

Fixed Point Theorem For Weakly Compatible Maps In Fuzzy Metric Space

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Abstract:

In this paper, the concept of weakly compatible maps in hausdorff fuzzy metric space has been applied to prove fuzzy fixed point theorems. A fixed point theorems for six self maps has been established using the concept of compatible maps of type hausdorff, which generalizes the result of Cho [1].

Keywords: Fuzzy metric space, fuzzy fixed point, t-norm and weak compatible map.

1Introduction:

The concept of Fuzzy sets was initially investigated by Zadeh [11] as a new way to represent vagueness in everyday life. Subsequently, it was developed by many authors and used in various fields. To use this concept in Topology and Analysis, several researchers have defined Fuzzy metric space in various ways. In this paper we deal with the Fuzzy metric space defined by Kramosil and Michalek [8] and modified by George and Veeramani [4]. Recently, Grebiec [5] has proved fixed point results for Fuzzy metric space. In the sequel, Singh and Chauhan [10] introduced the concept of compatible mappings of Fuzzy metric space and proved the common fixed point theorem. Phiangsungnoen et. al. [9] introduced the concept of compatible maps in hausdorff fuzzy metric space and proved fixed point theorems. Cho [2, 3] introduced the concept of compatible maps of type (α) and compatible maps of type (β) in fuzzy metric space. Using the concept of compatible maps of type hausdorff, Jain et. al. [6] proved a fixed point theorem for six self maps in a fuzzy metric space. Using the concept of compatible maps of type (β), Jain et. al. [7] proved a fixed point theorem in fuzzy metric space. In this paper, a fixed point theorem for six self maps has been established using the concept of compatible maps of type (β) and weak compatible maps, which generalizes the result of Cho [1]. For the sake of completeness, we recall some definitions and known results in Fuzzy metric space.

2 Preliminaries:

2.1. Definition: A binary operation *: [0, 1] × [0, 1] → [0, 1] is a continuous t-norm if * is satisfying the following conditions:

(a) * is commutative and associative;

(b) * is continuous;

(c) a * b = a for all a ∈ [0, 1];

(d) a* b ≤ c * d whenever a ≤ c and b ≤ d and a, b, c, d ∈ [0, 1].

2.2. Definition: A 3-tuple (X, M, *) is said to be a fuzzy metric space if X is an arbitrary set, * is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions; for all x, y, z ∈ X, s, t > 0.

(1) M (x, y, t) > 0;

(2) M(x, y, t) =1 if and only if x = y;

(3) M(x, y, t) = M(y, x, t);

(4) M(x, y, t)* M(y, z, s) ≤ M(x, z, t + s);

(5) M(x, y, .): (0, ∞)→[0, 1] is continuous.

Then M is called a fuzzy metric on X. The function M(x, y, t) denote the degree of nearness between x and y with respect to t.

2.3. Example: Let (X, d) be a metric space. Denote $a * b = a\,b$ for a, b ∈ [0, 1] and let M_d be a fuzzy set on $X^2 \times (0, \infty)$ defined as follows: M_d (x, y, t) $= \frac{t}{t+d(x,\ y)}$

Then (X, M_d, *) is a fuzzy metric space, we call this fuzzy metric induced by a metric d the standard intuitionistic fuzzy metric.

2.4. Definition: Let (X, M,*) be a fuzzy metric space, then

(a) A sequence $\{x_n\}$ in X is said to be convergent to x in X if for each ε > 0 and each t > 0, there exists n_0 ∈ N such that M (x_n, x, t) > 1 − ε for all n ≥ n_0.

(b) A sequence $\{x_n\}$ in X is said to be Cauchy if for each ε > 0 and each t > 0, there exist n_0 ∈ N such that M (x_n, x_m, t) > 1 − ε for all n, m ≥ n_0 .

(c) A fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

2.5. Proposition: In a fuzzy metric space (X, M, *), if a * a ≥ a for a ∈ [0, 1] then a * b = min {a, b} for all a, b ∈ [0, 1].

2.6. Definition: Two self-mappings A and S of a fuzzy metric space (X, M, *) are called compatible if $\lim_{n\to\infty}$ M (ASx_n, SAx_n, t) = 1 whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$ = x for some x in X.

2.7. Definition: Two self-maps A and B of a fuzzy metric space (X, M, *) are called weakly compatible (or coincidentally commuting) if they commute at their coincidence points, i.e. if $Ax = Bx$ for some $x \in X$ then $ABx = BAx$.

2.8. Remark: If self-maps A and B of a fuzzy metric space (X, M, *) are compatible then they are weakly compatible. Let (X, M,*) be a fuzzy metric space with the following condition: (6) $\lim_{t\to\infty}$ M (x, y, t) =1 for all x, y ∈ X.

2.9. Lemma: Let (X, M, *) be a fuzzy metric space. If there exists k ∈[0, 1] such that M(x, y, kt) ≥ M(x, y, t) then x = y.

2.10. Lemma: Let $\{x_n\}$ be a sequence in a fuzzy metric space (X, M, *) with the condition (6). If there exists k ∈[0, 1] such that M (y_n,y_{n+1}, kt) ≥ M (y_{n-1}, y_n, t) for all t > 0 and n ∈ N. Then { y_n } is a Cauchy sequence in X.

Main Result:

(1) Theorem. Let A, B, S, T, L and N be self-maps on a (X, M, *) complete fuzzy metric space and α: X → (0, 1] be a maping such that $[Sx]_{(\propto x)}$, $[Tx]_{(\propto x)}$are nonempty compact subset of X for all x ϵ X. Suppose that S, T: X → F(x) is a fuzzy mapping such that

(a). L(X) ⊆ ST(x), N(x) ⊆ AB(x);

(b). There exists a constant k ϵ [0, 1] such that

$$H_{M^2}\left([Lx]_{\propto(x)},[Ny]_{\propto(y)},kt\right)*\left\{\begin{matrix} H_M\left([ABx]_{\propto(x)},[Lx]_{\propto(x)},kt\right) \\ * H_M\left([STy]_{\propto(y)},[Ny]_{\propto(y)},kt\right)\end{matrix}\right\} \geq \left\{\begin{matrix} H_M\left([STx]_{\propto(x)},[Ny]_{\propto(y)},t\right) \\ * H_M\left([ABx]_{\propto(x)},[Ny]_{\propto(y)},t\right) \\ * H_M\left([ABy]_{\propto(y)},[Lx]_{\propto(x)},t\right) \\ * H_M\left([STx]_{\propto(x)},[Ny]_{\propto(y)},t\right) \\ * H_M\left([STx]_{\propto(x)},[ABy]_{\propto(y)},t\right)\end{matrix}\right\}$$

(c). AB=BA, ST=TS, LB=BL, NT=TN.

(d). Either AB or L is continuous.

(e). The pair (L, AB) is compatible and (N, ST) is weakly compatible.

The A, B, S, T, L and N have a common fixed point.

Proof: Let x_0 be an arbitrary point of X. By (a) there exists $x_1, x_2 \epsilon X$ such that

$[Lx_0]_{\propto(x)} = [STx_1]_{\propto(x)} = y_0$ and $[Nx_1]_{\propto(x)} = [ABx_2]_{\propto(x)} = y_1$.

Inductively we can construct sequence $\{x_n\}$ and $\{y_n\}$ such hat

$[Lx_{2n}]_{\propto(x)} = [STx_{2n+1}]_{\propto(x)} = [y_{2n}]_{\propto(x)}$ and $[Nx_{2n+1}]_{\propto(x)} = [ABx_{2n+2}]_{\propto(x)} = [y_{2n+1}]_{\alpha(x)}$

for n = 0, 1, 2, 3, 4……..

Step-1 by taking $x = x_{2n}$ and $y = x_{2n+1}$ in (b) we have

$$H_{M^2}\left([Lx_{2n}]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},kt\right)*\left\{\begin{matrix} H_M\left([ABx_{2n}]_{\propto(x)},[Lx_{2n}]_{\propto(x)},kt\right) \\ * H_M\left([STx_{2n+1}]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},kt\right)\end{matrix}\right\}$$

$$\geq \left\{\begin{matrix} H_M\left([STx_{2n}]_{\propto(x)},[Lx_{2n}]_{\propto(x)},t\right) * H_M\left([ABx_{2n+1}]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},t\right) \\ * H_M\left([ABx_{2n+1}]_{\propto(x)},[Lx_{2n}]_{\propto(x)},t\right) * H_M\left([STx_{2n}]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},t\right) \\ * H_M\left([STx_{2n}]_{\propto(x)},[ABx_{2n+1}]_{\propto(x)},t\right)\end{matrix}\right\}$$

$$H_{M^2}\left([y_{2n}]_{\propto(x)},[y_{2n+1}]_{\propto(x)},kt\right)*\left\{\begin{matrix} H_M\left([y_{2n-1}]_{\propto(x)},[y_{2n}]_{\propto(x)},kt\right) \\ * H_M\left([y_{2n}]_{\propto(x)},[y_{2n+1}]_{\propto(x)},kt\right)\end{matrix}\right\}$$

$$\geq \left\{\begin{matrix} H_M\left([y_{2n-1}]_{\propto(x)},[y_{2n}]_{\propto(x)},t\right) * H_M\left([y_{2n}]_{\propto(x)},[y_{2n+1}]_{\propto(x)},t\right) \\ * H_M\left([y_{2n}]_{\propto(x)},[y_{2n}]_{\propto(x)},t\right) * H_M\left([y_{2n-1}]_{\propto(x)},[y_{2n+1}]_{\propto(x)},t\right) \\ * H_M\left([y_{2n-1}]_{\propto(x)},[y_{2n}]_{\propto(x)},t\right)\end{matrix}\right\}$$

$$H_{M^2}\left([y_{2n}]_{\propto(x)},[y_{2n+1}]_{\propto(x)},kt\right) \geq H_M\left([y_{2n-1}]_{\propto(x)},[y_{2n}]_{\propto(x)},t\right)$$

In general

$$H_{M^2}([y_{2n+1}]_{\propto(x)}, [y_{2n+2}]_{\propto(x)}, kt) \geq H_M([y_{2n}]_{\propto(x)}, [y_{2n+1}]_{\propto(x)}, t)$$

In general for al n even or odd we have

$H_M([y_n]_{\propto(x)}, [y_{n+1}]_{\propto(x)}, kt) \geq H_M([y_{n-1}]_{\propto(x)}, [y_n]_{\propto(x)}, t)$ for k ϵ (0, 1) and all t >0.

Thus by Lemma 2.2.10, it $\{y_n\}$ is a Cauchy sequence in X. Since (X, M, *) is complete, it converges to a point z in X, and also its sub sequences converges as follows.

$\{[Lx_{2n}]_{\propto(x)}\} \rightarrow [z]_{\propto(x)}, [ABx_{2n}]_{\propto(x)}\} \rightarrow [z]_{\propto(x)}\{[Nx_{2n+1}]_{\propto(x)}\} \rightarrow [z]_{\propto(x)}$ and $\{[STx_{2n+1}]_{\propto(x)}\} \rightarrow [z]_{\propto(x)}$.

Case-1: AB is continuous.

Since AB is continuous. $AB[(AB)x_{2n}]_{\propto(x)} \rightarrow [ABz]_{\propto(x)}$ *and* $AB[Lx_{2n}]_{\propto(x)} \rightarrow [ABz]_{\propto(x)}$.

Since (L, AB) is complete.

$$[L(AB)x_{2n}]_{\propto(x)} \rightarrow [ABz]_{\propto(x)}.$$

Step 2: By taking $x = [(AB)x_{2n}]_{\propto(x)}$ *and* $y = x_{2n+1}$ in (b) we have

$$H_{M^2}([L(AB)x_{2n}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, kt) * \begin{Bmatrix} H_M([AB(AB)x_{2n}]_{\propto(x)}, [L(AB)x_{2n}]_{\propto(x)}, kt) \\ * H_M([STx_{2n+1}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, kt) \end{Bmatrix}$$
$$\geq \begin{Bmatrix} H_M([ST(ABx_{2n})]_{\propto(x)}, [L(AB)x_{2n}]_{\propto(x)}, t) * H_M([ABx_{2n+1}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, t) \\ * H_M([ABx_{2n+1}]_{\propto(x)}, [L(AB)x_{2n}]_{\propto(x)}, t) * H_M([ST(AB)x_{2n}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, t) \\ * H_M([ST(AB)x_{2n}]_{\propto(x)}, [ABx_{2n+1}]_{\propto(x)}, t) \end{Bmatrix}$$

This implies that as n→ ∞

$$H_{M^2}([ABz]_{\propto(x)}, z, kt) * \begin{Bmatrix} H_M[ABz]_{\propto(x)}, [ABz]_{\propto(x)}, kt) \\ * H_M([z]_{\propto(x)}, [z]_{\propto(x)}, kt) \end{Bmatrix} \geq \begin{Bmatrix} H_M([z]_{\propto(x)}, [ABz]_{\propto(x)}, t) \\ * H_M([ABz]_{\propto(x)}, [z]_{\propto(x)}, t) \\ * H_M([ABz]_{\propto(x)}, [ABz]_{\propto(x)}, t) \\ * H_M([z]_{\propto(x)}, [z]_{\propto(x)}, t) \\ * H_M([z]_{\propto(x)}, [ABz]_{\propto(x)}, t) \end{Bmatrix}$$

$$H_{M^2}\left([ABz]_{\propto(x)},[z]_{\propto(x)},kt\right)*\{1*1\}\geq\left\{\begin{matrix}H_M\left([ABz]_{\propto(x)},[z]_{\propto(x)},t\right)\\ *H_M([ABz]_{\propto(x)},[z]_{\propto(x)},t)\\ *1*1*H_M([ABz]_{\propto(x)},[z]_{\propto(x)},t)\end{matrix}\right\}$$

$H_M([ABz]_{\propto(x)},[z]_{\propto(x)},kt)\geq 1$ Thus we get $[ABz]_{\propto(x)}=[z]_{\propto(x)}$.

Step 3: By taking x = z and y = x_{2n+1} in (b) and we take $n\to\infty$ we have

$$H_{M^2}\left([Lz]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},kt\right)*\left\{\begin{matrix}H_M\left([ABz]_{\propto(x)},[Lz]_{\propto(x)},kt\right)\\ *H_M\left([STx_{2n+1}]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},kt\right)\end{matrix}\right\}$$

$$\geq\left\{\begin{matrix}H_M\left([STz]_{\propto(x)},[Lz]_{\propto(x)},t\right)*H_M([ABx_{2n+1}]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},t)\\ *H_M\left([ABx_{2n+1}]_{\propto(x)},[Lz]_{\propto(x)},t\right)*H_M([STz]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},t)\\ *H_M([STz]_{\propto(x)},[ABx_{2n+1}]_{\propto(x)},t)\end{matrix}\right\}$$

$$H_{M^2}\left([Lz]_{\propto(x)},[z]_{\propto(x)},kt\right)\geq H_M\left([Lz]_{\propto(x)},[z]_{\propto(x)},t\right)$$

$$[Lz]_{\propto(x)}=[z]_{\propto(x)}=[ABz]_{\propto(x)}$$

Step 4: By taking $x=Bz\ and\ y=x_{2n+1}$ and take $n\to\infty$we have

$$H_{M^2}\left([L(Bz)]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},kt\right)*\left\{\begin{matrix}H_M\left([AB(Bz)]_{\propto(x)},[L(Bz)]_{\propto(x)},kt\right)\\ *H_M\left([STx_{2n+1}]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},kt\right)\end{matrix}\right\}$$

$$\geq\left\{\begin{matrix}H_M\left([ST(Bz)]_{\propto(x)},[L(Bz)]_{\propto(x)},t\right)*H_M([ABx_{2n+1}]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},t)\\ *H_M\left([ABx_{2n+1}]_{\propto(x)},[L(Bz)]_{\propto(x)},t\right)*H_M([ST(BZ)]_{\propto(x)},[Nx_{2n+1}]_{\propto(x)},t)\\ *H_M([ST(Bz)]_{\propto(x)},[ABx_{2n+1}]_{\propto(x)},t)\end{matrix}\right\}$$

$$H_{M^2}\left([Bz]_{\propto(x)},[z]_{\propto(x)},kt\right)\geq H_M\left([Bz]_{\propto(x)},[z]_{\propto(x)},t\right)$$

$$H_M\left([Bz]_{\propto(x)},[z]_{\propto(x)},kt\right)\geq 1$$

Thus we have $[Bz]_{\propto(x)}=[z]_{\propto(x)}$

Since $z=ABz$ we also have $z=Az$ therefore $a=Az=Bz=Lz$.

Step 5: Since L(X) ⊆ ST(X) there exists v ϵ X such that $[z]_{\propto(x)}=[Lz]_{\propto(x)}=[STv]_{\propto(x)}$.

By taking $x=x_{2n},y=v$ in (b) and take $n\to\infty$we have

$$H_{M^2}\left([Lx_{2n}]_{\propto(x)},[Nv]_{\propto(x)},kt\right)*\left\{\begin{matrix}H_M\left([ABx_{2n}]_{\propto(x)},[Lx_{2n}]_{\propto(x)},kt\right)\\ *H_M\left([STv]_{\propto(x)},[Nv]_{\propto(x)},kt\right)\end{matrix}\right\}$$

$$\geq \begin{Bmatrix} H_M([STx_{2n}]_{\propto(x)}, [Lx_{2n}]_{\propto(x)}, t) \\ * H_M([ABv]_{\propto(x)}, [Nv]_{\propto(x)}, t) \\ * H_M([ABv]_{\propto(x)}, [Lx_{2n}]_{\propto(x)}, t) \\ * H_M([STx_{2n}]_{\propto(x)}, [Nv]_{\propto(x)}, t) \\ * H_M([STx_{2n}]_{\propto(x)}, [ABv]_{\propto(x)}, t) \end{Bmatrix}$$

$$H_{M^2}([z]_{\propto(x)}, [Nv]_{\propto(x)}, kt) \geq H_M([z]_{\propto(x)}, [Nv]_{\propto(x)}, t)$$

$$H_M([z]_{\propto(x)}, [Nv]_{\propto(x)}, kt) \geq 1$$

Thus we have $z = Nv \text{ and so } z = Nv = STv$.

Since (N, ST) is weakly compatible we have $ST(Nv) = N(STv).Thus\ STz = Nz$.

Step 6: By taking $x = x_{2n} \text{ and } y = [z]_{\propto(x)}$ in (b) and using step 5 we have

$$H_{M^2}([Lx_{2n}]_{\propto(x)}, [Nz]_{\propto(x)}, kt) * \begin{Bmatrix} H_M([ABx_{2n}]_{\propto(x)}, [Lx_{2n}]_{\propto(x)}, kt) \\ * H_M([STz]_{\propto(x)}, [Nz]_{\propto(x)}, kt) \end{Bmatrix}$$

$$\geq \begin{Bmatrix} H_M([STz_{2n}]_{\propto(x)}, [Lx_{2n}]_{\propto(x)}, t) * H_M([ABz]_{\propto(x)}, [Nz]_{\propto(x)}, t) \\ * H_M([ABz]_{\propto(x)}, [Lx_{2n}]_{\propto(x)}, t) * H_M([STx_{2n}]_{\propto(x)}, [Nz]_{\propto(x)}, t) \\ * H_M([STx_{2n}]_{\propto(x)}, [ABz]_{\propto(x)}, t) \end{Bmatrix}$$

$$H_{M^2}([z]_{\propto(x)}, [Nz]_{\propto(x)}, kt) * \begin{Bmatrix} H_M([z]_{\propto(x)}, [z]_{\propto(x)}, kt) \\ * H_M([z]_{\propto(x)}, [Nz]_{\propto(x)}, kt)\} \end{Bmatrix} \geq \begin{Bmatrix} H_M([z]_{\propto(x)}, [z]_{\propto(x)}, t) \\ * H_M([z]_{\propto(x)}, [Nz]_{\propto(x)}, t) \\ * H_M([z]_{\propto(x)}, [z]_{\propto(x)}, t) \\ * H_M([z]_{\propto(x)}, [Nz]_{\propto(x)}, t) \\ * H_M([z]_{\propto(x)}, [z]_{\propto(x)}, t) \end{Bmatrix}$$

$$H_M([z]_{\propto(x)}, [Nz]_{\propto(x)}, kt) \geq 1$$

Thus we have $[z]_{\propto(x)} = [Nz]_{\propto(x)}$ and therefore $[z]_{\propto(x)} = [Az]_{\propto(x)} = [Bz]_{\propto(x)} = [Lz]_{\propto(x)} = [Nz]_{\propto(x)} = [STz]_{\propto(x)}$

Step 7: By taking $x = [x_{2n}]_{\propto(x)} \text{ and } y = [Tz]_{\propto(x)}$ in (b) Since $NT = TN \text{ and } ST = TS$, we have $NTz = TNz = Tz \text{ and } ST(Tz) = Tz \text{ letting } n \to \infty$ we have

$$H_{M^2}([Lx_{2n}]_{\propto(x)}, [N(Tz)]_{\propto(x)}, kt) * \begin{Bmatrix} H_M([ABx_{2n}]_{\propto(x)}, [Lx_{2n}]_{\propto(x)}, kt) \\ * H_M([ST(Tz)]_{\propto(x)}, [N(Tz)]_{\propto(x)}, kt) \end{Bmatrix}$$

$$\geq \left\{ \begin{array}{c} H_M([STx_{2n}]_{\propto(x)}, [Lx_{2n}]_{\propto(x)}, t) * H_M([AB(Tz)]_{\propto(x)}, [N(Tz)]_{\propto(x)}, t) \\ * H_M([AB(Tz)]_{\propto(x)}, [Lx_{2n}]_{\propto(x)}, t) * H_M([STx_{2n}]_{\propto(x)}, [N(Tz)]_{\propto(x)}, t) \\ * H_M([STx_{2n}]_{\propto(x)}, [AB(Tz)]_{\propto(x)}, t) \end{array} \right\}$$

$$H_M\big([z]_{\propto(x)}, [Tz]_{\propto(x)}, kt\big) \geq 1$$

Thus $[z]_{\propto(x)} = [Tz]_{\propto(x)}$. Since $[Tz]_{\propto(x)} = [STz]_{\propto(x)}$ we also have $[z]_{\propto(x)} = [Sz]_{\propto(x)}$.

Therefore $[z]_{\propto(x)} = [Az]_{\propto(x)} = [Bz]_{\propto(x)} = [Lz]_{\propto(x)} = [Nz]_{\propto(x)} = [Sz]_{\propto(x)} = [Tz]_{\propto(x)}$, that is $[z]_{\propto(x)}$ is the common fixed point of the six maps.

Case-2: L is continuous. Since L is continuous

$[LLx_{2n}]_{\propto(x)} \rightarrow [Lz]_{\propto(x)} and\ [L(AB)x_{2n}]_{\propto(x)} \rightarrow [Lz]_{\propto(x)}$.

Since (L, AB) is compatible, $[(AB)Lx_{2n}]_{\propto(x)} \rightarrow [Lz]_{\propto(x)}$.

Step 8: By taking $x = [Lx_{2n}]_{\propto(x)}\ and\ y = [x_{2n+1}]_{\propto(x)}$ in (b) we have

$$H_{M^2}\big([LLx_{2n}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, kt\big) * \left\{ \begin{array}{c} H_M([ABLx_{2n}]_{\propto(x)}, [LLx_{2n}]_{\propto(x)}, kt) \\ * H_M([STx_{2n+1}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, kt) \end{array} \right\}$$

$$\geq \left\{ \begin{array}{c} H_M([STLx_{2n}]_{\propto(x)}, [LLx_{2n}]_{\propto(x)}, t) * H_M([ABx_{2n+1}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, t) \\ * H_M([ABx_{2n+1}]_{\propto(x)}, [LLx_{2n}]_{\propto(x)}, t) * H_M([STLx_{2n}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, t) \\ * H_M([STLx_{2n}]_{\propto(x)}, [ABx_{2n+1}]_{\propto(x)}, t) \end{array} \right\}$$

$$H_{M^2}\big([Lz]_{\propto(x)}, [z]_{\propto(x)}, kt\big) * \left\{ \begin{array}{c} H_M\big([Lz]_{\propto(x)}, [Lz]_{\propto(x)}, kt\big) \\ * H_M\big([z]_{\propto(x)}, [z]_{\propto(x)}, kt\big) \end{array} \right\} \geq \left\{ \begin{array}{c} H_M\big([z]_{\propto(x)}, [z]_{\propto(x)}, t\big) \\ * H_M([z]_{\propto(x)}, [z]_{\propto(x)}, t) \\ * H_M\big([z]_{\propto(x)}, [Lz]_{\propto(x)}, t\big) \\ * H_M([Lz]_{\propto(x)}, [z]_{\propto(x)}, t) \\ * H_M([Lz]_{\propto(x)}, [z]_{\propto(x)}, t) \end{array} \right\}$$

$$H_M\big([Lz]_{\propto(x)}, [z]_{\propto(x)}, kt\big) \geq 1$$

Thus we have $[z]_{\propto(x)} = [Lz]_{\propto(x)}$ and using step 5-7 we have

$[z]_{\propto(x)} = [Lz]_{\propto(x)} = [Nz]_{\propto(x)} = [Sz]_{\propto(x)} = [Tz]_{\propto(x)}$.

Step 9: Since N(X) $\subseteq AB(X)$ there exists$v \in X$ such that $[z]_{\propto(x)} = [Nz]_{\propto(x)} = [Sz]_{\propto(x)} = [Tz]_{\propto(x)}$. By taking$x = v, y = [x_{2n+1}]_{\propto(x)}$ in (b) and take $n \to \infty$ we have

$$H_{M^2}([Lv]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, kt) * \begin{Bmatrix} H_M([ABv]_{\propto(x)}, [Lv]_{\propto(x)}, kt) \\ * H_M([STx_{2n+1}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, kt) \end{Bmatrix}$$

$$\geq \begin{Bmatrix} H_M([STv]_{\propto(x)}, [Lv]_{\propto(x)}, t) * H_M([ABx_{2n+1}]_{\propto(x)}, [Nx_{2n+1}]_{\propto(x)}, t) \\ * H_M([ABx_{2n+1}]_{\propto(x)}, [Lv]_{\propto(x)}, t) * H_M([STv]_{\propto(x)}, [Lv]_{\propto(x)}, t) \\ * H_M([STv]_{\propto(x)}, [ABx_{2n+1}]_{\propto(x)}, t) \end{Bmatrix}$$

$$H_M\big([Lv]_{\propto(x)}, [z]_{\propto(x)}, \mathrm{k}t\big) \geq 1$$

Thus we have $[z]_{\propto(x)} = [Lv]_{\propto(x)} = [ABv]_{\propto(x)}$

Since (L, AB) is weakly compatible, we have $[Lz]_{\propto(x)} = [ABz]_{\propto(x)}$ and using step 4, we have $[z]_{\propto(x)} = [Bz]_{\propto(x)}$. Therefore $[z]_{\propto(x)} = [Az]_{\propto(x)} = [Bz]_{\propto(x)} = [Sz]_{\propto(x)} = [Tz]_{\propto(x)} = [Lz]_{\propto(x)} = [Nz]_{\propto(x)}$

That is z is the common random fixed point of the six maps in this case also.

Step 10: For uniqueness, let $(w \neq z)$ be another common fixed point of A, B, S, T, L and N taking $x = [z]_{\propto(x)}, y = [w]_{\propto(x)}$ in (b) we have

$$H_{M^2}\big([Lz]_{\propto(x)}, [\mathrm{Nw}]_{\propto(x)}, kt\big) * \begin{Bmatrix} H_M\big([\mathrm{ABz}]_{\propto(x)}, [Lz]_{\propto(x)}, kt\big) \\ * H_M\big([\mathrm{STw}]_{\propto(x)}, [Nw]_{\propto(x)}, kt\big) \end{Bmatrix}$$

$$\geq \begin{Bmatrix} H_M\big([\mathrm{STz}]_{\propto(x)}, [Lz]_{\propto(x)}, t\big) * H_M\big([\mathrm{AB}w]_{\propto(x)}, [Nw]_{\propto(x)}, t\big) \\ * H_M\big([\mathrm{AB}w]_{\propto(x)}, [Lz]_{\propto(x)}, t\big) * H_M\big([\mathrm{STz}]_{\propto(x)}, [Nw]_{\propto(x)}, t\big) \\ * H_M\big([\mathrm{STz}]_{\propto(x)}, [ABw]_{\propto(x)}, t\big) \end{Bmatrix}$$

$$H_M\big([z]_{\propto(x)}, [w]_{\propto(x)}, kt\big) \geq 1$$

Thus we have $[z]_{\propto(x)} = [w]_{\propto(x)}$. This completes the proof of the theorem. If we take $B = T = Ix$ (The identity map on X) in the main theorem we have the following.

Corollary3.1: Let A, S, L and N be self-maps on a complete fuzzy metric space (X, M, *) with $t * t \geq t \; for \; all \; t \in [0, 1]$ satisfying

(a) $L(X) \subseteq S(X), N(X) \subseteq A(X)$

(b) There exists a constant $k \in (0,1)$ such that

$$H_{M^2}\left([Lx]_{\propto(x)},[\mathrm{Ny}]_{\propto(x)},kt\right)*\begin{Bmatrix}H_M\left([Ax]_{\propto(x)},[Lz]_{\propto(x)},kt\right),\\ H_M\left([\mathrm{Sy}]_{\propto(x)},[Ny]_{\propto(x)},kt\right)\end{Bmatrix}\geq\begin{Bmatrix}H_M\left([\mathrm{Sx}]_{\propto(x)},[Lx]_{\propto(x)},t\right)\\ *H_M\left([Ay]_{\propto(x)},[Ny]_{\propto(x)},t\right)\\ *H_M\left([Ay]_{\propto(x)},[Lx]_{\propto(x)},t\right)\\ *H_M\left([Sx]_{\propto(x)},[Ny]_{\propto(x)},t\right)\\ *H_M\left([Sx]_{\propto(x)},[Ay]_{\propto(x)},t\right)\end{Bmatrix}$$

For all $x,y\in X\ and\ t>0$.

(c)Either A or L is continuous.

(d) The pair (L, A) is compatible and (N, S) is weakly compatible. Then A, S if we take $A=S, L=N\ and\ B=T=Lx$ is the main theorem, we have the following:

Corollary 3.2: Let (X ,M ,∗) be a compatible fuzzy metric space with$t\ *t\ \geq t\ for\ all\ t\in$ [0,1]and let A and L be compatible maps on X such that $L\ (X)\ \subset\ A\ (X)$, if A is continuous and there exists a constant k ∈ (0, 1) such that

$$H_{M^2}\left([Lx]_{\propto(x)},[Ly]_{\propto(y)},kt\right)*\begin{Bmatrix}H_M\ \left([Ax]_{\propto(x)},[Lx]_{\propto(x)},kt\right),\\ H_M\ \left([Ay]_{\propto(y)},[Ly]_{\propto(y)},kt\right)\end{Bmatrix}\geq\begin{Bmatrix}H_M\left([Ax]_{\propto(x)},[Lx]_{\propto(x)},t\right)\\ *H_M\left([Ay]_{\propto(y)},[Ly]_{\propto(y)},t\right)\\ *H_M\left([Ay]_{\propto(y)},[Ly]_{\propto(y)},t\right)\\ *H_M\left([Ax]_{\propto(x)},[Ly]_{\propto(y)},t\right)\\ *H_M\ \left([Ax]_{\propto(x)},[Ay]_{\propto(y)},t\right)\end{Bmatrix}$$

For all $x,y\ \in\ X\ and\ t>0$ then A and L have a unique fixed point.

4 Conclusions

In the present work we introduced a new concept of fuzzy mappings in the fuzzy metric space on compact sets, which is a partial generalization of fuzzy contractive mappings in the sense of GeorgeandVeeramani. Also, we derived the existence of fixed point theorem for weakly compatible maps in fuzzy metric space. Moreover, we reduced our result from fuzzy mappings in fuzzy metric spaces. Finally, we showed some relation of multivalued mappings and fuzzy mappings, which can be utilized to derive fixed point for multivalued mappings.

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