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A New Approach to Solve Intuitionistic Fuzzy Linear Programming Problems with Symmetric Trapezoidal Intuitionistic Fuzzy Numbers

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Abstract

Parvathi & Malathi (Intuitionistic fuzzy simplex method, International Journal of Computer Applications, 48, 39-48, 2012) proposed an intuitionistic fuzzy simplex algorithm to solve Intuitionistic Fuzzy Linear Programming Problems (IFLPPs) with Symmetric Trapezoidal Intuitionistic Fuzzy Numbers (STIFNs) by using a special ranking function and used the linearity property to obtain the desired results. In this paper, it is proved that the linearity property, used by authors, is not satisfied for given ranking function. So, to overcome this drawback, a new method is proposed to solve the same type of intuitionistic fuzzy linear programming problems.
Keywords: Intuitionistic fuzzy linear programming problems, Symmetric trapezoidal intuitionistic fuzzy numbers, Ranking function.

1. Introduction

Linear programming is one of the most successively applied operation research techniques. Real world situations are represented by using any linear programming model which involves a lot of parameters, whose values are assigned by experts. However, both experts and decision maker frequently do not precisely know the value of those parameters. Therefore, it is useful to consider the knowledge of experts about the parameters as fuzzy data (Zadeh 1965).

Atanassov (1999) introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the concept of fuzzy set proposed by Zadeh (1965). In IFS, the degree of non-membership denoting the non-belongingness to a set is also associated with the degree of membership of belongingness to a set. Unlike the fuzzy set where the non-membership degree is taken as one minus the membership degree, in IFS, the membership and non-membership degrees are more or less independent and related only by that the sum of two degrees must not exceed one.

Nehi (2010) proposed the concept of Trapezoidal (triangular) intuitionistic fuzzy numbers and also proposed a method for ranking of intuitionistic fuzzy numbers based on the characteristic values of membership and non- membership functions of an intuitionistic fuzzy number (IFN).

Dubey & Mehra (2011) proposed a more general definition of triangular intuitionistic fuzzy numbers (TIFN) than defined in (Li 2010) and defined a ranking function based on value and ambiguity indexes.

Parvathi & Malathi (2012 a) introduced symmetric trapezoidal intuitionistic fuzzy numbers (STIFNs) and also proposed the arithmetic operations of STIFNs based on (α, β) cuts.

Nagoorgani & Ponnalagu (2012) defined the division operation for triangular intuitionistic fuzzy number

(TIFN) using (α,β) cut and also defined scoring and accuracy function to rank TIFN. Based on this approach, Nagoorgani & Ponnalagu (2012) obtained the solution of intuitionistic fuzzy linear programming problems.

Parvathi & Malathi (2012 b) proposed intuitionistic fuzzy simplex method using ranking function to solve intuitionistic fuzzy linear programming problems with symmetric trapezoidal intuitionistic fuzzy numbers.

Suresh *et al.* (2014) proposed the ranking of triangular intuitionistic fuzzy numbers by means of magnitude and solved intuitionistic fuzzy linear programming problems using this ranking.

In this paper, it is pointed out that ranking function defined by Parvathi & Malathi (2012 b) is not satisfying the linearity property and a new method is proposed to solve intuitionistic fuzzy linear programming problems with symmetric trapezoidal intuitionistic fuzzy numbers.

This paper is organized as follows: In Section 2, some basic definitions and arithmetic operations are given. In Section 3, ranking function proposed by Parvathi & Malathi (2012 b) is presented. The drawbacks of the existing method are pointed out in Section 4. In Section 5, a new method is proposed for solving intuitionistic fuzzy linear programming problems with symmetric trapezoidal intuitionistic fuzzy numbers. Illustrative example is given in Section 6, concluding remarks are given in Section 7.

2. Preliminaries

In this section, some basic definitions and arithmetic operations on STIFNs, used in the paper are presented (Parvathi & Malathi 2012 b).

Definition 1: An intuitionistic fuzzy set (IFS) $\tilde{A}$ in X is defined as an object of the form $\tilde{A}=\{<x,\mu_{\tilde{A}}(x),\nu_{\tilde{A}}(x)>: x\in X\}$ where the functions $\mu_{\tilde{A}}: X\to[0,1]$ and $\nu_{\tilde{A}}: X\to[0,1]$ define the degree of membership and the degree of non- membership of the element $x\in X$ respectively and for every $x\in X$ in $\tilde{A}$, $0\le\mu_{\tilde{A}}(x)+\nu_{\tilde{A}}(x)\le 1$ holds.

Definition 2: For every common fuzzy subset $\tilde{A}$ on X, intuitionistic fuzzy index of x in $\tilde{A}$ is defined as $\pi_{\tilde{A}}(x)=1-\mu_{\tilde{A}}(x)-\nu_{\tilde{A}}(x)$. It is also known as degree of hesitancy or degree of uncertainty of the element x in $\tilde{A}$. Therefore, for every $x\in X, 0\le\pi_{\tilde{A}}(x)\le 1$.

Definition 3: An IFS $\tilde{A}^I=\{<x,\mu_{\tilde{A}}(x),\nu_{\tilde{A}}(x)>: x\in X\}$ is called IF-normal if there exist at least two points $x_0,x_1\in X$ such that $\mu_{\tilde{A}}(x_0)=1,\nu_{\tilde{A}}(x_1)=1$.

Definition 4: An intuitionistic fuzzy number (IFN) $\tilde{A}^I$ is

a) an intuitionistic fuzzy subset of the real line.

b) normal, i.e., there is some $x_0\in R$ such that $\mu_{\tilde{A}^I}(x_0)=1,\nu_{\tilde{A}^I}(x_0)=0$.

c) convex for the membership function $\mu_{\tilde{A}^I}(x)$, i.e., $\mu_{\tilde{A}^I}(\lambda x_1+(1-\lambda)x_2)\ge\min\left(\mu_{\tilde{A}^I}(x_1),\mu_{\tilde{A}^I}(x_2)\right)$ for every $x_1,x_2\in R,\lambda\in[0,1]$.

d) concave for the non-membership function $\nu_{\tilde{A}^I}(x)$, i.e., $\nu_{\tilde{A}^I}(\lambda x_1+(1-\lambda)x_2)\le \max\left(\nu_{\tilde{A}^I}(x_1),\nu_{\tilde{A}^I}(x_2)\right)$ for every $x_1,x_2\in R,\lambda\in[0,1]$.

Definition 5: A trapezoidal intuitionistic fuzzy number (TIFN) $\tilde{A}^I$ is an IFS in R with membership function and non-membership function as follows:

$$\mu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{x-(a_1-\alpha)}{\alpha} & \text{for } x\in[a_1-\alpha,a_1]\\ 1 & \text{for } x\in[a_1,a_2]\\ \dfrac{a_2+\beta-x}{\beta} & \text{for } x\in[a_2,a_2+\beta]\\ 0 & \text{otherwise}\end{cases}$$

and

$$\nu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{a_1-x}{\alpha'} & \text{for } x\in[a_1-\alpha',a_1]\\ 0 & \text{for } x\in[a_1,a_2]\\ \dfrac{x-a_2}{\beta'} & \text{for } x\in[a_2,a_2+\beta']\\ 1 & \text{otherwise}\end{cases}$$

A TIFN is denoted by $\tilde{A}^I_{TIFN}=[a_1,a_2,\alpha,\beta;a_1,a_2,\alpha',\beta']$.

Definition 6: A TIFN is said to be Symmetric Trapezoidal Intuitionistic Fuzzy Number (STIFN) if $\alpha=\beta$ (say h) and $\alpha'=\beta'$ (say h'). Hence, the definition of STIFN is as follows:

A TIFS $\tilde{A}^I$ in R is said to be a STIFN if there exist real numbers a_1,a_2,h,h' where $a_1\le a_2, h\le h'$ and $h,h'>0$ such that the membership and non-membership functions are as follows:

$$\mu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{x-(a_1-h)}{h} & \text{for } x\in[a_1-h,a_1]\\ 1 & \text{for } x\in[a_1,a_2]\\ \dfrac{a_2+h-x}{h} & \text{for } x\in[a_2,a_2+h]\\ 0 & \text{otherwise}\end{cases}$$

and

$$\nu_{\tilde{A}^I}(x)=\begin{cases}\dfrac{a_1-x}{h'} & \text{for } x\in[a_1-h',a_1]\\ 0 & \text{for } x\in[a_1,a_2]\\ \dfrac{x-a_2}{h'} & \text{for } x\in[a_2,a_2+h']\\ 1 & \text{otherwise}\end{cases}$$

A STIFN is denoted by $\tilde{A}^I_{STIFN}=[a_1,a_2,h,h;a_1,a_2,h',h']$.

Arithmetic Operations on STIFNs (Parvathi & Malathi 2012 a)

If $\tilde{A}^I = [a_1, a_2, h, h; a_1, a_2, h', h']$ and $\tilde{B}^I = [b_1, b_2, k, k; b_1, b_2, k', k']$ are two STIFNs, then

$$\tilde{A}^I + \tilde{B}^I = [a_1 + b_1, a_2 + b_2, h+k, h+k; a_1 + b_1, a_2 + b_2, h' + k', h' + k']$$

$$\tilde{A}^I - \tilde{B}^I = [a_1 - b_2, a_2 - b_1, h+k, h+k; a_1 - b_2, a_2 - b_1, h' + k', h' + k']$$

If $k \in R$ then

$$k\tilde{A}^I = \begin{cases} [ka_1, ka_2, kh, kh; ka_1, ka_2, kh', kh'] & \text{if k} \geq 0 \\ [ka_2, ka_1, -kh, -kh; ka_2, ka_1, -kh', -kh'] & \text{if k<0} \end{cases}$$

3. Ranking Function (Parvathi & Malathi 2012 b)

A ranking function is used to order IFN and it is a mapping from the set of STIFNs to the set of real numbers *R*. In fact, an efficient approach for ordering the elements of *F(R)*, the set of STIFNs on *R*, is to define a ranking function $\Re : F(R) \to R$ which maps each STIFN into the real line. Orders on *F(R)* are defined as follows:

$\tilde{A}^I \succeq \tilde{B}^I$ if and only if $\Re(\tilde{A}^I) \geq \Re(\tilde{B}^I)$

$\tilde{A}^I \succ \tilde{B}^I$ if and only if $\Re(\tilde{A}^I) > \Re(\tilde{B}^I)$

$\tilde{A}^I \preceq \tilde{B}^I$ if and only if $\Re(\tilde{A}^I) \leq \Re(\tilde{B}^I)$

$\tilde{A}^I \approx \tilde{B}^I$ if and only if $\Re(\tilde{A}^I) = \Re(\tilde{B}^I)$

where $\tilde{A}^I = [a_1, a_2, h, h; a_1, a_2, h', h']$, $\tilde{B}^I = [b_1, b_2, k, k; b_1, b_2, k', k'] \in F(R)$ and

$$\Re(\tilde{A}^I) = a_1 + a_2 + \frac{1}{2}(h' - h), \Re(\tilde{B}^I) = b_1 + b_2 + \frac{1}{2}(k' - k).$$

4. Drawbacks in the existing method

Parvathi & Malathi (2012 b) proposed an intuitionistic fuzzy simplex method to solve the intuitionistic fuzzy linear programming problem with symmetric trapezoidal intuitionistic fuzzy numbers (P_1).

$$\text{Maximize } \sum_{j=1}^{n} c_j \tilde{x}_j^{\,I}$$

$$\text{Subject to } \sum_{j=1}^{n} a_{ij} \tilde{x}_j^{\,I} \preceq \tilde{b}_i^{\,I}; \quad i = 1, 2, ..., m, \qquad (P_1)$$

$$\tilde{x}_j^{\,I} \succeq 0. \qquad j = 1, 2, ..., n.$$

Parvathi & Malathi (2012 b) used the linearity property $\Re(\tilde{A}^I - \tilde{B}^I) = \Re(\tilde{A}^I) - \Re(\tilde{B}^I)$ to solve IFLPP with STIFNs. However, this linearity property is not satisfied for the given ranking function. The proof of this is given as follows:

Proof: Let $\tilde{A}^I = [a_1, a_2, h, h; a_1, a_2, h', h']$ and $\tilde{B}^I = [b_1, b_2, k, k; b_1, b_2, k', k']$ be two STIFNs.

then $\Re\left(\tilde{A}^I\right)=a_1+a_2+\frac{1}{2}(h'-h)$ and $\Re\left(\tilde{B}^I\right)=b_1+b_2+\frac{1}{2}(k'-k)$.

$$\Re\left(\tilde{A}^I\right)-\Re\left(\tilde{B}^I\right)=\left[a_1+a_2+\frac{1}{2}(h'-h)\right]-\left[b_1+b_2+\frac{1}{2}(k'-k)\right]=a_1+a_2-b_1-b_2+\frac{1}{2}\left(h'-h-k'+k\right)$$

But $\Re\left(\tilde{A}^I-\tilde{B}^I\right)=\Re\left[\left(a_1,a_2,h,h;a_1,a_2,h',h'\right)-\left(b_1,b_2,k,k;b_1,b_2,k',k'\right)\right]$

$$=\Re\left[\left(a_1-b_2,a_2-b_1,h+k,h+k;a_1-b_2,a_2-b_1,h'+k',h'+k'\right)\right]$$

$$=a_1-b_2+a_2-b_1+\frac{1}{2}\left(h'+k'-h-k\right)=a_1+a_2-b_1-b_2+\frac{1}{2}\left(h'-h-k+k'\right)\neq\Re\left(\tilde{A}^I\right)-\Re\left(\tilde{B}^I\right)$$

Example:

Let $\tilde{A}^I=[5,7,2,2;5,7,4,4]$ and $\tilde{B}^I=[3,5,4,4;3,5,6,6]$

$$\Re\left(\tilde{A}^I\right)=5+7+\frac{1}{2}(4-2)=12+1=13$$

$\Re\left(\tilde{B}^I\right)=3+5+\frac{1}{2}(6-4)=8+1=9$, $\Re\left(\tilde{A}^I\right)-\Re\left(\tilde{B}^I\right)=13-9=4$

$$\tilde{A}^I-\tilde{B}^I=[5-5,7-3,2+4,2+4;5-5,7-3,4+6,4+6]=[0,4,6,6;0,4,10,10]$$

$\Re\left(\tilde{A}^I-\tilde{B}^I\right)=0+4+\frac{1}{2}(10-6)=4+2=6$, Hence, $\Re\left(\tilde{A}^I\right)-\Re\left(\tilde{B}^I\right)\neq\Re\left(\tilde{A}^I-\tilde{B}^I\right)$

5. Proposed method to find the solution of IFLPPs with STIFNs

In this section, a new method is proposed to find the intuitionistic fuzzy optimal solution of IFLPPs with STIFNs. The steps of the proposed method are as follows:

Step 1: Substituting $\tilde{x}_j^I=\left[\left(x_j,y_j,h_j,h_j\right);\left(x_j,y_j,h_j',h_j'\right)\right]$ and $\tilde{b}_i^I=\left[\left(b_i,g_i,k_i,k_i\right);\left(b_i,g_i,k_i',k_i'\right)\right]$ in problem (P_1). It can be transformed into problem (P_2):

$$\text{Maximize } \sum_{j=1}^{n} c_j\left[\left(x_j,y_j,h_j,h_j\right);\left(x_j,y_j,h_j',h_j'\right)\right]$$

$$\text{Subject to } \sum_{j=1}^{n} a_{ij}\left[\left(x_j,y_j,h_j,h_j\right);\left(x_j,y_j,h_j',h_j'\right)\right]\preceq\left[\left(b_i,g_i,k_i,k_i\right);\left(b_i,g_i,k_i',k_i'\right)\right] \quad (P_2)$$

$$\left[\left(x_j,y_j,h_j,h_j\right);\left(x_j,y_j,h_j',h_j'\right)\right]\geq 0.$$

Step 2: Assuming the values of $c_j\left[\left(x_j,y_j,h_j,h_j\right);\left(x_j,y_j,h_j',h_j'\right)\right]=\left[\left(p_j,q_j,r_j,r_j\right);\left(p_j,q_j,r_j',r_j'\right)\right]$ and $a_{ij}\left[\left(x_j,y_j,h_j,h_j\right);\left(x_j,y_j,h_j',h_j'\right)\right]=\left[\left(d_j,e_j,f_j,f_j\right);\left(d_j,e_j,f_j',f_j'\right)\right]$, the problem ($P_2$) can be transformed into problem (P_3):

$$\text{Maximize } \left(\sum_{j=1}^{n}\left[\left(p_j,q_j,r_j,r_j\right);\left(p_j,q_j,r_j^{'},r_j^{'}\right)\right]\right)$$

$$\text{Subject to } \sum_{j=1}^{n}\left[\left(d_j,e_j,f_j,f_j\right);\left(d_j,e_j,f_j^{'},f_j^{'}\right)\right]\preceq\left[\left(b_i,g_i,k_i,k_i\right);\left(b_i,g_i,k_i^{'},k_i^{'}\right)\right] \quad (P_3)$$

$$\left[\left(x_j,y_j,h_j,h_j\right);\left(x_j,y_j,h_j^{'},h_j^{'}\right)\right]\succeq 0.$$

Step 3: The problem (P_3) can be transformed into problem (P_4):

$$\text{Maximize } \left[\left(\sum_{j=1}^{n}p_j,\sum_{j=1}^{n}q_j,\sum_{j=1}^{n}r_j,\sum_{j=1}^{n}r_j\right);\left(\sum_{j=1}^{n}p_j,\sum_{j=1}^{n}q_j,\sum_{j=1}^{n}r_j^{'},\sum_{j=1}^{n}r_j^{'}\right)\right] \quad (P_4)$$

$$\text{Subject to } \left[\left(\sum_{j=1}^{n}d_j,\sum_{j=1}^{n}e_j,\sum_{j=1}^{n}f_j,\sum_{j=1}^{n}f_j\right);\left(\sum_{j=1}^{n}d_j,\sum_{j=1}^{n}e_j,\sum_{j=1}^{n}f_j^{'},\sum_{j=1}^{n}f_j^{'}\right)\right] \preceq \left[\left(b_i,g_i,k_i,k_i\right);\left(b_i,g_i,k_i^{'},k_i^{'}\right)\right]$$

$$\left[\left(x_j,y_j,h_j,h_j\right);\left(x_j,y_j,h_j^{'},h_j^{'}\right)\right] \succeq 0.$$

Step 4: Applying the ranking function, defined in Section 3, the problem (P_4) can be transformed into problem (P_5):

$$\text{Maximize } \Re\left[\left(\sum_{j=1}^{n}p_j,\sum_{j=1}^{n}q_j,\sum_{j=1}^{n}r_j,\sum_{j=1}^{n}r_j\right);\left(\sum_{j=1}^{n}p_j,\sum_{j=1}^{n}q_j,\sum_{j=1}^{n}r_j^{'},\sum_{j=1}^{n}r_j^{'}\right)\right] \quad (P_5)$$

$$\text{Subject to } \Re\left[\left(\sum_{j=1}^{n}d_j,\sum_{j=1}^{n}e_j,\sum_{j=1}^{n}f_j,\sum_{j=1}^{n}f_j\right);\left(\sum_{j=1}^{n}d_j,\sum_{j=1}^{n}e_j,\sum_{j=1}^{n}f_j^{'},\sum_{j=1}^{n}f_j^{'}\right)\right] \le \Re\left[\left(b_i,g_i,k_i,k_i\right);\left(b_i,g_i,k_i^{'},k_i^{'}\right)\right]$$

$$\Re\left[\left(x_j,y_j,h_j,h_j\right);\left(x_j,y_j,h_j^{'},h_j^{'}\right)\right] \ge 0.$$

Step 5: Applying the ranking formula, the problem (P_5) can be transformed into problem (P_6):

$$\text{Maximize } \left(\sum_{j=1}^{n}p_j+\sum_{j=1}^{n}q_j+\frac{1}{2}\left(\sum_{j=1}^{n}r_j^{'}-\sum_{j=1}^{n}r_j\right)\right)$$

$$\text{Subject to } \left(\sum_{j=1}^{n}d_j+\sum_{j=1}^{n}e_j+\frac{1}{2}\left(\sum_{j=1}^{n}f_j^{'}-\sum_{j=1}^{n}f_j\right)\right)\le\left(b_i+g_i+\frac{1}{2}(k_i^{'}-k_i)\right) \quad (P_6)$$

$$x_j+y_j+\frac{1}{2}\left(h_j^{'}-h_j\right)\ge 0$$

$$x_j\le y_j, h_j\le h_j^{'}, x_j, y_j, h_j, h_j^{'} \ge 0.$$

Step 6: Solve the problem (P_6) by using any existing method or by software TORA to find the values of $x_j, y_j, h_j, h_j^{'}$ and put these values in $\tilde{x}_j^I=\left(x_j,y_j,h_j,h_j^{'}\right)$ to find the intuitionistic fuzzy optimal solution.

Step 7: Find the intuitionistic fuzzy optimal value by putting the values of $\tilde{x}_j^I$ in $\sum_{j=1}^{n}c_j\tilde{x}_j^I$.

6. Illustrative Example

In this section, to illustrate the proposed method, a numerical example is solved.

Example 1[Parvathi & Malathi 2012 b, Section 5.5, pp. 45] Reddy Mikks produces both interior and exterior paints from two raw materials, M_1 and M_2. The following table provides the required data of the problem:

	Tones of raw material of	
	Exterior paints	Interior paints
Raw material M_1	6	4
Raw material M_2	1	2
Profit per ton	5	4

The maximum daily availability of raw materials can vary from day to day due to breakdown of machines, delay supply, etc. At the same time, the maximum daily availability is somewhat close to 24 tons for M_1 and 6 tons for M_2. Also, a market survey indicates that the daily demand of exterior and interior paints can also vary due to variations in the requirement of consumers. But it is expected that the daily demand for interior paint cannot exceed that for exterior paint by more than 4 tons approximately. Also, the maximum daily demand for interior paint is somewhat close to 2 tons.

Reddy Mikks wants to determine the optimum (best) product mix of interior and exterior paints that maximizes the total daily profit. For the Reddy Mikks problem, it is necessary to determine the daily amounts to be produced of exterior and interior paints.

Since the maximum daily availability of raw materials $M_1$and M_2 are uncertain, the daily amounts to be produced of exterior and interior paints will also be uncertain. So, the problem can be modelled as an intuitionistic fuzzy linear programming problem by using symmetric trapezoidal intuitionistic fuzzy numbers for each uncertain value.

Maximum daily availability for raw material M_1 which is close to 24 can be modelled as [23, 25, 1, 1; 23, 25, 3, 3]. Similarly, the other parameters are also modelled as STIFNs taking into account the nature of the problem and other requirements.

The given IFLPP can be formulated as:

$$\begin{aligned}
&\text{Maximize } \left(5\tilde{x}_1^I + 4\tilde{x}_2^I\right)\\
&\text{Subject to } 6\tilde{x}_1^I + 4\tilde{x}_2^I \preceq [23, 25, 1, 1; 23, 25, 3, 3]\\
&\tilde{x}_1^I + 2\tilde{x}_2^I \preceq [5, 7, 2, 2; 5, 7, 4, 4]\\
&-\tilde{x}_1^I + \tilde{x}_2^I \preceq [3, 5, 4, 4; 3, 5, 6, 6]\\
&\tilde{x}_2^I \preceq [1, 3, 2, 2; 1, 3, 4, 4]\\
&\tilde{x}_1^I, \tilde{x}_2^I \succeq 0.
\end{aligned} \qquad (P_7)$$

Solution: The intuitionistic fuzzy optimal solution of the problem (P_7) can be obtained as follows:

Step 1: Substituting the values of $\tilde{x}_1^I = \left[\left(x_1, y_1, h_1, h_1\right); \left(x_1, y_1, h_1^{'}, h_1^{'}\right)\right]$ and $\tilde{x}_2^I = \left[\left(x_2, y_2, h_2, h_2\right); \left(x_2, y_2, h_2^{'}, h_2^{'}\right)\right]$ in the problem (P_7), it can be transformed into problem (P_8):

$$\begin{aligned}
&\text{Maximize}\left(5\left[\left(x_1, y_1, h_1, h_1\right); \left(x_1, y_1, h_1^{'}, h_1^{'}\right)\right] + 4\left[\left(x_2, y_2, h_2, h_2\right); \left(x_2, y_2, h_2^{'}, h_2^{'}\right)\right]\right)\\
&\text{Subject to } 6\left[\left(x_1, y_1, h_1, h_1\right); \left(x_1, y_1, h_1^{'}, h_1^{'}\right)\right] + 4\left[\left(x_2, y_2, h_2, h_2\right); \left(x_2, y_2, h_2^{'}, h_2^{'}\right)\right] \preceq [23, 25, 1, 1; 23, 25, 3, 3]\\
&\left[\left(x_1, y_1, h_1, h_1\right); \left(x_1, y_1, h_1^{'}, h_1^{'}\right)\right] + 2\left[\left(x_2, y_2, h_2, h_2\right); \left(x_2, y_2, h_2^{'}, h_2^{'}\right)\right] \preceq [5, 7, 2, 2; 5, 7, 4, 4]\\
&-\left[\left(x_1, y_1, h_1, h_1\right); \left(x_1, y_1, h_1^{'}, h_1^{'}\right)\right] + \left[\left(x_2, y_2, h_2, h_2\right); \left(x_2, y_2, h_2^{'}, h_2^{'}\right)\right] \preceq [3, 5, 4, 4; 3, 5, 6, 6]\\
&\left[\left(x_2, y_2, h_2, h_2\right); \left(x_2, y_2, h_2^{'}, h_2^{'}\right)\right] \preceq [1, 3, 2, 2; 1, 3, 4, 4]\\
&\left[\left(x_1, y_1, h_1, h_1\right); \left(x_1, y_1, h_1^{'}, h_1^{'}\right)\right] \succeq 0, \left[\left(x_2, y_2, h_2, h_2\right); \left(x_2, y_2, h_2^{'}, h_2^{'}\right)\right] \succeq 0.
\end{aligned} \qquad (P_8)$$

Step 2: Using Step 2 of proposed method, the problem (P_8) can be transformed into problem (P_9):

$$\begin{aligned}
&\text{Maximize } \left(\left[\left(5x_1,5y_1,5h_1,5h_1\right);\left(5x_1,5y_1,5h_1^{'},5h_1^{'}\right)\right]+\left[\left(4x_2,4y_2,4h_2,4h_2\right);\left(4x_2,4y_2,4h_2^{'},4h_2^{'}\right)\right]\right)\\
&\text{Subject to}\\
&\left[\left(6x_1,6y_1,6h_1,6h_1\right);\left(6x_1,6y_1,6h_1^{'},6h_1^{'}\right)\right]+\left[\left(4x_2,4y_2,4h_2,4h_2\right);\left(4x_2,4y_2,4h_2^{'},4h_2^{'}\right)\right]\preceq\left[23,25,1,1;23,25,3,3\right]\\
&\left[\left(x_1,y_1,h_1,h_1\right);\left(x_1,y_1,h_1^{'},h_1^{'}\right)\right]+\left[\left(2x_2,2y_2,2h_2,2h_2\right);\left(2x_2,2y_2,2h_2^{'},2h_2^{'}\right)\right]\preceq\left[5,7,2,2;5,7,4,4\right]\\
&\left[\left(-y_1,-x_1,h_1,h_1\right);\left(-y_1,-x_1,h_1^{'},h_1^{'}\right)\right]+\left[\left(x_2,y_2,h_2,h_2\right);\left(x_2,y_2,h_2^{'},h_2^{'}\right)\right]\preceq\left[3,5,4,4;3,5,6,6\right] \qquad (P_9)\\
&\left[\left(x_2,y_2,h_2,h_2\right);\left(x_2,y_2,h_2^{'},h_2^{'}\right)\right]\preceq\left[1,3,2,2;1,3,4,4\right]\\
&\left[\left(x_1,y_1,h_1,h_1\right);\left(x_1,y_1,h_1^{'},h_1^{'}\right)\right]\succeq 0,\\
&\left[\left(x_2,y_2,h_2,h_2\right);\left(x_2,y_2,h_2^{'},h_2^{'}\right)\right]\succeq 0.
\end{aligned}$$

Step 3: Using Step 3 of the proposed method, the problem (P_9) can be transformed into problem (P_{10}):

$$\begin{aligned}
&\text{Maximize } \left[5x_1+4x_2,5y_1+4y_2,5h_1+4h_2,5h_1+4h_2;5x_1+4x_2,5y_1+4y_2,5h_1^{'}+4h_2^{'},5h_1^{'}+4h_2^{'}\right]\\
&\text{Subject to } \left[6x_1+4x_2,6y_1+4y_2,6h_1+4h_2,6h_1+4h_2;6x_1+4x_2,6y_1+4y_2,6h_1^{'}+4h_2^{'},6h_1^{'}+4h_2^{'}\right]\preceq\left[23,25,1,1;23,25,3,3\right]\\
&\left[x_1+2x_2,y_1+2y_2,h_1+2h_2,h_1+2h_2;x_1+2x_2,y_1+2y_2,h_1^{'}+2h_2^{'},h_1^{'}+2h_2^{'}\right]\preceq\left[5,7,2,2;5,7,4,4\right]\\
&\left[-y_1+x_2,-x_1+y_2,h_1+h_2,h_1+h_2;-y_1+x_2,-x_1+y_2,h_1^{'}+h_2^{'},h_1^{'}+h_2^{'}\right]\preceq\left[3,5,4,4;3,5,6,6\right] \qquad (P_{10})\\
&\left[\left(x_2,y_2,h_2,h_2\right);\left(x_2,y_2,h_2^{'},h_2^{'}\right)\right]\preceq\left[1,3,2,2;1,3,4,4\right]\\
&\left[\left(x_1,y_1,h_1,h_1\right);\left(x_1,y_1,h_1^{'},h_1^{'}\right)\right]\succeq 0,\\
&\left[\left(x_2,y_2,h_2,h_2\right);\left(x_2,y_2,h_2^{'},h_2^{'}\right)\right]\succeq 0.
\end{aligned}$$

Step 4: Using Step 4 of the proposed method, the problem (P_{10}) can be transformed into problem (P_{11}):

$$\begin{aligned}
&\text{Maximize } \Re\left(\left[5x_1+4x_2,5y_1+4y_2,5h_1+4h_2,5h_1+4h_2;5x_1+4x_2,5y_1+4y_2,5h_1^{'}+4h_2^{'},5h_1^{'}+4h_2^{'}\right]\right)\\
&\text{Subject to } \Re\left(\left[6x_1+4x_2,6y_1+4y_2,6h_1+4h_2,6h_1+4h_2;6x_1+4x_2,6y_1+4y_2,6h_1^{'}+4h_2^{'},6h_1^{'}+4h_2^{'}\right]\right)\le\Re\left(\left[23,25,1,1;23,25,3,3\right]\right)\\
&\Re\left(\left[x_1+2x_2,y_1+2y_2,h_1+2h_2,h_1+2h_2;x_1+2x_2,y_1+2y_2,h_1^{'}+2h_2^{'},h_1^{'}+2h_2^{'}\right]\right)\le\Re\left(\left[5,7,2,2;5,7,4,4\right]\right) \qquad (P_{11})\\
&\Re\left(\left[-y_1+x_2,-x_1+y_2,h_1+h_2,h_1+h_2;-y_1+x_2,-x_1+y_2,h_1^{'}+h_2^{'},h_1^{'}+h_2^{'}\right]\right)\le\Re\left(\left[3,5,4,4;3,5,6,6\right]\right)\\
&\Re\left(\left[\left(x_2,y_2,h_2,h_2\right);\left(x_2,y_2,h_2^{'},h_2^{'}\right)\right]\right)\le\Re\left(\left[1,3,2,2;1,3,4,4\right]\right)\\
&\Re\left[\left(x_1,y_1,h_1,h_1\right);\left(x_1,y_1,h_1^{'},h_1^{'}\right)\right]\ge 0,\\
&\Re\left[\left(x_2,y_2,h_2,h_2\right);\left(x_2,y_2,h_2^{'},h_2^{'}\right)\right]\ge 0.
\end{aligned}$$

Step 5: Using Step 5 of the proposed method, the problem (P_{11}) can be transformed into problem (P_{12}):

$$\begin{aligned}
&\text{Maximize } \left(5x_1+4x_2+5y_1+4y_2+\frac{1}{2}\left[5h_1^{'}+4h_2^{'}-5h_1-4h_2\right]\right)\\
&\text{Subject to } 6x_1+4x_2+6y_1+4y_2+\frac{1}{2}\left[6h_1^{'}+4h_2^{'}-6h_1-4h_2\right]\le 49\\
&x_1+2x_2+y_1+2y_2+\frac{1}{2}\left[h_1^{'}+2h_2^{'}-h_1-2h_2\right]\le 13\\
&-y_1+x_2-x_1+y_2+\frac{1}{2}\left[h_1^{'}+h_2^{'}-h_1-h_2\right]\le 9 \qquad (P_{12})\\
&x_2+y_2+\frac{1}{2}\left[h_2^{'}-h_2\right]\le 5
\end{aligned}$$

$$x_1+y_1+\frac{1}{2}\left(h_1^{'}-h_1\right)\geq 0, x_2+y_2+\frac{1}{2}\left(h_2^{'}-h_2\right)\geq 0,$$
$$x_1\leq y_1, x_2\leq y_2, h_1\leq h_1^{'}, h_2\leq h_2^{'}; x_1, x_2, y_1, y_2, h_1, h_1^{'}, h_2, h_2^{'}\geq 0.$$

Step 6: Solving the problem (P_{12}) by the software TORA, the obtained values of $x_1, y_1, h_1, h_1^{'}$ are $0,0,\frac{3}{4},\frac{49}{4}$ and $x_2, y_2, h_2, h_2^{'}$ are $0,0,0,\frac{29}{4}$ respectively. Hence, the intuitionistic fuzzy optimal solution is $\tilde{x}_1^I=\left[\left(0,0,\frac{3}{4},\frac{3}{4}\right);\left(0,0,\frac{49}{4},\frac{49}{4}\right)\right]$ and $\tilde{x}_2^I=\left[\left(0,0,0,0\right);\left(0,0,\frac{29}{4},\frac{29}{4}\right)\right]$.

Step 7: Putting the values of $\tilde{x}_1^I$ and $\tilde{x}_2^I$ in $5\tilde{x}_1^I+4\tilde{x}_2^I$, the intuitionistic fuzzy optimal value is $\left[\left(0,0,\frac{15}{4},\frac{15}{4}\right);\left(0,0,\frac{361}{4},\frac{361}{4}\right)\right]$.

7. Conclusion

In this paper, drawback of the existing method is pointed out and to resolve this drawback, a new method is proposed to solve intuitionistic fuzzy linear programming problems with symmetric trapezoidal intuitionistic fuzzy numbers.

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