Fredholm Equation of First Kind in Borehole Gamma Ray Measurements - Cubic Spline Weighing

S. S. Raghuwanshi (Corresponding author), HPG (Retd), AMD, Hyderabad
Satyanand Panda,Department of Mathematics , National Institute of Technology, Calicut, Kerala
Sameer Sharma, Department of Mathematics, DAV College , Jalandhar, Punjab, India
raghu2257@yahoo.com

Abstract

The problem in the processing of gamma-ray borehole data is to find out the radioactive zones down the bore depths which vary from 50m to 1000m in various geological formations and subsequent evaluations. The radiation field in the boreholes in cylindrical geometries follows the Fredholm Integral Equation of first kind. And in this work it has been dealt numerically using cubic spline method to compute the second derivatives of the measured gamma ray intensity at various depths in the borehole. This is achieved by expressing this integral equation as a sum of products of the measured gamma ray intensities and some unknown coefficients which can be computed using Lagrange interpolation method. A finite window is selected for this summation which can be varied. In this paper the three windows have been selected. The logging probe is calibrated in primary borehole standard and then the real data for radiation field in the boreholes is acquired. Three such windows were selected and results for 3, 5, 7 point windows have been presented in this paper.

Keywords: Fredholm Equation, Borehole Gamma Ray Measurements, Cubic Spline Weighing, logging probe

1. Introduction

For Uranium and other explorations borehole gamma ray measurements are carried out all over the world and its principal goal is to evaluate the depth-wise concentration distribution of uranium ore grade (%eU_3O_8). This method of borehole logging provides quantitatively the radioactive material present in the various horizons of the rock material down the borehole. For this purpose the logging probe which is made up of either gas filled detector (GM Tubes) or scintillator detector (NaI(Tl), BGO, LaBr etc) for the total or spectral loggings ha to be calibrated. This can be done either by directly calibrating in primary boreholes (Raghuwanshi and Sarma, 2009) or by secondary standards. The grades of primary as well as secondary and real borehole have been confirmed both by chemical methods and physical methods (Core analysis, Beta-gamma analysis and gamma ray spectrometry).

2. Methodology of Measurements

The gamma ray measurements are done by lowering the probe in the bore holes and gathering counts in the counting system by staying for sampling time Δt at equal interval Δz along the borehole depth z. The borehole measurements at different depths give rise a set of varying values of counts per sampling time from which we compute the grade of uranium at different depths all along the profile of such measurements.

2.1 Fredholm Integral Equation of first kind

Let us assume that the radioactive matter of ore grade q is uniformly present in the borehole down the depth.

Since there is no variation in the ore concentration q at different sampling depths, then the measured radioactivity intensity I(z) at depth z will remain same at all depths. In that case therefore the measured intensity of radiation field I(z) will simply be equal to product kq , where k is calibration constant of the measuring logging instrument.

But in reality the ore grade and therefore the radioactivity varies with borehole depth. In this real case we need therefore to express the measured radioactivity intensity I(z) by the following definite integral expression (having infinite limits) :

$$I(z) = k \int \Phi(z- z')q(z')dz' \qquad - (1)$$

In this integral the function $\Phi(z- z')$ defines the shape function of the gamma ray field intensity for a unit ore grade in the borehole (frequently referred to as the geological impulse response) and is determined in practice in a model borehole containing an infinite thick ore zone adjacent to an infinitely thick barren zone.

The equation (1) is called Fredholm Integral Equation of first kind. Here the term $q(z')$ is the ore grade at any depth z' and is unknown while I(z) and $\Phi(z- z')$ are known functions in this integral equation. I(z) is measured quantity while the shape function is empirically determined in known bore holes such as primary bore holes (Raghuwanshi and Sarma,2009). And $q(z')$ will be determined by the inversion method after the proper calibration constant is known and shape parameter too determined, it is here we needed windowing and evaluating the coefficients usinf cubic spline method.

2.2 Methodology

There is a vast literature for the techniques for solving Fredholm integral equation of first kind. However, for solving this specific integral equation (1) various attempts have been made and efforts have been done first to evaluate the kernel Φ (Czubek, 1978; Killeen and Conway, 1979a; Killeen and Conway, 1979b). The evaluation of this shape function plays a pivotal role in ore evaluation and therefore its solution becomes a major step in this effort. This will finally tell us the direction of true interpretation of the borehole gamma ray logging data.

Various guesses were attempted for the shape parameters (Czubek,1966) but using the exponential expression such as $\Phi(z) = 0.5\alpha\exp(-\alpha|z|)$, (where α is some constant) Killeen and Conway(1979) found simple to solve the equation by deconvolution method. They expressed the following formula for ore grade at depth Zi as follows:

$$q(Zi) \quad = \quad I(Z) – [I(Zi+1) + I(Zi-1)-2\, I(Zi)](1/\alpha)^2$$

In this work the cubic spline method is employed to compute the second derivatives of the measured gamma ray intensity I(z) and using the expansion method of solving the Fredholm Integral Equation of first kind the ore grade at depth z can be expressed as

$$q(Zi) \quad = \quad \Sigma W_p I_{i+p}(Zi)$$

where W_p are weights to be determined which is summed from –p to +p. Weights have been computed and are given in Table. 1.

3. Results and Conclusion

For demonstrating the efficacy of the procedure gamma ray borehole logging data of primary standard (Fig.1) and also real field borehole data from Chitrial outlier area of Devarakonda, Nalgonda District, AP, India has been treated with our method and the results are shown in Fig.2 and the efficacy of this technique delineating the discontinuities in the grade is noticed, which would otherwise be appear uniform. The anomalies after getting weighted become more clear and conspicuous and are therefore useful for interpretation by the field exploration scientist.

References

Conaway, J.G., 1980a. Uranium concentrations and the system response function in gamma-ray logging. Geol. Surv, Can., Papers, 80-1a, pp. 77-87.

Conaway, J.G., 1980b. Deconvolution of gamma-ray logs in the case of dipping radioactive zones. Geophysics, v(45). In press.

Czubek, J.A. Lenda, A: G-Function in gamma-ray transport problems, Krakow, August 1978. Report No. 1022/PL

Raghuwanshi, S S , Sarma,C V R, 2009, National Conference on SSNTD, GDNU, Amritsar.

Table 1.

Weighing factors computed by the Cubic Spline method

3 Point Window	5 Point Window	7 Point Window
-1.091273	0.135889139	-0.020924463
3.1825596	-1.634836329	0.261435922
-1.091273	3.99789439	-1.948703295
	-1.634836329	4.416383676
	0.135889139	-1.948703295
		0.261435922
		-0.020924463

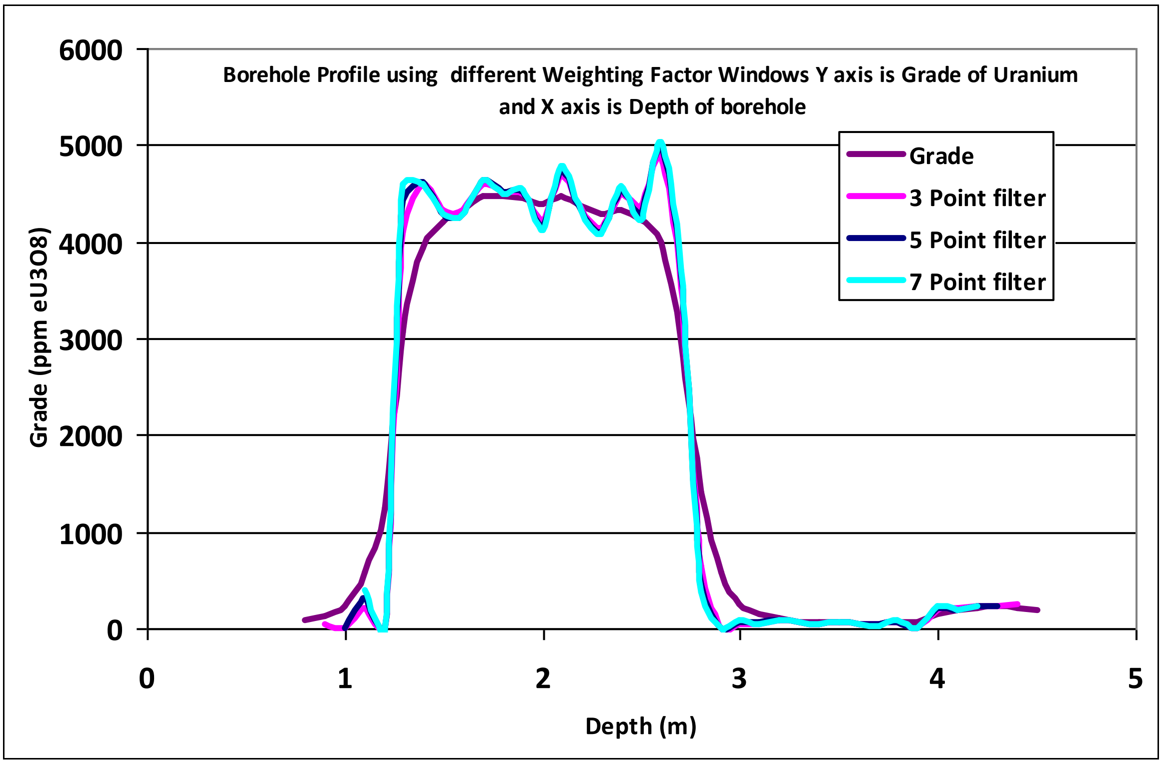

Fig 1. Profile of Standard borehole (HQ Hyderabad) along with 3, 5, 7 point filters

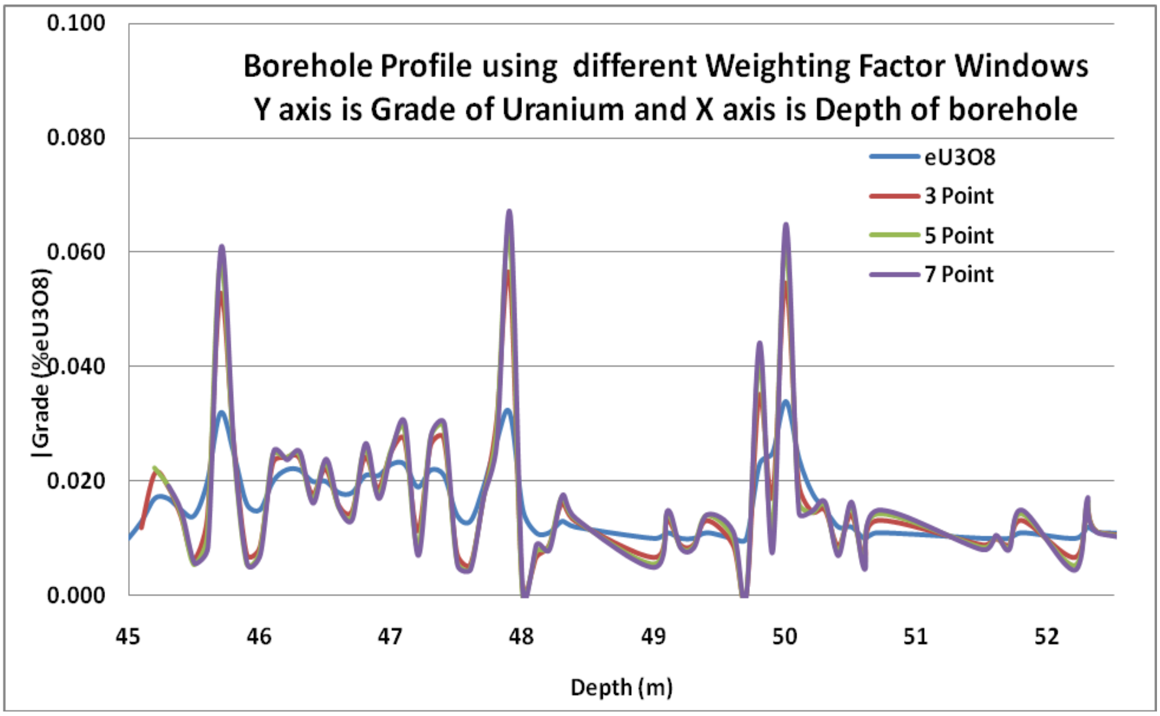

Fig 2: Profile of Field borehole (Chitrial area, Hyderabad) along with 3, 5, 7 point filters.