Mathematical Theory and Modeling
ISSN 2224-5804 (Paper) ISSN 2225-0522 (Online)
Vol.5, No.6, 2015

Modified Homotopy Perturbation Method (MHPM) for Dynamics Gas Equation

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Abstract
In this paper, Modified Homotopy Perturbation Method (MHPM) is applied to solve the nonlinear homogeneous dynamics gas equation and analytic solution is found. The proposed method proves to be a powerful scheme for solving the strictly nonlinear partial differential equations as it reduces the computational work and the solution converges rapidly to the exact solution.

1. Introduction:

The phenomenon arising from the real world is in the form of nonlinear equations which has a great interest to the researchers. For the last two decades, a remarkable work has been done to handle such nonlinear equations whether these are ordinary or partial. Many methods are developed to solve these equations such as Homotopy Perturbation Method (HPM) [1], Adomian Decomposition Method (ADM) [2], Variational Iteration Method (VIM) [3], Differential Transform Method (DTM) [4], Homotopy Analysis Method (HAM) [5] and Reduced Differential Transform Method (RDTM) [6].

Gas dynamics are the conversation laws which are in the form of mathematical expressions existing in the applications of different branches of engineering for example the conversation of momentum, mass and energy etc. Different kinds of dynamics gas models were solved by different researchers such as Jawad at al. [7], Evans and Bulut [2], Jafari et al. [8] with help of different methods and numerical techniques. From the last two or three years different mathematicians such as Biazar, Eslami and Kumar have been used method of differential transform (DTM) and Homotopy Perturbation Transform Method (HPTM) for solving the gas dynamics equation and finding their exact solutions [9,10].

In this present work, we use Modified Homotopy Perturbation Method (MHPM) to solve the non linear homogeneous dynamics gas equation and find its exact solution. The main advantage of this method is that it requires very simple mathematical calculations that converge rapidly to the exact solution. The nonlinear homogeneous dynamics gas equation is

$$\frac{\partial u}{\partial t}+\frac{1}{2}(u^2)_x-u(u-1)=0, \tag{1}$$

with the initial condition, $u(x,0)=f(x).$ (2)

Different researchers solve this equation by different methods see [2-3, 6-7].

2. Basic Idea of Modified Homotopy Perturbation Method (MHPM):

To understand the basic ideas of MHPM, we rewrite dynamics gas equation (1) in the following form

$$Lu+Nu=0, \tag{3}$$

with the initial condition

$$u(x,0)=f(x).$$

The linear and nonlinear operators are given by

$$Lu=\frac{\partial u}{\partial t}, \qquad Nu=\frac{1}{2}\frac{\partial u^2}{\partial x}-u(1-u). \tag{4}$$

The variables of $u_0(x, t)$ can be separated as

$$u_0(x,t) = u(x,0)c_1(t) + \frac{\partial u(x,0)}{\partial x} c_2(t), \tag{5}$$

and the initial condition is given by

$$u(x,0)c_1(0) + \frac{\partial u(x,0)}{\partial x} c_2(0) = f(x). \tag{6}$$

We obtain $c_1(t)$ and $c_2(t)$ by Eq. (5).

According to the Homotopy Perturbation Technique (HPM), a homotopy can be constructed as follows

$$H(v,p) = (1-p)(Lv - Lu_0) + p(Lv + Nv), \qquad p \in [0,1], \tag{7}$$

where $p \in [0, 1]$ is an embedding parameter and $u_0(x, t)$ is an initial approximation of Eq. (1) Now we have,

$$H(v,0) = Lv - Lu_0 = 0, \qquad H(v,1) = Lv + Nv = 0.$$

The deformation of p from zero to unity is just that of v from u_0 to u, and $L(v - u_0)$ and $Lv + Nv$ are called homotopy. According to the HPM, we first use the embedding parameter p as a "small parameter", and assume that the solution to Eq. (7) may be expressed as a series in p

$$v = v_0 + pv_1 + p^2 v_2 + p^3 v_3 + \ldots \tag{8}$$

Setting $p = 1$, the approximate solution to Eq. (7) is then

$$u = \lim_{p \to 1} v = v_0 + v_1 + v_2 + v_3 + \ldots$$

Substituting (8) into (7) and equating the terms with the same power of p, we get

$$\begin{aligned} &p^0, \quad Lv_0 - Lu_0 = 0 \\ &p^1, \quad Lv_1 + Lu_0 + Nv_0 = 0 \\ &p^{k+1}, \quad Lv_{k+1} + Nv_k = 0, \qquad k \geq 1. \end{aligned} \tag{9}$$

Combining the initial approximation u_0 with the above equations, we identify v_n for $n = 1, 2, \cdots$;

and obtain the n-th approximation of the exact solution as $u_n = \sum_{k=0}^{n} v_k$.

If there exists some $v_n = 0,\ n \geq 1,$ then the exact solution to the equation can be denoted as $u(x,t) = \sum_{k=0}^{n-1} v_k$. For simplicity, in this paper we assume that $v_1(x,t) = 0,$ namely, the exact solution is denoted as $u(x,t) = v_0(x,t)$. since $u(x,t)$ satisfies the initial condition, we get

$$c_1(0) = 1, \qquad c_2(0) = 0.$$

Thus we have

$$Lv_1 = -Lu_0 - Nv_0 = -L\left[u(x,0)c_1(t) + \frac{\partial u(x,0)}{\partial x} c_2(t)\right] - N\left[u(x,0)c_1(t) + \frac{\partial u(x,0)}{\partial x} c_2(t)\right] \equiv 0.$$

From the above formula, we get the proper $c_1(t)$ and $c_2(t)$. Furthermore, the appropriate initial approximation $u_0(x, t)$ may be obtained. The detailed process will be displayed in the next example.

3. Method Implementation:

In this section, we consider the following homogeneous dynamics gas equation

$$\frac{\partial u}{\partial t} + \frac{1}{2}(u^2)_x - u(1-u) = 0, \tag{10}$$

with the initial condition

$$u(x,0) = e^{-x}.$$

Let us choose

$$u_0(x,t)=e^{-x}c_1(t)-e^{-x}c_2(t). \tag{11}$$

Then

$$\frac{\partial u_1}{\partial t}=-\frac{\partial u_0}{\partial t}-\frac{1}{2}\frac{\partial}{\partial x}(u_0^2)+u_0(1-u_0)$$

$$=-e^{-x}c_1'(t)+e^{-x}c_2'(t)-\frac{1}{2}\left[e^{-x}c_1(t)-e^{-x}c_2(t)\right]_x^2+\left[e^{-x}c_1(t)-e^{-x}c_2(t)\right]\left[1-e^{-x}c_1(t)+e^{-x}c_2(t)\right]$$

$$=-e^{-x}c_1'(t)+e^{-x}c_2'(t)-\frac{1}{2}(2)\left[e^{-x}c_1(t)-e^{-x}c_2(t)\right]\left[-e^{-x}c_1(t)+e^{-x}c_2(t)\right]+\left[e^{-x}c_1(t)-e^{-x}c_2(t)\right]$$

$$=-\left[e^{-x}c_1(t)-e^{-x}c_2(t)\right]^2$$

$$=-e^{-x}c_1'(t)+e^{-x}c_2'(t)+\left[e^{-x}c_1(t)-e^{-x}c_2(t)\right]^2-\left[e^{-x}c_1(t)-e^{-x}c_2(t)\right]^2+e^{-x}c_1(t)-e^{-x}c_2(t)\equiv 0$$

$$=-e^{-x}\left[c_1'(t)-c_1(t)\right]+e^{-x}\left[c_2'(t)-c_2(t)\right]\equiv 0$$

From above equation, we have a system of differential equations as

$$c_1'(t)-c_1(t)=0. \qquad c_2(t)=0, \qquad c_1(0)=1, \qquad c_2(0)=0$$

Solving this system we get

$$c_1(t)=e^t.$$

So solution of the Eq. (10) is

$$u(x,t)=e^{-x}e^t=e^{t-x}.$$

This is exact solution of the given nonlinear homogeneous equation as given by [2, 3, 6, 8].

4. Conclusion:

In this paper, we have applied Modified Homotopy Perturbation Method (MHPM) for getting the solution of the nonlinear homogeneous dynamics gas equation. The proposed method proves to be a powerful technique for solving the nonlinear partial differential equations and gives efficient and reliable solutions. The solution obtained from MHPM converges rapidly to the exact solution.

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