.0).(1.4).(4.1)\} = A(0.3)$$

$$m_0\ (\Phi)\ \{(0.4).(4.0).\} = A(0.4)$$

Consider the following set

$\{1.\ 2.\ 3.\ 4.\ 0\}$ $\{1.\ 3.\ 0.\ 2.\ 4\}$ $\{1.\ 4.\ 2.\ 0.\ 3\}$ $\{1.\ 0.\ 4.\ 3.\ 2\}$

$\{1.\ 2.\ 3\}$ $\{1.\ 3.\ 0\}$ $\{1.\ 4.\ 2\}$ $\{1.\ 0.\ 4\}$

$\{2.\ 3.\ 4\}$ $\{2.\ 4.\ 1\}$ $\{2.\ 0.\ 3\}$ $\{2.\ 1.\ 0\}$

$\{3.\ 4.\ 0\}$ $\{3.\ 0.\ 2\}$ $\{3.\ 1.\ 4\}$ $\{3.\ 2.\ 1\}$

$\{4.\ 0.\ 1\}$ $\{4.\ 1.\ 3\}$ $\{4.\ 2.\ 0\}$ $\{4.\ 3.\ 2\}$

$\{0.\ 1.\ 2\}$ $\{0.\ 2.\ 4\}$ $\{0.\ 3.\ 1\}$ $\{0.\ 4.\ 2\}$

All can be represented as thin cyclic design of index 5.each

4Conclusion

The underline procedure have yielded promising result on application and theoretic relationship between the Aunu permutation as group theoretic element and the thin cyclic design as block of orbit with special transition properties.

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