Mathematical Modeling of River Blindness Disease with Demography Using Euler Method

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Abstract

The study focused on the mathematical modelling of river blindness (Onchocerciasis) infectious disease using SIR model with demography and Euler method as the analytical procedure in Excel programming. Onchocerciasis is discussed, assumptions are made and basic deterministic features are studied. The interaction between the susceptibility and infection decline drastically to 0.01%, in the 90 days simulated diseases about 52% of the population are susceptible to the disease and 50% on average infection rate is recorded within the 14 dyas before the infection starts decreasing from 55% and dies out. The recovery rate is 0.37(37%) and seemingly constant.

Keywords: Euler method, Simulation, Infectious disease, SIR, Demography

1. Introduction

Mathematical modelling is a mathematical process of investigating the concept and theory of infectious disease transmission to forecast the further occurrence in order to device possible control of the epidemic. Mathematical model of epidemic was first developed by Daniel Bernoulli on the monitoring of smallpox outbreak and confirmed that vaccination against the disease can help improve life expectancy. The modern and better improve mathematical model of epidemic is due to G. McKendrick and W. O. Kermack (1927) who designed a simple deterministic model to study the behaviour of outbreak of many infectious disease and made records of them over time.

1.1 Type of mathematical epidemic models

There are basically two types of mathematical epidemic models. These are stochastic model and Deterministic model. The stochastic model deals with the random study of the epidemic process using probability techniques to estimate the epidemic outcomes and the measures the probability of extinction time and the size based on mean, variance and distribution. Deterministic model of epidemic focuses on the rate of transmission base on stages that is derived mathematically using differential function with respect to time in limiting large population. There are time dependent and non-time dependent known as autonomous. This transition rate is defined as compartments. Hence, deterministic mathematical model of epidemic is described as deterministic compartmental models. It uses limiting population, time of occurrence within the subgroup to enable the approximation of the deterministic dynamics of the epidemics.

1.2 The Mathematical classical Model Reviews of deterministic model

In determining the Epidemic Models such as the deterministic compartmental models in epidemiological study, what should come to mind of the modeller is the nature of the epidemic based on demography or without demography. Demography is very important in the study of the transition rate of infectious diseases.

Demography deals with population dynamics of the susceptible or infected population in a closed system with characteristics such as age, sex, location, birth, death, race or social and economic status of the population understudy. Deterministic compartmental model without demography may be SIR, SIS, SIRS models and with demography SIR model sometimes may involve model with E compartment (Keeling, Matt, 2008).

The variables are described as S-Susceptible, I-Infection, E-Exposed period and R-Recovery. Deterministic compartmental models of SIR is due to Kermack and McKendrick (1927) where S–the number of susceptible, I–the number of infective, R–the number of recoveries β –contact rate γ –recovery rate. In situation of model concerning demography the population parameter, μ , is included during the model compartment designed (Anderson, 1992; Hackborn, 2008, Iannelli, 2005).

1.3 Modeling River Blindness Disease

River blindness is popularly known as Onchocerciasis. It is an infection caused by the parasite *Onchocerca volvulus* (worm) spread by the bite of an infected blackfly. River Blindness transmission is most common with intense outbreak in remote African, Southern American and in the Middle East especially in Yemen rural agricultural villages usually located near rapidly flowing streams. Adventure travellers, missionaries, and Peace Corps volunteers who are at risk of blackfly bites in endemic areas are **susceptible.** Persons with heavy river blindness infections usually have one or more of the three conditions: dermatitis, eye lesions, and/or subcutaneous nodules. Superficial skin biopsies will identify the parasite microscopically. The disease is mostly treated with oral medication (Ivermectin) and the use of insecticides such as DEET, and Wearing long sleeve shirts and pants.

2. Epidemiology

The World Health Organization's (WHO) on onchocerciasis estimates the global prevalence is 17.7 million, of whom about 270,000 are blind and another 500,000 have a visual impairment. About 99% of infected persons are in Africa and 11% in Nigeria and it more prevalence in Mubi Village Gombe State where about 89% of the entire village suffers one form of blindness or the other. This hyper-endemic earn the community the village of the blind. The occurrence is also found in Yemen and some countries in the southern Americas. Onchocerciasis is locally transmitted in thirty countries of Africa, 13 foci in the Americas (Mexico, Guatemala, Ecuador, Colombia, Venezuela, Brazil) and in Yemen. These countries are classified as Endemic, Meso and Hyper endermic by the OCP countries categories (Available at http://www.wellnessproposals.com/health-care/handouts/parasitic-zoonotic-diseases/onchocerciasis-factsheet.pdf)

2.2 Transmission

Onchocerciasis infection is transmitted when a blackfly bites a person who has onchocerciasis, microscopic worms (called microfilariae) in the infected person's skin and can then be transmitted among person with the population. The microfilariae develop over 2 weeks to a stage where they are infectious to a human that is the contact transmission rate. An infectious person will typically transmit the disease to another person with infection rate of 2.12 on average (available at http://www.wellnessproposals.com/health-care/handouts/parasitic-zoonotic-diseases/onchocerciasis-factsheet.pdf).

3. S-I-R Model of River Blindness Disease with demography

This section focuses on the assumptions and black bus model of the disease. It also shows the model differential behaviour, the steady state calculation, disease free equilibrium in terms of R_0 .

3.1. Assumptions

The SIR Model with demography is used in case study of River blindness disease to compute the amount of susceptible, infected, recovered people in a dynamic population. This model considered as best to be used based on the following assumptions:

1) The population is dynamic
2) People are born into the system to be susceptible and they die as a result of non-recovery or natural death as the case may be.
3) The people coming into the system are susceptible and the only process to leave the susceptible compartment is to become infected from the disease. They leave the infected group to recovery stage. Once they are recovered, they become immune.
4) Only birth and death are consider with no consideration to age, sex, social status, and race affecting the probability of being infected.
5) There is no inherited immunity.
6) The population is mixed homogenous base on level of interaction.
7) There are no changes in the population of black fly bite.

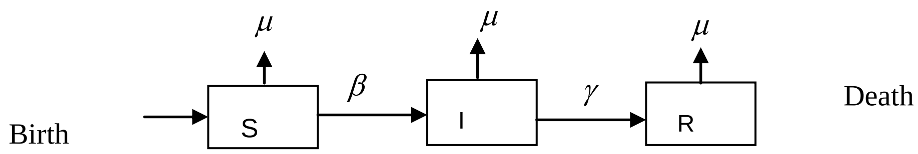
3.2 The SIR Model with demography formulation

The formulation of model requires the description of the parameters involve:

Let $S(t)$ be the number of susceptible individuals at time t is, $I(t)$ be the number of infected individuals at time t, $R(t)$ is the number of recovered individuals at time t and N is the total population size $S(t)+I(t)+R(t)=N$. Therefore, the assumptions lead to compartment black box model and a set of differential equations.

3.2.1 The compartment black box model

The SIR Model with demography is expressed as:



$$\frac{dS}{dt}=\mu-\beta SI-\mu I \qquad 1$$

$$\frac{dI}{dt}=\beta SI-\gamma I-\mu I \qquad 2$$

$$\frac{dR}{dt}=\gamma I-\mu R \qquad 3$$

To interpret the parameters (Diekmann and Heesterbeek, 2000) explain that S is the population that are susceptible, I is the number of people that are infected with the river blindness the disease where γ is the recovery rate (with greater or equal to zero), is the probability of becoming infected, is the number of people infected person comes in contact with in each period of time on average, β is the average number of transmissions from an infected person in a time period (with greater or equal to zero), and μ is the life expectancy. From these equations (1, 2, 3), we can see from equation (1), that the susceptible group will decrease over time and approach zero. From equation (3), we know that the recovered group increase and will approach N over time (Hackborn, 2008; Iannelli, 2005).

3.2.2 Steady State Analysis

To determine the steady state of the SIR model with demography, let the rate of susceptibility,

infection and recovery equal to zero; $\frac{dS}{dt}=0$ $\frac{dI}{dt}=0$ and $\frac{dR}{dt}=0$, hence we assume that $S>0$, $R=0$ and $I=0$ therefore $S+I+R=1$ for $R=1-S-I$
Setting the equation 1, 2 and 3 to zero

$\mu-\beta SI-\mu I=0$ (1.1)

$\beta SI-\gamma I-\mu I=0$ (2.1)

$\gamma I-\mu R=0$ (3.1)

From the equation 2.1 then factoring out I we get $\beta SI-I(\gamma+\mu)=0$ and $I(\beta S-(\gamma+\mu))=0$ Then $I=0$ and $\beta S-(\gamma+\mu)=0$ $\beta S=(\gamma+\mu)$ dividing both sides by β we have the steady state of the susceptibility to be

$$S=\frac{(\gamma+\mu)}{\beta} \quad \ldots 4$$

To find the steady state of I, taking equation 1.1 and substitute the value of $\frac{(\gamma+\mu)}{\beta}$ for S to get in $\mu-\beta SI-\mu S=0$; $\mu-(\beta I+\mu)S=0$

$(\beta I+\mu)S=\mu$ Dividing though by $(\beta I+\mu)$ we get $S=\frac{\mu}{(\beta I+\mu)}$ substituting the value of $\frac{(\gamma+\mu)}{\beta}$ for S to get I then $\frac{(\gamma+\mu)}{\beta}=\frac{\mu}{(\beta I+\mu)}$. Multiplying both side by $\frac{(\beta I+\mu)}{(\gamma+\mu)}$ and cancelling out $(\gamma+\mu)$ in the LHS and $(\beta I+\mu)$ in the right hand side, it becomes $\frac{(\beta I+\mu)}{(\gamma+\mu)}.\frac{(\gamma+\mu)}{\beta}=\frac{\mu}{(\beta I+\mu)}.\frac{(\beta I+\mu)}{(\gamma+\mu)}$, multiply by β to get

$(\beta I+\mu)=.\frac{\mu}{(\gamma+\mu)}.\beta$. Therefore, $\beta I=.\frac{\mu}{(\gamma+\mu)}.\beta-\mu$ and dividing both side by β to get I,

$I=\frac{\beta\mu}{\beta(\gamma+\mu)}-\frac{\mu}{\beta}$ re-arranging the expression, we get

$$I=\frac{\mu}{\beta}\left[\frac{\beta}{(\gamma+\mu)}-1\right] \quad \ldots 5$$

The steady state of R is obtained using the function of $R=1-S-I$

$$R=1-\frac{(\gamma+\mu)}{\beta}-\frac{\beta\mu}{\beta(\gamma+\mu)}-\frac{\mu}{\beta}$$

$\therefore$

$$R=1-\frac{(\gamma+\mu)}{\beta}-\frac{\mu}{\beta}\left[\frac{\beta}{(\gamma+\mu)}-1\right] \quad \ldots 6$$

3.2.3 The Disease Free Equilibrium

From the steady state of $S=\frac{(\gamma+\mu)}{\beta}=\frac{1}{R_0}$ where $S=\frac{1}{R_0}$ for $R_0=\frac{\beta}{(\gamma+\mu)}$ which the probability of transmission rate multiple by the period of infection with demography $(\gamma+\mu)$. The SIR model compartment in terms of reproductive number is expressed as $S=\frac{(\gamma+\mu)}{\beta}=\frac{1}{R_0}$ where

$S = \frac{1}{R_0}$ for $R_0 = \frac{\beta}{(\gamma + \mu)}$ is the probability of transmission rate with the in period of infection of $\frac{1}{(\gamma + \mu)}$ is the SIR model with demography value of μ making the rate of infection to be $(\gamma + \mu)$.

The equilibrium of the expression of the non-zero steady states in terms of R_0 of S^*, I^* and R^* in terms of the value of R_0 (see Smith, David, and Lang, 2008).

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$$S = \frac{(\gamma + \mu)}{\beta} = \frac{1}{R_0}$$

$$\therefore \qquad S^* = \frac{1}{R_0} \quad \ldots 7$$

$$I^* = \frac{\mu}{\beta}[R_o - 1] \quad \ldots 8$$

$$R^* = 1 - \frac{1}{R_o} - \frac{\mu}{\beta}[R_o - 1] \quad \ldots 9$$

The equilibrium in terms of R_0 are:

$$S^* = \frac{1}{R_0}, \quad I^* = \frac{\mu}{\beta}[R_o - 1] \text{ and } R^* = 1 - \frac{1}{R_o} - \frac{\mu}{\beta}[R_o - 1]$$

4. Analysis

4.1 Computation Procedure

Applying the numerical analysis using Euler method, the numerical expression for The equilibrium of S^*, I^* and R^* in terms of the value of S, I, R and the year tn

$$t_{n+1} = t_n + h$$

$$S_{n+1} = S_n + h(\mu - \beta S_n I_n - \mu S_n)$$

$$I_{n+1} = I_n + h(\beta S_n I_n - \gamma I_n - \mu I_n)$$

$$R_{n+1} = R_n + h(\gamma I_n - \mu R_n)$$

Where h is the step size and entering the function into Excel using the programming formulae in the cell 7 and key parameters are simulated using Euler numerical method see appendix A for the simulated data table: The parameters are defined as follow:

$$\mu = 556 \quad \frac{1}{\mu} = 0.0017, \ \gamma = 2.12 weeks\, in\ \ days = 15$$

$$\frac{1}{\gamma} = 0.0067 \quad \beta = 856\ per\ year = \frac{856}{365} = 2.3451,$$

$$S = 0.65, \quad R(o) = 0, I(0) = 1\ x10^{-3} = 0.001\ and\ \ h = 0.5$$

4.2 Euler Method Computation

$$t_{n+1} = t_n + h = A6 + \$C\$2$$

$$S_{n+1} = S_n + h(\mu - \beta S_n I_n - \mu S_n) = B6 + \$C\$2 * (\$H\$2 - \$F\$1 * B6 * C6 - \$H\$2 * C6)$$

$$I_{n+1} = I_n + h(\beta S_n I_n - \gamma I_n - \mu I_n) = C6 + \$C\$2 * (\$F\$1 * B6 * C6 - \$H\$1 * C6 - \$H\$2 * C6)$$

$$R_{n+1} = R_n + h(\gamma I_n - \mu R_n) = D6 + \$C\$2 * (\$H\$1 * C6 - \$H\$2 * D6)$$

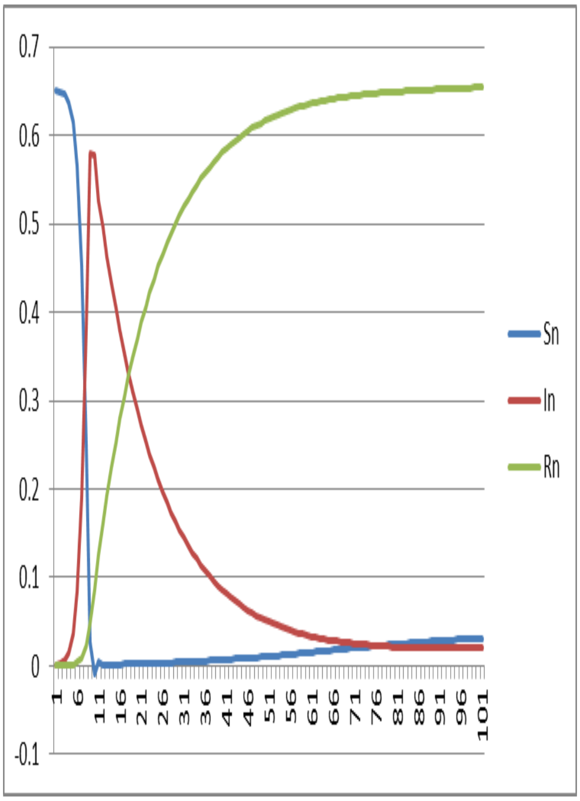

Fig 1. **Graphical Result of Simulated Analysis 90 days**

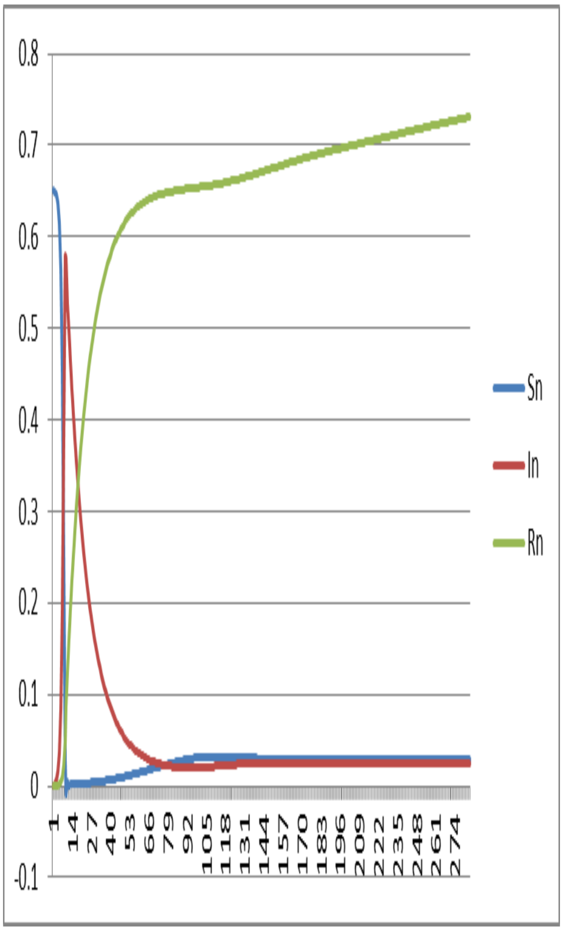

Fig 2. **Graphical Result of Simulated Analysis 180 days**

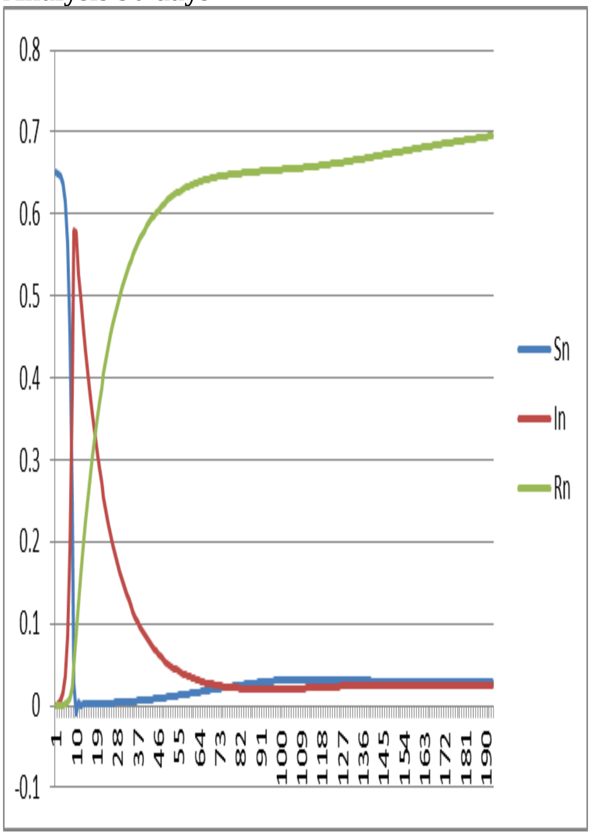

Fig 3. **Graphical Result of Simulated Analysis 270 days**

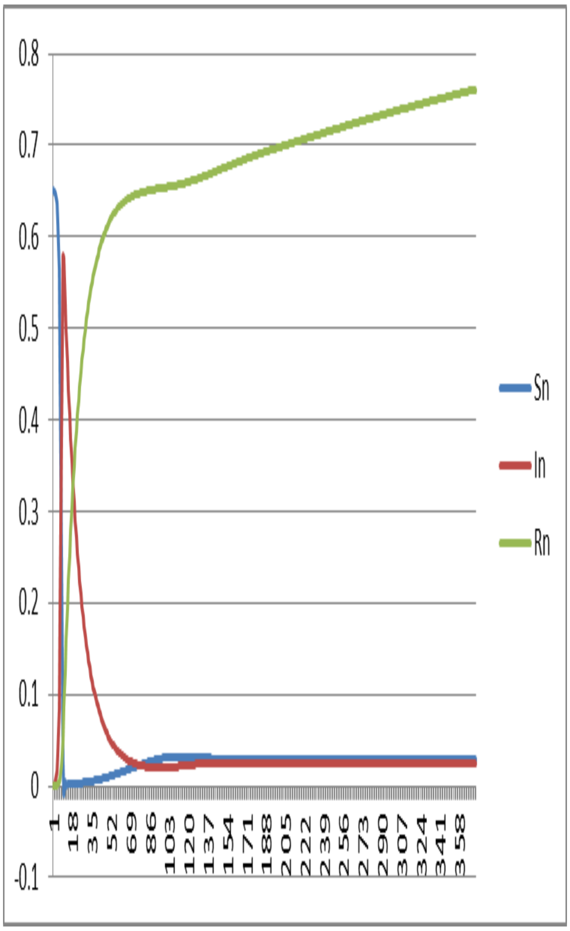

Fig 3. **Graphical Result of Simulated Analysis 365 days**

5. Discussion

The fig 1 illustrates about 0.47(47%) and 47% are infected by the river blindness in the first ten days of the three months simulated period of the disease. About negligible number of the population recovered. From the 19 days of the first three month the infection begins to decline and recovery rate starts increasing from 0.37(37%) in the 21days. The interaction between the susceptibility and infection decline drastically to 0.01%, in the 90 days simulated diseases about 52% of the population are susceptible to the disease and 50% on average infection rate is recorded within the 14 days before the infection starts decreasing from 55% and dies out. The recovery rate is 0.37(37%) which is seemingly constant. This finding suggests endemic onchocerciasis representing a serious health risk to the endemic community in northern Nigeria as the susceptibility level is 0.55(55%) with critical point of infection level at 50% on average and begins to decrease drastically. This may be as a result of intervention by the health workers in terms of treatment and awareness. In the subsequent months, that is, 9 to 12 months the disease susceptibility, infection rate and recovery assume the same behavioural pattern see fig 3 and 4 above.

6. Conclusion

The SIR Model is used in epidemiology to compute the amount of susceptible, infected, and recovered people in a population. It is also used to explain the change in the number of people needing medical attention during an epidemic. It is important to note that this model does not work with all diseases and can be improved using contact and number of black fly population in the disease environment to determine the stochastic nature of the disease. This model can be used to explain the change in the number of people needing medical attention during an epidemic. It is important to note that this model does not work with all diseases. For the SIR model to be appropriate, once a person has recovered from the disease, they would receive lifelong immunity. The SIR model is also not appropriate if a person was infected but is not infectious. This finding suggests endemic onchocerciasis representing a serious health risk to the endemic community in northern Nigeria as the susceptibility level is 0.55(55%) with critical point of infection level at 56% and begins to decrease drastically. This may be as a result of intervention by the health workers in terms of treatment and awareness.

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Appendix A

s(0)	0.65	beta	2.3451	gamma	0.067	I0	0.001	R0	0
h	1			Mu	0.0017				

tn	**Sn**	**In**	**Rn**
0	0.65	0.001	0
1	0.64907	0.00246	6.7E-05
2	0.64593	0.00603	0.00023
3	0.63741	0.01474	0.00064
4	0.61599	0.03575	0.00162
5	0.565	0.08494	0.00401
6	0.45319	0.19166	0.0097
7	0.25043	0.38218	0.02252
8	0.02726	0.58037	0.04809
9	-0.0082	0.5776	0.08689
10	0.00462	0.52683	0.12545
11	0.00061	0.49634	0.16053
12	0.0016	0.46294	0.19351
13	0.00156	0.43288	0.2242
14	0.00167	0.40472	0.25282
15	0.00178	0.37851	0.27951
16	0.0019	0.35408	0.30439
17	0.00202	0.33133	0.3276
18	0.00215	0.31014	0.34924
19	0.00228	0.29039	0.36943
20	0.00242	0.272	0.38825
21	0.00257	0.25486	0.40582
22	0.00273	0.23889	0.4222
23	0.0029	0.22401	0.43749
24	0.00307	0.21014	0.45176
25	0.00325	0.19722	0.46507
26	0.00344	0.18517	0.47749
27	0.00364	0.17394	0.48909
28	0.00385	0.16348	0.49991
29	0.00407	0.15372	0.51001
30	0.00429	0.14463	0.51944
31	0.00453	0.13615	0.52825
32	0.00478	0.12824	0.53647

33	0.00503	0.12087	0.54416
34	0.0053	0.11399	0.55133
35	0.00557	0.10758	0.55803
36	0.00586	0.10159	0.56429
37	0.00615	0.09601	0.57013
38	0.00646	0.0908	0.5756
39	0.00677	0.08593	0.5807
40	0.00709	0.0814	0.58547
41	0.00743	0.07716	0.58993
42	0.00777	0.0732	0.5941
43	0.00812	0.06951	0.59799
44	0.00849	0.06606	0.60163
45	0.00886	0.06283	0.60504
46	0.00924	0.05982	0.60822
47	0.00963	0.05701	0.61119
48	0.01002	0.05438	0.61397
49	0.01043	0.05192	0.61657
50	0.01084	0.04962	0.619
51	0.01126	0.04747	0.62127
52	0.01169	0.04547	0.6234
53	0.01212	0.04359	0.62538
54	0.01256	0.04183	0.62724
55	0.01301	0.04019	0.62898
56	0.01346	0.03866	0.6306
57	0.01392	0.03722	0.63212
58	0.01438	0.03588	0.63354
59	0.01484	0.03462	0.63487
60	0.01531	0.03345	0.63611
61	0.01579	0.03235	0.63727
62	0.01626	0.03133	0.63835
63	0.01674	0.03037	0.63936
64	0.01722	0.02948	0.64031
65	0.0177	0.02864	0.6412
66	0.01818	0.02786	0.64203
67	0.01866	0.02714	0.6428
68	0.01914	0.02646	0.64353
69	0.01962	0.02583	0.64421
70	0.0201	0.02524	0.64484
71	0.02058	0.0247	0.64544
72	0.02105	0.0242	0.646
73	0.02152	0.02373	0.64652
74	0.02199	0.02329	0.64701
75	0.02245	0.0229	0.64747

76	0.0229	0.02253	0.6479
77	0.02335	0.02219	0.64831
78	0.0238	0.02188	0.6487

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